Erratum: Global Deadline-Monotonic Scheduling of Arbitrary-Deadline Sporadic Task Systems

Jian-Jia Chen
Department of Informatics, TU Dortmund University, Germany

Abstract. This paper presents an error in the schedulability test for global deadline-monotonic scheduling of arbitrary-deadline sporadic task systems in identical multiprocessor systems proposed by Baruah and Fisher in OPODIS 2007. This erratum provides a simple fix. Fortunately, the speedup bound $2 + \sqrt{3}$ claimed in their paper remains valid with this simple fix.

In the sporadic task model, a task $\tau_i$ is characterized by its relative deadline $D_i$, its minimum inter-arrival time (period) $T_i$, and its worst-case execution time $C_i$. An arbitrary-deadline sporadic task set does not assume any relation between the relative deadlines and the periods of the tasks. Baruah and Fisher [1] considered an arbitrary-deadline sporadic task set executed on $m \geq 2$ identical processors based on global deadline-monotonic (DM) scheduling, in which $D_1 \leq D_2 \leq \ldots \leq D_n$. They proposed a schedulability test that is the state of the art of this problem with respect to the resource augmentation (speedup) bound. We here recall their notation as follows:

- density $\delta_i$ of task $\tau_i$: $C_i / \min(D_i,T_i)$;
- maximum density $\delta_{\text{max}}(k)$ among the first $k$ tasks: $\max_{i=1}^{k} (\delta_i)$;
- demand bound function of task $\tau_i$: $DBF(\tau_i,t) = \max \left( 0, \left( \left\lfloor \frac{t-D_i}{T_i} \right\rfloor + 1 \right) C_i \right)$;
- load $\text{LOAD}(k)$ of the first $k$ tasks: $\text{LOAD}(k) = \max_{t>0} \sum_{i=1}^{k} DBF(\tau_i,t)$.

The schedulability test of task $\tau_k$ under global DM by Baruah and Fisher [1] is as follows:

\[
\left( 1 + \frac{D_k}{\Delta} \right) \text{LOAD}(k) + \left( \lceil \mu_k \rceil - 1 \right) \delta_{\text{max}}(k) \leq \mu_k \tag{1}
\]

since

\[
\iff D_k \leq \Delta \Rightarrow 2\text{LOAD}(k) + \left( \lceil \mu_k \rceil - 1 \right) \delta_{\text{max}}(k) \leq \mu_k \tag{2}
\]

where $\mu_k$ is defined as $m - (m - 1)\delta_k$.

The schedulability test in Eqs. (1) and (2) is based on an incorrect Lemma 3 from the original analysis [1], stated as follows: “The total remaining execution requirement of all the carry-in jobs of each task $\tau_i$ (that has carry-in jobs at time-instant $t_0$) is $< \Delta \times \delta_{\text{max}}(k)$.”

There was one unsound step in the second part of the equation set (5) in the original proof [1]. They stated that the condition $m\phi_i - (m - 1)y_i < (m - (m - 1)\delta_k)\phi_i$ implies

* This paper has been supported by DFG, as part of the Collaborative Research Center SFB876 (http://sfb876.tu-dortmund.de/).
that $y_i > \phi_i \delta_{\text{max}}(k)$. The fact is that $m \phi_i - (m - 1) y_i < (m - (m - 1) \delta_k) \phi_i$ only implies $y_i > \phi_i \delta_k$. The original implication holds when $\delta_k \geq \delta_i$, i.e., $\delta_k$ is equal to $\delta_{\text{max}}(k)$. Without such an implication, the remaining execution requirement of task $\tau_i$ can only be safely stated as $< \Delta \delta_i + \phi_i (\delta_i - \delta_k)$ in their proof.

However, this correct inequality introduces an unknown variable $\phi_i$. One simple solution to fix their analysis is to define $\mu_k$ as $m - (m - 1) \delta_{\text{max}}(k)$. With this solution, all the analysis steps are valid and their Lemma 3 is correct. Therefore, the schedulability test in Eq. (1) and Eq. (2) both remain valid if $\mu_k$ is defined as $m - (m - 1) \delta_{\text{max}}(k)$.

Based on the above discussion, the (sufficient) schedulability test in Corollary 1 by Baruah and Fisher [1] should be restated as

$$\text{LOAD}(k) \leq \frac{1}{2} (m - (m - 1) \delta_{\text{max}}(k)) (1 - \delta_{\text{max}}(k)).$$  \hspace{1cm} (3)

Fortunately, the above schedulability test in Eq. (3) still leads to the speedup bound $2 + \sqrt{3}$, as the procedure in the proof of Lemma 5 in the original analysis remains valid by using only the condition $\delta_{\text{max}}(k) \leq x$.

References