technische universität dortmund | fakultät für informatik | computer science 12

Prof. Jian-Jia Chen

Real-Time Systems (SS 2014)

Exercise 2: Schedulability Analysis

Discussion Date: 14/21, May 2014

Exercise 2.1

Please sketch the necessary and sufficient schedulability (namely exact) tests for RM and EDF scheduling policies for task sets with constrained deadlines and implicit deadlines. Please also shortly explain how the tests work with pseudo-codes.

Exercise 2.2

What are the differences between the the efficient utilization-based schedulability tests and the time-demand schedulability tests for RM? Please use one example to illustrate their differences.

Exercise 2.3

Suppose that we are given the following 3 sporadic real-time tasks with constrained deadlines.

	τ_1	τ_2	τ_3
C_i	2	2	4
T_i	6	8	12
D_i	5	4	12

(a) Is the deadline-monotonic (DM) schedule feasible?

(b) Is the EDF schedule feasible?

(c) Suppose that all the tasks arrive at time 0. Please construct the schedules of DM and EDF.

(d) Suppose that the system has only two priority levels by using fixed-priority scheduling. How do you assign the priority levels of these three tasks? In general, can you think of a general method to handle such a problem when the available priority levels are less than the number of the tasks when considering task sets with constrained deadline? What is the corresponding schedulability analysis?

Exercise 2.4

Provide pseudo code and analyze the complexity and resource augmentation factor for the schedulability test of RM so that

- the algorithm runs in polynomial time with respect to $\frac{1}{\delta}$ and the number of tasks, and
- the resource augmentation factor is $\frac{1}{1-\delta}$,

where $0 < \delta < 1$ is given as an input.

Exercise 2.5

Given a set $\mathcal{T}$ of n independent, preemptable, and periodic tasks with implicit deadlines, they can be partitioned into $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_k$ task sets, in which $\bigcup_{j=1}^{k} \mathcal{T}_j$ is $\mathcal{T}$. Moreover, for $j = 1, 2, \ldots, k$, the periods of the tasks in each task set $\mathcal{T}_j$ are *simply periodic* or *harmonic*. That is, for any $\tau_i, \tau_\ell \in \mathcal{T}_j$, $\frac{T_i}{T_\ell}$ is a positive integer if $T_i \geq T_\ell$.

Prove that if

$$\sum_{i=1}^{n} U_i \leq k(2^{\frac{1}{k}} - 1),$$

rate-monotonic scheduling is a feasible schedule.

Challenge 2.6

Given n independent, preemptable, and periodic tasks with implicit deadlines, there are two types of executions for each task τ_i. It is known that $C_{i,2} \leq C_{i,1} \leq 2C_{i,2}$, in which $C_{i,1}$ is the WCET of the first version, and $C_{i,2}$ is the WCET of the second version.

Suppose that the tasks are ordered in a rate-monotonic order, in which $T_i \leq T_{i+1}$ $i = 1, 2, \ldots, n-1$ and $2T_1 \geq T_n$. For any two consecutive jobs of task τ_i, at most one of them is executed by using the first-version (i.e., $C_{i,1}$).

The task set is schedulable under rate monotonic scheduling if

$$\sum_{i=1}^{n} \frac{C_{i,2}}{T_i} \leq n(1.5^{\frac{1}{n}} - 1).$$

Prove the above statement, and also generalize the above bound for another condition $C_{i,2} \leq C_{i,1} \leq \alpha C_{i,2}$ where $\alpha \geq 1$.

(**Hint:** The critical instant theorem for this case is as follows: If the worst-case response time of task τ_k is no more than T_k, then, the response time of a job of task τ_k, arriving at time t, by releasing all the higher-priority tasks τ_is with execution time $C_{i,1}$ at time t and the subsequent job with execution time $C_{i,2}$ at time $t + T_i$ as early as possible by respecting the period, is the worst-case response time of task τ_k.)