Real-Time Calculus

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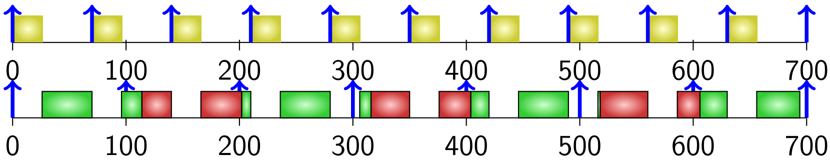
Arbitrary Deadlines

The worst-case response time of τ_i by only considering the first job of τ_i at the critical instant is too optimistic when the relative deadline of τ_i is larger than the period.



Consider two tasks:

- τ_1 has period 70 and execution time 26 and τ_2 is with period 100 and execution time 62.
- τ_2's seven jobs have the following response times, respectively: 114, 102, 116, 104, 118, 106, 94.
- Note that the first job's response time is not the longest.

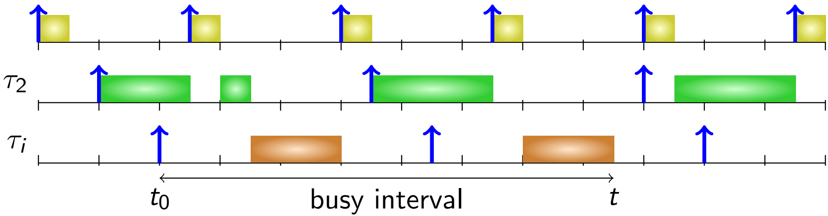
Busy Intervals

Definition

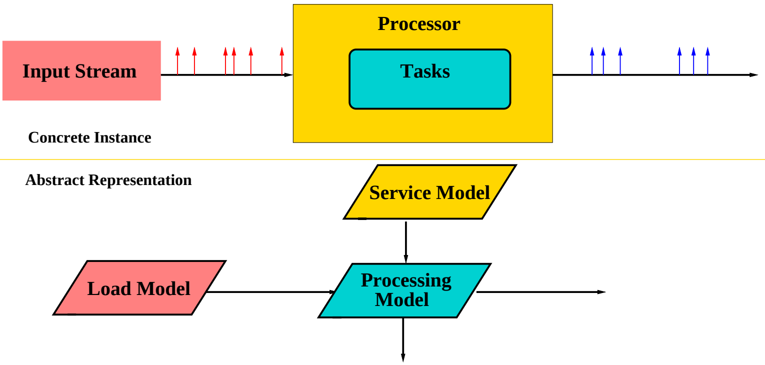
A τ_i-level busy interval $(t_0, t]$ of task τ_i begins at an instant t_0 when

1. all jobs in τ_i released before t have completed, and
2. a job of τ_i releases.

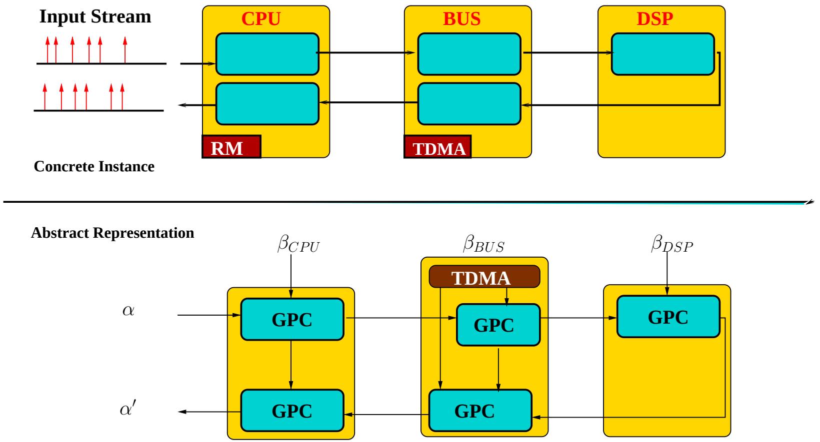
The interval ends at the first instant t after t0 when all jobs in τ_i released since t_0 are complete.



Abstract Models for Real-Time Calculus



Abstract Models for Module Performance Analysis



Overview

System View	Module Performance Analysis (MPA)
Math. View	Real-Time Calculus (RTC)
	Min-Plus Calculus, Max-Plus Calculus

Backgrounds

- Real-Time Calculus can be regarded as a worst-case/best-case variant of classical queuing theory. It is a formal method for the analysis of distributed real-time embedded systems.
- Related Work:
  - Min-Plus Algebra: F. Baccelli, G. Cohen, G. J. Olster, and J. P. Quadrat, Synchronization and Linearity —An Algebra for Discrete Event Systems, Wiley, New York, 1992.
  - Network Calculus: J.-Y. Le Boudec and P. Thiran, Network Calculus -A Theory of Deterministic Queuing Systems for the Internet, Lecture Notes in Computer Science, vol. 2050, Springer Verlag, 2001.

Plus-Times and Min-Plus Algebras

- Algebraic structure
  - a set of (finite or infinite) elements S
  - one or more operators defined on the elements of this set
- Plus-Times Algebra: Two operators $+$ and $\times$, denoted by $(S, +, \times)$
- Min-Plus Algebra
  - Two operators $\oplus$ (min) and $\otimes$ (plus), denoted by $(S \cup \{+\infty\}, \inf, +)$
  - Infimum:
    - The infimum of a subset of some set is the greatest element, not necessarily in the subset, that is less than or equal to all other elements of the subset.
    - For example, $\inf\{[a, b]\} = a$, $\inf\{(a, b]\} = a$, where $\min\{[a, b]\} = a$, $\min\{(a, b]\} =$ undefined.
  - Supremum:
    - The supremum of a subset of some set is the smallest element, not necessarily in the subset, that is more than or equal to all other elements of the subset.
    - For example, $\sup\{[a, b]\} = b$, $\sup\{[a, b)\} = b$, where $\max\{[a, b]\} = b$, $\max\{[a, b)\} =$ undefined.

Definition of Arrival Curves and Service Curves

- For a specific trace:
  - Data streams: $R(t)$ = number of events in $[0, t)$
  - Resource stream: $C(t)$ = available resource in $[0, t)$
- For the worst cases and the best cases in any interval with length Δ:
  - Arrival Curve $[\alpha^l, \alpha^u]$:

$$\alpha^l(\Delta) = \inf_{\lambda \geq 0, \forall R} \{R(\Delta + \lambda) - R(\lambda)\}$$

$$\alpha^u(\Delta) = \sup_{\lambda \geq 0, \forall R} \{R(\Delta + \lambda) - R(\lambda)\}$$

  - Service Curve $[\beta^l, \beta^u]$:

$$\beta^l(\Delta) = \inf_{\lambda \geq 0, \forall C} \{C(\Delta + \lambda) - C(\lambda)\}$$

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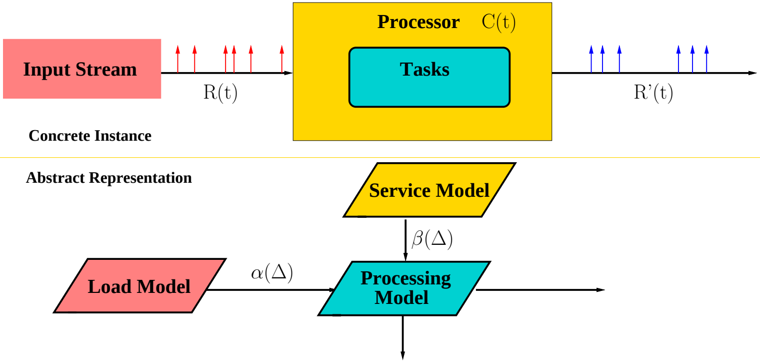
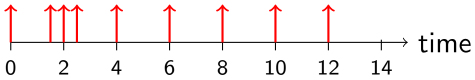
Abstract Models for Real-Time Calculus



Arrival Curve: An Example

Use a sliding window to get the upper bound of the number of events in a specified interval length.



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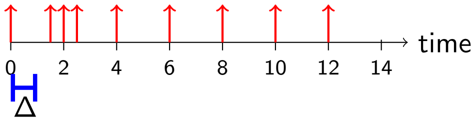

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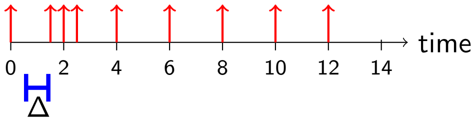

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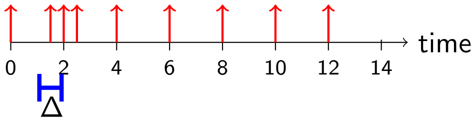

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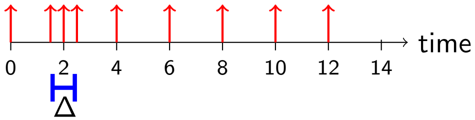

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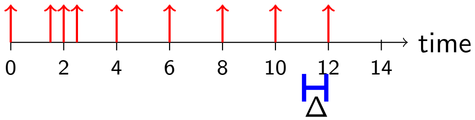

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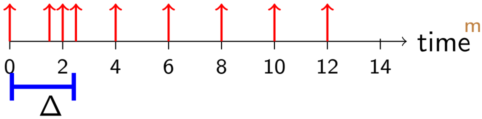

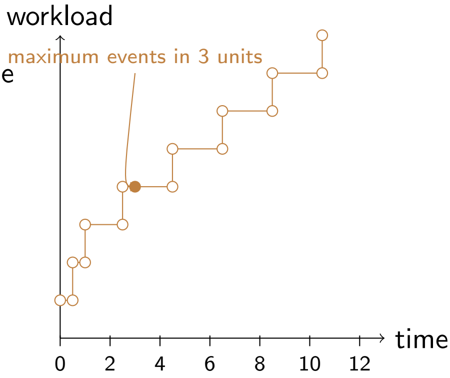

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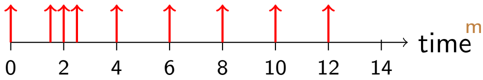

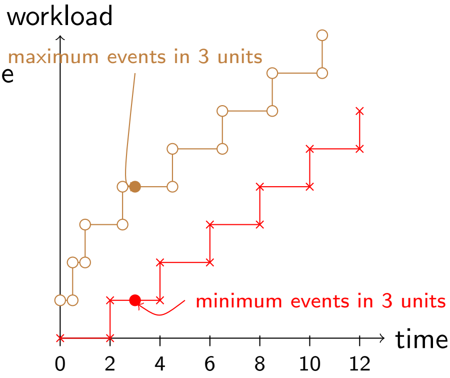

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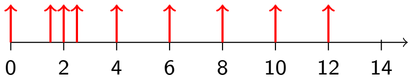
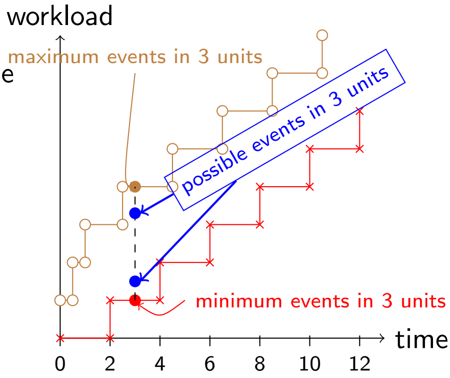
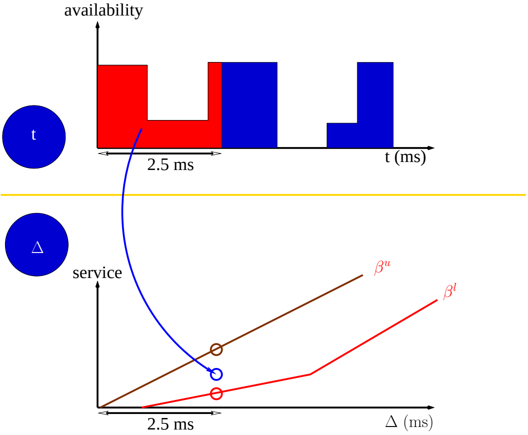




Service Curve: An Example

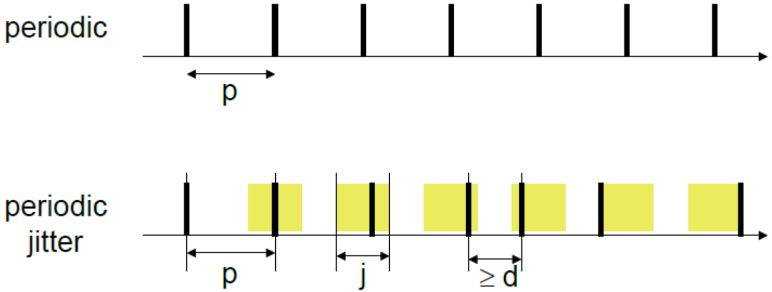
Resource Availability

Service Curves
$\beta = [\beta^l, \beta^u]$



Example 1: Periodic with Jitter

A common event pattern that is used in literature can be specified by the parameter triple (p, j, d), where p denotes the period, j the jitter, and d the minimum inter-arrival distance of events in the modeled stream.



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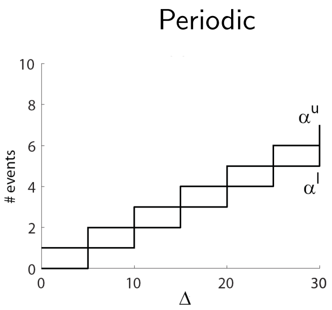
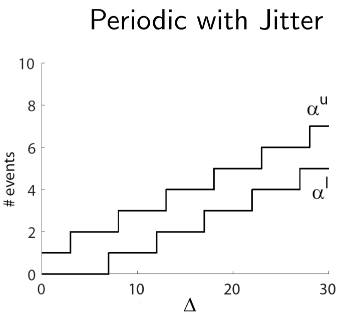




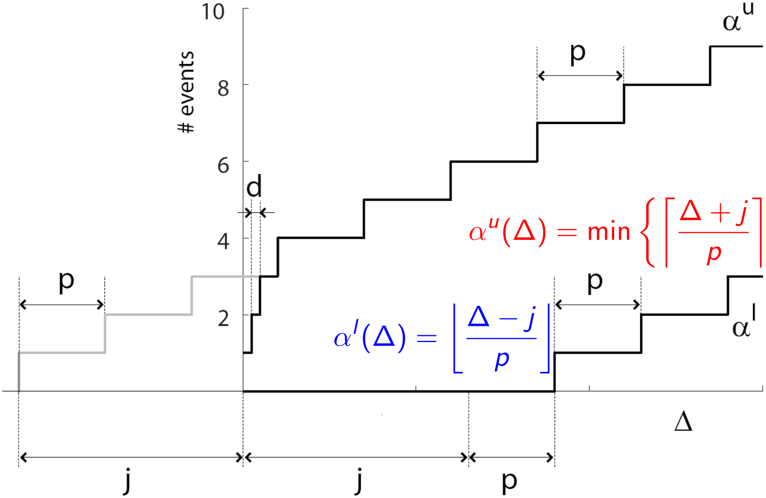
$$\alpha^u(\Delta) = \left\lceil \frac{\Delta}{p} \right\rceil$$

$$\alpha^l(\Delta) = \left\lfloor \frac{\Delta}{p} \right\rfloor$$

$$\alpha^u(\Delta) = \left\lceil \frac{\Delta + j}{p} \right\rceil$$

$$\alpha^l(\Delta) = \left\lfloor \frac{\Delta - j}{p} \right\rfloor$$

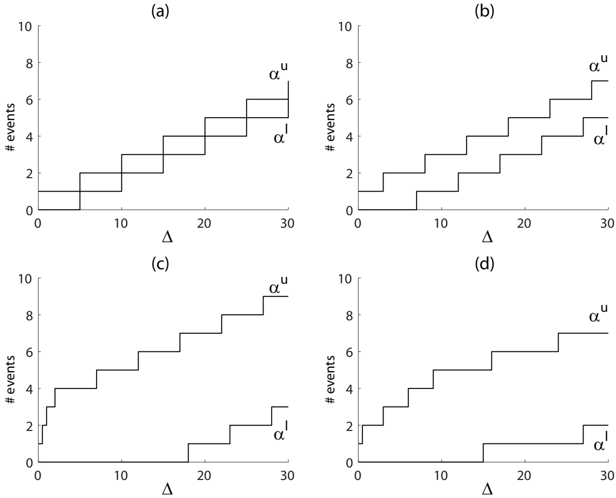
Example 1: Periodic with Jitter



$$\alpha^u(\Delta) = \min\left\{ \left\lceil \frac{\Delta + j}{p} \right\rceil, \left\lceil \frac{\Delta}{d} \right\rceil \right\}$$

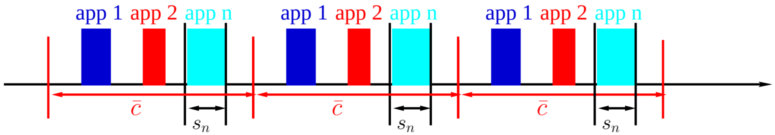
$$\alpha^l(\Delta) = \left\lfloor \frac{\Delta - j}{p} \right\rfloor$$

More Examples on Arrival Curves



Example 2: TDMA Resource

- Consider a real-time system consisting of n applications that are executed on a resource with bandwidth B that controls resource access using a TDMA (Time Division Multiple Access) policy.
- Analogously, we could consider a distributed system with n communicating nodes, that communicate via a shared bus with bandwidth B, with a bus arbitrator that implements a TDMA policy.
- TDMA policy: In every TDMA cycle of length $\bar{c}$, one single resource slot of length s_i is assigned to application i.



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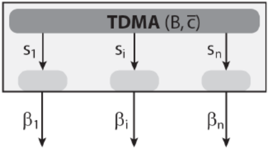
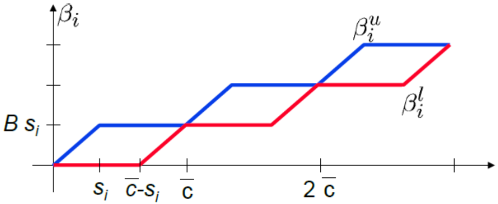
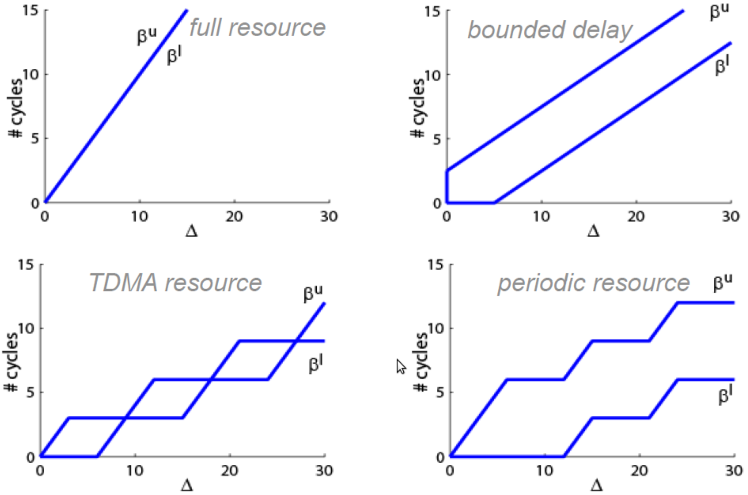




$$\beta^u(\Delta) = B \min\left\{ \left\lceil \frac{\Delta}{\bar{c}} \right\rceil s_i, \Delta - \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor (\bar{c} - s_i) \right\}$$

$$\beta^l(\Delta) = B \max\left\{ \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor s_i, \Delta - \left\lceil \frac{\Delta}{\bar{c}} \right\rceil (\bar{c} - s_i) \right\}$$

More Examples on Service Curves



On-Site Exercise

Consider a stream of events. There are at most three events and at least 2 events arriving within a time interval $[k \cdot p, (k+1) \cdot p)$ for $k = 0, 1, 2, \ldots$. These 2 or 3 events within a period p are defined as follows:

- At time $k \cdot p$, there is one event.
- At time $k \cdot p + 0.3p$, there is another event.
- Between time $k \cdot p + 0.4p$ and $k \cdot p + 0.5p$, there may be one event.

How do the upper and lower arrival curves look like?

Convolutions

- Plus-times system theory: signals f, impulse response g, convolution in time domain:

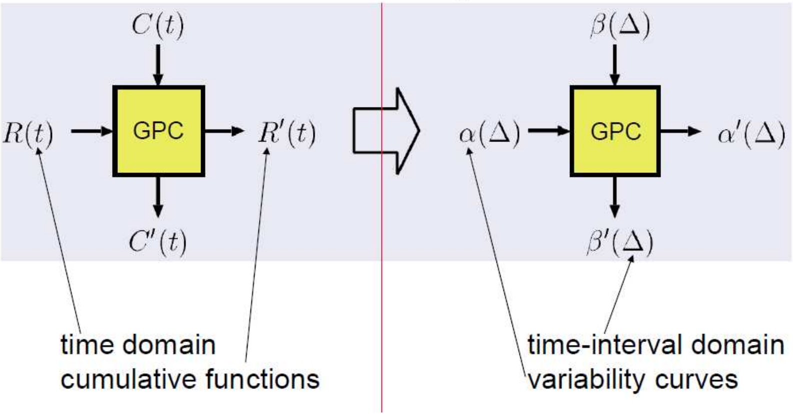
$$h(t) = (f \times g)(t) = \int_0^t f(t-s)g(s)ds,$$

where f, g can be thought as signals and impulse response, respectively.

- Min-Plus system theory: streams R, variability curves g, convolution in time-interval domain:

$$R'(t) \geq (R \otimes g)(t) = \inf_{0 \leq \lambda \leq t} \{R(t-\lambda) + g(\lambda)\}.$$

Abstraction



Convolution and De-convolution

- $f \otimes g$ is called *min-plus convolution*

$$(f \otimes g)(t) = \inf_{0 \leq \lambda \leq t} \{f(t-\lambda) + g(\lambda)\}$$

- $f \oslash g$ is called *min-plus de-convolution*

$$(f \oslash g)(t) = \sup_{0 \leq \lambda} \{f(t+\lambda) - g(\lambda)\}$$

- $f \bar{\otimes} g$ is called *max-plus convolution*

$$(f \bar{\otimes} g)(t) = \sup_{0 \leq \lambda \leq t} \{f(t-\lambda) + g(\lambda)\}$$

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$$(f \bar{\oslash} g)(t) = \inf_{0 \leq \lambda} \{f(t+\lambda) - g(\lambda)\}$$

Arrival and Service Curves Revisit

$$\alpha^l(t-s) \leq R(t) - R(s) \leq \alpha^u(t-s) \forall s \leq t.$$

$$\beta^l(t-s) \leq C(t) - C(s) \leq \beta^u(t-s) \forall s \leq t.$$

Therefore, by using the convolution and de-convolution, we know that

$$\alpha^u = R \oslash R; \qquad \alpha^l = R \bar{\oslash} R; \qquad \beta^u = C \oslash C; \qquad \beta^l = C \bar{\oslash} C;$$

The proof for α^u:

$$\alpha^u(\Delta) = \sup_{\lambda \geq 0} \{R(\Delta + \lambda) - R(\lambda)\} \geq R(\Delta + \lambda) - R(\lambda), \ \forall \lambda \geq 0.$$

Tight Curves

A curve f is *sub-additive*, if

$$f(a) + f(b) \geq f(a+b) \quad \forall a, b \geq 0.$$

The sub-additive *closure* $\overline{f}$ of a curve f is the largest sub-additive curve with $\overline{f} \leq f$ and is computed as

$$\overline{f} = \min\{f, (f \otimes f), (f \otimes f \otimes f), \ldots\}.$$

If f is interpreted as an arrival curve, then any trace R that is upper bounded by f is also upper bounded by the sub-additive closure $\overline{f}$.

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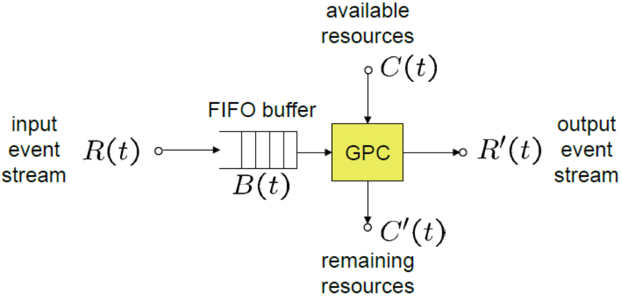
A tight upper arrival curve should satisfy the sub-additive property.

Greedy Processing Component (GPC)



- Component is triggered by incoming events.
- A fully preemptable task is instantiated at every event arrival to process the incoming event.
- Active tasks are processed in a greedy fashion in FIFO order.
- Processing is restricted by the availability of resources.

Some Relations (only for your reference)

- The output upper arrival curve of a component satisfies

$$\alpha^{u\prime} \leq (\alpha^u \oslash \beta^l)$$

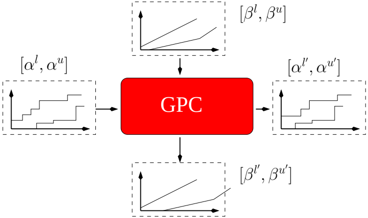
with a simple and pessimistic calculation.

- The remaining lower service curve of a component satisfies

$$\beta^{l\prime}(\Delta) = \sup_{0 \leq \lambda \leq \Delta} (\beta^l(\lambda) - \alpha^u(\lambda))$$

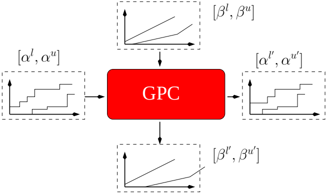
More Relations (only for your reference)

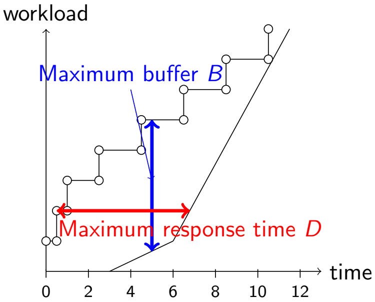
$$\alpha^{u'} = [(\alpha^u \otimes \beta^u) \oslash \beta^l] \wedge \beta^u$$
$$\alpha^{l'} = [(\alpha^u \oslash \beta^l) \otimes \beta^l] \wedge \beta^l$$
$$\beta^{u'} = (\beta^u - \alpha^l)\bar{\oslash}0$$
$$\beta^{l'} = (\beta^l - \alpha^u)\bar{\otimes}0$$



Without formal proofs....

Graphical Interpretation



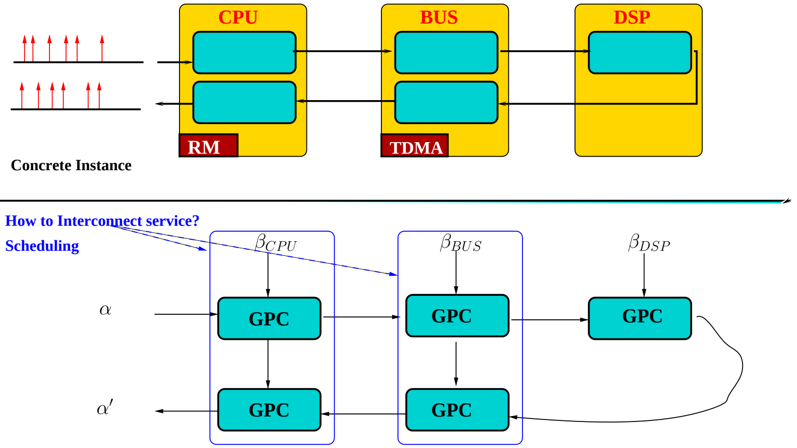


$$B = \sup_{t \geq 0}\{R(t) - R'(t)\} \leq \sup_{\lambda \geq 0}\{\alpha^u(\lambda) - \beta^l(\lambda)\}$$

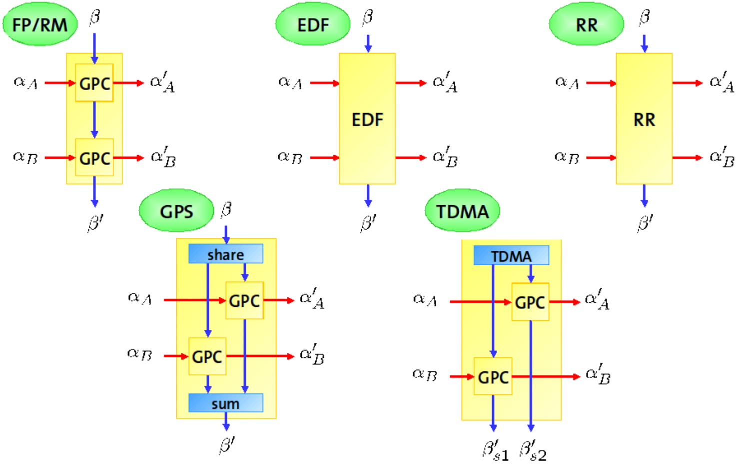
$$D = \sup_{t \geq 0}\{\inf\{\tau \geq 0 : R(t) \leq R'(t+\tau)\}\}$$

$$= \sup_{\Delta \geq 0}\{\inf\{\tau \geq 0 : \alpha^u(\Delta) \leq \beta^l(\Delta+\tau)\}\}$$

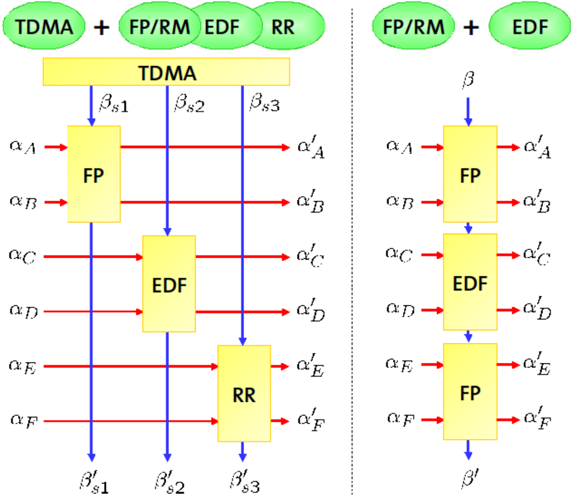
System Composition



Scheduling and Arbitration



Mixed Hierarchical Scheduling

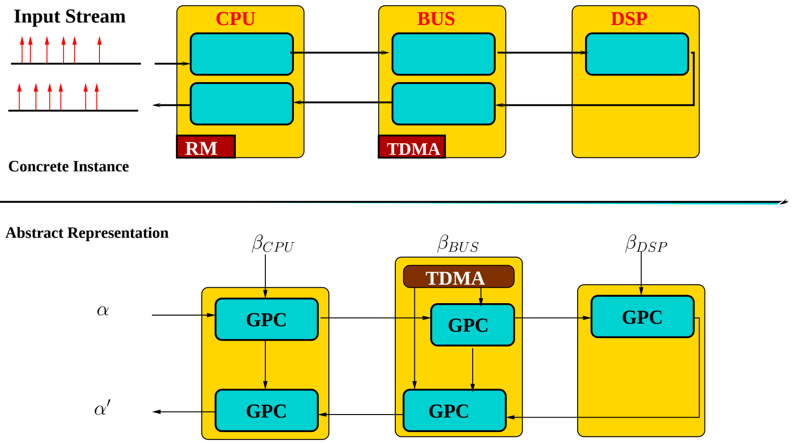


...and many other combinations:

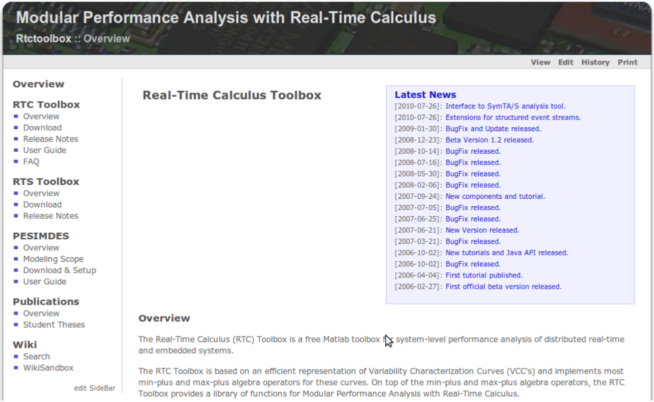
- RR + EDF
- FP/RM + RR
- FP/RM + GPS
- GPS + EDF

. . .

Complete System Composition



RTC Toolbox (http://www.mpa.ethz.ch/Rtctoolbox)


Modular Performance Analysis with Real-Time Calculus

Rtctoolbox :: Overview

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Overview

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Real-Time Calculus Toolbox

Latest News

[2010-07-26]: Interface to SymTA/S analysis tool.
[2010-07-26]: Extensions for structured event streams.
[2009-01-30]: BugFix and Update released.
[2008-12-23]: Beta Version 1.2 released.
[2008-10-14]: BugFix released.
[2008-07-16]: BugFix released.
[2008-05-30]: BugFix released.
[2008-02-06]: BugFix released.
[2007-09-24]: New components and tutorial.
[2007-07-05]: BugFix released.
[2007-06-25]: BugFix released.
[2007-06-21]: New Version released.
[2007-03-21]: BugFix released.
[2006-10-02]: New tutorials and Java API released.
[2006-10-02]: BugFix released.
[2006-04-04]: First tutorial published.
[2006-02-27]: First official beta version released.

Overview

The Real-Time Calculus (RTC) Toolbox is a free Matlab toolbox f system-level performance analysis of distributed real-time and embedded systems.

The RTC Toolbox is based on an efficient representation of Variability Characterization Curves (VCC's) and implements most min-plus and max-plus algebra operators for these curves. On top of the min-plus and max-plus algebra operators, the RTC Toolbox provides a library of functions for Modular Performance Analysis with Real-Time Calculus.

Advantages and Disadvantages of RTC and MPA

- Advantages
  - More powerful abstraction than "classical" real-time analysis
  - Resources are first-class citizens of the method
  - Allows composition in terms of (a) tasks, (b) streams, (c) resources, (d) sharing strategies.
- Disadvantages
  - Needs some effort to understand and implement
  - Extension to new arbitration schemes not always simple
  - *Not applicable for schedulers that change the scheduling policies dynamically.*