Erratum: Global Deadline-Monotonic Scheduling of Arbitrary-Deadline Sporadic Task Systems*

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Abstract. This paper presents an error in the schedulability test for global deadline-monotonic scheduling of arbitrary-deadline sporadic task systems in identical multiprocessor systems proposed by Baruah and Fisher in OPODIS 2007. This erratum provides a simple fix. Fortunately, the speedup bound $2+\sqrt{3}$ claimed in their paper remains valid with this simple fix.

In the sporadic task model, a task τ_i is characterized by its relative deadline D_i, its minimum inter-arrival time (period) T_i, and its worst-case execution time C_i. An arbitrary-deadline sporadic task set does not assume any relation between the relative deadlines and the periods of the tasks. Baruah and Fisher [1] considered an arbitrary-deadline sporadic task set executed on $m \geq 2$ identical processors based on global deadline-monotonic (DM) scheduling, in which $D_1 \leq D_2 \leq \ldots, \leq D_n$. They proposed a schedulability test that is the state of the art of this problem with respect to the resource augmentation (speedup) bound. We here recall their notation as follows:

- density δ_i of task τ_i: $C_i / \min(D_i, T_i)$;
- maximum density $\delta_{\max}(k)$ among the first k tasks: $\max_{i=1}^{k}(\delta_i)$;
- demand bound function of task τ_i: $\text{DBF}(\tau_i, t) = \max\left(0, \left(\left\lfloor \frac{t-D_i}{T_i} \right\rfloor + 1\right) C_i\right)$;
- load $\text{LOAD}(k)$ of the first k tasks: $\text{LOAD}(k) = \max_{t>0} \frac{\sum_{i=1}^{k} \text{DBF}(\tau_i,t)}{t}$.

The schedulability test of task τ_k under global DM by Baruah and Fisher [1] is as follows:

$$\left(1 + \frac{D_k}{\Delta}\right) \text{LOAD}(k) + (\lceil \mu_k \rceil - 1)\delta_{\max}(k) \leq \mu_k \tag{1}$$

$$\overset{\text{since } D_k \leq \Delta}{\Leftarrow} \quad 2\text{LOAD}(k) + (\lceil \mu_k \rceil - 1)\delta_{\max}(k) \leq \mu_k \tag{2}$$

where μ_k is defined as $m - (m-1)\delta_k$.

The schedulability test in Eqs. (1) and (2) is based on an incorrect Lemma 3 from the original analysis [1], stated as follows: *"The total remaining execution requirement of all the carry-in jobs of each task τ_i (that has carry-in jobs at time-instant t_0) is $< \Delta \times \delta_{max}(k)$."*

There was one unsound step in the second part of the equation set (5) in the original proof [1]. They stated that the condition $m\phi_i - (m-1)y_i < (m-(m-1)\delta_k)\phi_i$ implies

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that $y_i > \phi_i \delta_{\max}(k)$. The fact is that $m\phi_i - (m-1)y_i < (m - (m-1)\delta_k)\phi_i$ only implies $y_i > \phi_i \delta_k$. The original implication holds when $\delta_k \geq \delta_i$, i.e., δ_k is equal to $\delta_{\max}(k)$. Without such an implication, the remaining execution requirement of task τ_i can only be safely stated as $< \Delta\delta_i + \phi_i(\delta_i - \delta_k)$ in their proof.

However, this correct inequality introduces an unknown variable ϕ_i. One simple solution to fix their analysis is to define μ_k as $m-(m-1)\delta_{\max}(k)$. With this solution, all the analysis steps are valid and their Lemma 3 is correct. Therefore, the schedulability test in Eq. (1) and Eq. (2) both remain valid if μ_k is defined as $m - (m-1)\delta_{\max}(k)$.

Based on the above discussion, the (sufficient) schedulability test in Corollary 1 by Baruah and Fisher [1] should be restated as

$$\textsc{load}(k) \leq \frac{1}{2}\left(m - (m-1)\delta_{\max}(k)\right)\left(1 - \delta_{\max}(k)\right). \tag{3}$$

Fortunately, the above schedulability test in Eq. (3) still leads to the speedup bound $2 + \sqrt{3}$, as the procedure in the proof of Lemma 5 in the original analysis remains valid by using only the condition $\delta_{\max}(k) \leq x$.

References

1. S. K. Baruah and N. Fisher. Global deadline-monotonic scheduling of arbitrary-deadline sporadic task systems. In *Principles of Distributed Systems, 11th International Conference, OPODIS 2007, Guadeloupe, French West Indies, December 17-20, 2007. Proceedings*, pages 204–216, 2007.