Uniprocessor Scheduling Strategies for Self-Suspending Task Systems

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ABSTRACT

We study uniprocessor scheduling for hard real-time self-suspending task systems where each task may contain a single self-suspension interval. We focus on improving state-of-the-art fixed-relative-deadline (FRD) scheduling approaches, where an FRD scheduler assigns a separate relative deadline to each computation segment of a task. Then, FRD schedules different computation segments by using the earliest-deadline first (EDF) scheduling policy, based on the assigned deadlines for the computation segments. Our proposed algorithm, Shortest Execution Interval First Deadline Assignment (SEIFDA), greedily assigns the relative deadlines of the computation segments, starting with the task with the smallest execution interval length, i.e., the period minus the self-suspension time. We show that any reasonable deadline assignment under this strategy has a speedup factor of 3. Moreover, we present how to approximate the schedulability test and a generalized mixed integer linear programming (MILP) that can be formulated based on the tolerable loss in the schedulability test defined by the users. We show by both analysis and experiments that through designing smarter relative deadline assignment policies, the resulting FRD scheduling algorithms yield significantly better performance than existing schedulers for such task systems.

1. INTRODUCTION

Self-suspension has become increasingly important for many real-time applications, due to 1) the interactions with external devices, such as GPUs [19], I/O devices [16], and accelerators [4], 2) multicore systems with shared resources [15], 3) suspension-aware multiprocessor synchronization protocols [5, 22], etc. Introducing suspension delays may negatively impact real-time schedulability, particularly given that such delays can be quite lengthy in many scenarios.

Two models are studied in the literature: *dynamic* and *segmented* self-suspension (sporadic) task models. The *segmented* self-suspension model characterizes the lengths of the computation segments and suspension intervals as an array $(C_{i,1}, S_{i,1}, C_{i,2}, S_{i,2}, ..., S_{i,m_i-1}, C_{i,m_i})$, composed of m_i computation segments separated by $m_i - 1$ suspension intervals, in which $C_{i,j}$ is the worst-case execution time of a computation segment, and $S_{i,j}$ is the worst-case length of a self-suspension interval. The dynamic self-suspension model allows a job of task τ_i to suspend itself at any moment before it finishes as long as the worst-case self-suspension time S_i is not violated.

It was shown by Ridouard et al. [24] that the scheduler design problem for the segmented self-suspension task model is $\mathcal{NP}$-hard in the strong sense. Chen [7] has recently shown that deciding whether a segmented self-suspension task set can be schedulable by a fixed-priority scheduling policy is co$\mathcal{NP}$-hard in the strong sense. Although the computational complexity for the dynamic self-suspension task model is still unknown in most classes, it was shown by Chen [7] that a wide range of scheduling strategies are with unbounded speedup factors. For the computational complexity and the difficulty to handle self-suspension systems, please refer to [7, 24].

In this paper, we focus on segmented self-suspension task systems, in which a job of a task can suspend at most once. To resolve the computational complexity issues in many of these $\mathcal{NP}$-hard scheduling problems in real-time systems, approximation algorithms, and in particular, approximations based on *resource augmentation* (to quantify the worst-case speedup factors, detailed in Section 3.1) have attracted much attention. Designing scheduling algorithms and schedulability tests with bounded speedup factors (resource augmentation factors, equivalently) ensures a bounded gap between the derived solution and the optimal solution for such $\mathcal{NP}$-hard problems.

Overview of related work. The problem of scheduling and analyzing schedulability of real-time suspending tasks has received much attention. For more details, please refer to the recent review paper [11] for scheduling self-suspending tasks in real-time systems. Although the impact of self-suspension behaviour in real-time systems has been investigated since 1990, the literature of this research topic has been seriously flawed as reported in [11].

Here, we summarize the existing results that are directly related to segmented self-suspending task systems. Under fixed-priority scheduling, Rajkumar [23] proposed a *period enforcer* algorithm to handle the impact of self-suspensions. Although the period enforcer algorithm can be applied for self-suspending tasks with multiple computation segments, Chen and Brandenburg [8] have recently shown that period enforcement can be a cause of deadline misses for self-suspending tasks sets that are otherwise schedulable. Moreover, its schedulability test is also concluded as unknown in [8]. For task systems with at most one self-suspension interval per task, Lakshmanan and Rajkumar proposed two slack enforcement mechanisms in [18], in which the objectives are similar to the period enforcer by shaping the higher-priority jobs so that the higher-priority interference can behave like ordinary periodic tasks. The correctness of the slack enforcement mechanisms was classified as an open issue, since the proofs in [18] were incomplete.

Lakshmanan and Rajkumar [18] also proposed an pseudo-polynomial-time worst-case response time test (*recently shown unsafe by Nelissen et al. [20]*) for a special case, in which there are ordinary sporadic tasks without any self-suspension and one segmented self-suspending task as the lowest-priority task. The sufficient schedulability test by Nelissen et al. [20] requires exponential-time complexity even when the task system has *only one self-suspending task*. To handle multiple sporadic segmented self-suspending tasks, Nelissen et al. [20]

proposed to convert higher-priority tasks into sporadic tasks with jitters, which is unsafe.[1] The methods in [12, 17] assign each computation segment a fixed-priority level and an offset, which was shown incorrect in [11]. For details with respect to these issues, please refer to [11].

Chen and Liu [10] and Huang and Chen [14] proposed to use release time enforcement, called fixed-relative-deadline (FRD), under dynamic-priority scheduling and fixed-priority scheduling, respectively. An FRD scheduler assigns a separate relative deadline to each computation segment of a task and assigns different computation segments different relative deadlines. Thus, relative deadline assignment policies become critical to the performance of FRD scheduling. It is shown in [10] that a rather simple assignment policy, namely equal-deadline assignment (EDA), that assigns relative deadlines equally to the computation segments of a self-suspending task and uses EDF in [10] and fixed-priority in [14] for scheduling the computation segments, yields better performance w.r.t. resource augmentation bounds compared to traditional job-level or task-level fixed priority scheduling algorithms. Note that the study in [10] assumed only one self-suspension interval per task. EDA was later shown to have bounded speedup factors in [14] (under both EDF and fixed-priority scheduling) for multiple self-suspension intervals.

Contributions. In this paper we study the problem of scheduling a sporadic self-suspending hard real-time task system on a uniprocessor, where each self-suspending task may contain one suspension interval. We consider implicit-deadline task systems. Although EDA is shown to be superior to traditional real-time schedulers for such cases [10,14], its deadline assignment policy is rather straightforward and the potential of FRD scheduling seems not to be fully exploited under EDA.

Our proposed algorithm, Shortest Execution Interval First Deadline Assignment (SEIFDA), considers the deadline assignment starting with the task with the smallest execution interval length, i.e., the period minus the self-suspension time. When considering task τ_k, SEIFDA greedily chooses any feasible deadline only based on the interference from the other $k-1$ tasks with assigned deadlines, under an assumption that the shorter computation segment of task τ_k has a short relative deadline. This results in several strategies for the deadline selection, as presented in Section 5. We show that SEIFDA by adopting any deadline assignment has a speedup factor of 3 in Section 6. Moreover, we also present how to approximate the schedulability test in Section 7. Section 8 presents a generalized mixed integer linear programming (MILP) that can be formulated based on the tolerable loss in the schedulability test defined by the users. We show by both analysis and experiments that through designing smarter relative deadline assignment policies, the resulting FRD scheduling algorithms yield significantly better performance than existing schedulers for such task systems.

2. TASK MODEL

We consider n sporadic one-segment self-suspending real-time tasks $\mathbf{T} = \{\tau_1, \tau_2, \ldots, \tau_n\}$ in a uniprocessor system, in which the n tasks are independent. Each task can release an infinite number of jobs (or task instances) under a given minimum inter-arrival time (temporal) constraints T_i, also called the tasks period. This means if a job of task τ_i arrives at time θ_a the next instance of the task must arrive not earlier than $\theta_a + T_i$. For one-segment self-suspending tasks the execution of each job of τ_i is composed of two *computation* segments separated by one *suspension* interval. After the first computation segment is finished the job suspends itself, i.e., for the length of the suspension interval it is removed from the ready queue and the job in the ready queue with the highest priority is executed. The second computation segment is eligible to execute only after the completion of the suspension interval. That is, after the suspension interval of a jobs ends the job will be reentered into the ready queue. A one-segment self-suspending task τ_i is characterized by 3 tuples:

$$\tau_i = ((C_{i,1}, S_i, C_{i,2}), T_i, D_i)$$

where T_i denotes the minimum inter-arrival time of τ_i; D_i denotes the relative deadline of task τ_i; $C_{i,1}$ and $C_{i,2}$ denote the worst case execution time (WCET) of the first and second computation segment respectively; and S_i denotes the upper bound on the suspension time of τ_i. All these values are positive. For the simplicity of presentations, for the rest of this paper, we implicitly call such tasks as self-suspending tasks as the context is clear. In this work, we restrict our attention to *implicit-deadline* task systems, i.e., $D_i = T_i$.

We do not assume, that each task in the task set must be a self-suspending task. If a task has no self-suspension behavior, there is only one computation segment of task τ_i, which is equivalent to the conventional sporadic task model. In our solution, such ordinary sporadic tasks should still be scheduled by using their original deadlines and demand bound functions. However, for the simplicity of presentation, we do not consider these tasks in the paper.

For a self-suspending task we denote $C_i = C_{i,1} + C_{i,2}$ and assume that $C_i + S_i \leq D_i$ for any task $\tau_i \in \mathbf{T}$. Furthermore, we denote $C_i^{max} = \max\{C_{i,1},\ C_{i,2}\}$ and $C_i^{min} = \min\{C_{i,1},\ C_{i,2}\}$. The utilization of task τ_i is defined as $U_i = C_i/T_i$. Moreover, we also use $U_{i,1} = C_{i,1}/T_i$ and $U_{i,2} = C_{i,2}/T_i$ for notational brevity. We further assume that $\sum_{i=1}^{n} U_i \leq 1$. We use the following definitions of feasibility and schedulability in this paper:

- A schedule is *feasible* if there is no deadline miss and all the scheduling constraints are respected.
- A self-suspension task system $\mathbf{T}$ is called *schedulable* if there exists a feasible schedule for the task system for any release patterns under the temporal constraints.
- A self-suspension task system $\mathbf{T}$ is *schedulable* under a scheduling algorithm if the schedule produced by the algorithm for the task system is always feasible.

3. FIXED-RELATIVE-DEADLINE (FRD) STRATEGIES

In this paper, we adopt the Fixed-Relative-Deadline (FRD) strategies that have been already used in [10]. For each $\tau_i \in \mathbf{T}$ an FRD policy assigns relative deadlines $D_{i,1}$ and $D_{i,2}$ for the executions of the first subtask and the second subtask of τ_i, respectively. Specifically, when a job of τ_i arrives at time t,

- the first subjob (i.e., the first computation segment) has the release time t and its absolute deadline is $t + D_{i,1}$,
- the suspension has to be finished before $t + D_{i,1} + S_i$,
- the second subjob (i.e., the second computation segment) *is enforced* to be released at time $t + D_{i,1} + S_i$ and its absolute deadline is $t + D_{i,1} + S_i + D_{i,2}$.

[1] To our knowledge, the erratum is still prepared by the authors.

Based on the assigned relative deadlines, each subjob has its own absolute deadline, assigned when a job arrives. The underlying scheduling policy uses the standard earliest-deadline-first (EDF) scheduling to schedule the subjobs with dynamic-priority scheduling.

An FRD assignment is feasible if the worst-case response time of the first (second, respectively) computation segment of task τ_i is no more than $D_{i,1}$ ($D_{i,2}$, respectively). To ensure the feasibility of the resulting schedule, such a scheduling policy has to ensure that $D_{i,1} + D_{i,2} + S_i \leq T_i$. For the remaining parts of this paper we will always assume that $D_{i,1} + D_{i,2} + S_i = T_i$. Otherwise, if $D_{i,1} + D_{i,2} + S_i < T_i$, we can always increase $D_{i,2}$ by setting it to $T_i - S_i - D_{i,1}$ without jeopardizing (i.e., reducing) the schedulability of the task set.[2]

3.1 Resource Augmentation Factor

A common approach to quantify the quality of scheduling algorithms (or schedulability tests) is to bound the degree that the considered algorithm may under-perform a (maybe hypothetical) optimal one. To obtain such a bound, we adopt the concept of the resource augmentation factor or speedup factor. When the system is sped up by f, the worst-case execution times $C_{i,1}$ and $C_{i,2}$ become $\frac{C_{i,1}}{f}$ and $\frac{C_{i,2}}{f}$, respectively. However, in this paper, S_i remains the same. Note that there are also other practical scenarios which quantify the speedup factors by reducing the self-suspension time while speeding up. For detailed discussions with regard to this matter, please refer to [7]. Typically, the resource augmentation factor is defined, by referring to any arbitrarily feasible schedule under an optimal scheduling algorithm:

Definition 1. **Scheduling algorithm with respect to arbitrary schedules**: *We call such a factor the* arbitrary speedup factor. *Provided that the task set* $\mathbf{T}$ *can be feasibly scheduled, an algorithm* $\mathcal{A}$ *is called with an arbitrary speedup factor* α *when algorithm* $\mathcal{A}$ *guarantees to derive a feasible schedule by speeding up the system with a factor* α. □

3.2 Schedulability Test for FRD

Although FRD was introduced in [10], the schedulability tests provided in [10] were mainly for EDA. More general schedulability tests were not provided. Therefore, in this section, we will first explain how to perform schedulability tests under FRD. We use demand bound functions (DBF) to calculate the maximum cumulative execution time requirement of a task over a given interval $[t_0, t_0 + t)$ when the arrival time of the computation segments have to be within this interval. For the simplicity of presentation, we set t_0 to 0 for the illustrative example used in this section. The concrete appearance of the DBF for an FRD scheduling policy depends only on the value of $D_{i,1}$ as $D_{i,2} = T_i - S_i - D_{i,1}$ as discussed before.

One intuitive way to formulate the DBFs of a task for an FRD scheduling policy is to represent it as a generalized multiframe (GMF) task [2] with two frames depending on the values of $D_{i,1}$, $D_{i,2}$, and S_i. In the GMF task model (with two frames), task τ_i is represented by a 3-tuple of vectors $(\overrightarrow{C_i}, \overrightarrow{D_i}, \overrightarrow{T_i})$ where $\overrightarrow{C_i}$, $\overrightarrow{D_i}$, and $\overrightarrow{T_i}$ are vectors of identical length, for one segmented self-suspension length 2, representing the WCETs, relative deadlines, and inter-arrival times of the frames, respectively. The j-th frame of a task τ_i has the WCET, relative deadline, and interarrival time of the ($j \mod 2$)-th frame. For a one-segment self-suspending task, there are two frames in the GMF task model: $\tau_i = \{(C_{i,1}, D_i^1, T_i^1), (C_{i,2}, D_i^2, T_i^2)\}$.[3] As the second computation segment is released after the suspension interval we know that $D_i^1 = D_{i,1}$ and $T_i^1 = D_{i,1} + S_i$. Moreover, $T_i^2 = T_i - T_i^1 = T_i - D_{i,1} - S_i$ and $D_i^2 = D_{i,2} = T_i - D_{i,1} - S_i$. Now we can formulate the DBFs for the case where the segment released at time 0 is represented by the first frame and by the second frame in dbf_i^1 in Eq. (1) and dbf_i^2 in Eq. (2), respectively.

If the first computation segment is released at 0 the segment has to be finished at $t = D_{i,1}$ while the second segment has to be finished at $t = T_i$. This pattern repeats periodically and is formalized in Eq. (1):

$$dbf_i^1(t, D_{i,1}) = \left\lfloor \frac{t + (T_i - D_{i,1})}{T_i} \right\rfloor C_{i,1} + \left\lfloor \frac{t}{T_i} \right\rfloor C_{i,2} \quad (1)$$

If the second computation segment is released at 0 it has to be finished at $t = D_{i,2}$, the behavior is identical with releasing the first segment at $-(D_{i,1} + S_i)$. This means the first segment has to be finished at $T_i - S_i$. This pattern repeats periodically and is formalized in Eq. (2):

$$dbf_i^2(t, D_{i,1}) = \left\lfloor \frac{t + (D_{i,1} + S_i)}{T_i} \right\rfloor C_{i,2} + \left\lfloor \frac{t + S_i}{T_i} \right\rfloor C_{i,1} \quad (2)$$

The exact DBF for τ_i under an FRD assignment is the maximum of the two possible arrival patterns:

$$\text{DBF}_i^{\text{frd}}(t, D_{i,1}) = \max(dbf_i^1(t, D_{i,1}), dbf_i^2(t, D_{i,1})) \quad (3)$$

Using the DBF in Eq. (3) we can now formulate the exact schedulability test:

Theorem 1 (Exact Schedulability Test for FRD). *An FRD schedule is feasible if and only if*

$$\sum_{\tau_i \in \mathbf{T}} \text{DBF}_i^{frd}(t, D_{i,1}) \leq t, \qquad \forall t \geq 0.$$

Proof. This follows directly from Theorem 1 in [2], i.e., the schedulability condition for generalized multiframe task systems under EDF using demand bound functions. □

In Figure 1 we see examples of DBFs for a task with $C_{i,1} = 2$, $C_{i,2} = 3$, $S_i = 4$, and $T_i = 20$ for three different settings of $D_{i,1}$, i.e., $D_{i,1} = 2$ (grey, dashed), 4 (red, solid), and 8 (blue, dotted). For example, with $D_{i,1} = 4$ we get $D_{i,2} = 12$. We have to take care of two cases, depending on weather the computation segment released at time 0 is $C_{i,1}$ or $C_{i,2}$ and take the maximum of both cases as we are looking for the maximum possible workload. If $C_{i,1}$ is released at 0 the DBF equals 0 in the interval $[0, 4)$, as no workload has to be finished up until this point. The maximum workload after $t = 4$ is at least 2, as $C_{i,1}$ has to be finished, and at $t = 20$ it is 5 as both $C_{i,1}$ and $C_{i,2}$ have to be finished. When $C_{i,2}$ starts at $t = 0$ it has to be finished at 12 thus the total workload in $[0, 12)$ is 3. Then, $C_{i,1}$ is released at $t = 12$ with absolute deadline 16, followed by the suspension interval, thus the workload is 3 in $[12, 16)$ and 5 in $[16, 20)$ if $C_{i,2}$ is released first. In total, by taking the maximum

[2] This can be easily seen by the sufficient test shown in Theorem 1.

[3] Superscripts are used for terms when referring to the GMF task.

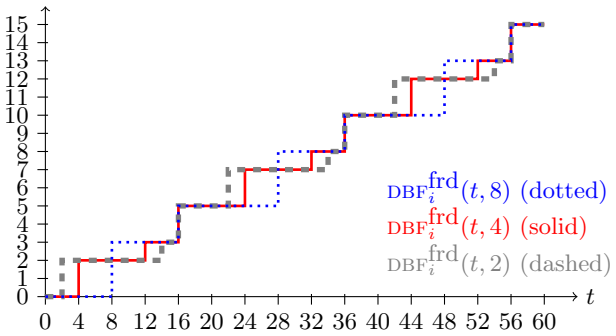

Figure 1: An example of $\text{DBF}_i^{\text{frd}}(t, D_{i,1})$ for different values of $D_{i,1}$, where $C_{i,1} = 2$, $C_{i,2} = 3$, $S_i = 4$, and $T_i = 20$.

of both cases, we get the red line in Figure 1 in $[0, 20)$. As the task is released periodically with period 20, the DBF is also periodic with period 20. This also shows, that we only have 3 jump points in each period as the jump at T_i by $dbf_i^1(t, D_{i,1})$ is already covered by the jump of $dbf_i^2(t, D_{i,1})$ at $T_i - S_i$.

In addition to the exact schedulability test we present two necessary conditions for the schedulability of the task set. One for the schedulability under an FRD assignment and one for any arbitrary scheduling algorithm. This allows to compare our approach to the best possible result any scheduling algorithm could provide.

LEMMA 1 (NECESSARY CONDITION FOR FRD). *If there exists an FRD schedule to feasibly schedule* $\mathbf{T}$*, then*

$$\sum_{\tau_i \in \mathbf{T}} \text{DBF}_i^{\textit{frd-nece}}(t) \leq t, \qquad \forall t \geq 0,$$

where

$$\text{DBF}_i^{\textit{frd-nece}}(t) = \left(\left\lfloor \frac{t - (T_i - S_i)}{T_i} \right\rfloor + 1 \right) (C_{i,1} + C_{i,2}) \qquad (4)$$

Proof. This was proved in Lemma 1 in [10] with a slightly different formulation of the equation. □

We now provide a necessary condition for any arbitrary scheduling algorithm for implicit-deadline one-segment self-suspension task sets, assuming that $C_{i,1}$, $C_{i,2}$, and S_i are given. We use Eq. (5) for a lower bound of the workload in the current period which together with the workload created in already finished periods leads to Eq. (6) to calculate a lower bound over a given time interval of length t:

$$\text{G}_i(t) = \begin{cases} 0 & \text{if } 0 \leq t < T_i - S_i \\ C_i^{max} & \text{if } T_i - S_i \leq t < T_i \end{cases} \qquad (5)$$

$$\text{DBF}_i^{\text{nece}}(t) = \left\lfloor \frac{t}{T_i} \right\rfloor (C_{i,1} + C_{i,2}) + \text{G}_i \left(t - \left\lfloor \frac{t}{T_i} \right\rfloor \right) \qquad (6)$$

LEMMA 2. *If task set* $\mathbf{T}$ *can be feasibly scheduled, then*

$$\sum_{\tau_i \in \mathbf{T}} \text{DBF}_i^{\textit{nece}}(t) \leq t, \qquad \forall t \geq 0.$$

Proof. It is easy to observe that independent from the concrete scheduling policy $C_{i,1} + C_{i,2}$ have to be scheduled after a complete interval of length T_i. What remains is to show, that Eq. (5) is a lower bound on the possible workload distributions over one period. [4] We have to look at the two cases $C_{i,1} \geq C_{i,2}$ and $C_{i,1} < C_{i,2}$. If for both cases a release pattern exists where the jump of the DBF for any arbitrary scheduling policy has to happen before $T_i - S_i$ the proof is done. If $C_{i,1} \geq C_{i,2}$ and we release $C_{i,1}$ at time $t_0 = 0$ the first subjob has to be finished before $T_i - S_i$ as S_i and the execution of $C_{i,2}$ still have to happen before T_i. If $C_{i,1} < C_{i,2}$ we release $C_{i,1}$ at $-S_i - C_{i,1}$ and thus $C_{i,2}$ has to be finished before $T_i - S_i$ independent from the scheduling policy. As both the release patterns and the DBF are periodic with period T_i this concludes the proof. □

3.3 Existing FRD Approaches

The general concept of FRD approaches was introduced in [10], in which two existing approaches were discussed:

- Proportional (Proportional relative deadline assignment): $D_{i,1} = \frac{C_{i,1}}{C_{i,1}+C_{i,2}} \cdot (T_i - S_i)$; $D_{i,2} = \frac{C_{i,2}}{C_{i,1}+C_{i,2}} \cdot (T_i - S_i)$.
- EDA (Equal relative Deadline Assignment): $D_{i,1} = D_{i,2} = (T_i - S_i)/2$.

At the first glance, Algorithm Proportional may seem very reasonable and Algorithm EDA may seem very pessimistic. Unfortunately, there exists a concrete input task set, as shown in [10], for which the arbitrary speedup factor of Algorithm Proportional is not even a constant. The reason why Algorithm Proportional does not have a constant speedup factor is due to the aggressive relative deadline assignment which greedily sets the $D_{i,1}$ as $\frac{C_{i,1}}{C_{i,1}+C_{i,2}} \cdot (T_i - S_i)$ without considering the interference from the other tasks.

Although Algorithm EDA only greedily assigns the relative deadline, it was already shown in [10] that the following condition $\text{DBF}_i^{\text{frd}}(t, (T_i - S_i)/2) \leq \text{DBF}_i^{\text{frd-nece}}(t)$ holds for any $t \geq 0$. Therefore, the spirit behind Algorithm EDA was to keep this constant factor by setting $D_{i,1}$ to $(T_i - S_i)/2$. Chen and Liu [10] showed that EDA has an arbitrary speedup factor of 3. However, there are still a few drawbacks in Algorithm EDA, even though it has constant speedup factors:

- First, it cannot handle any task set, in which there exists a task τ_i with $C_i^{max} > (T_i - S_i)/2$.
- Second, as shown in Figure 1, the demand bound function by setting $D_{i,1}$ to $(T_i - S_i)/2$ is not always the best option. Assigning $D_{i,1}$ to $(T_i - S_i)/2$ is pretty aggressive.

4. TRANSFORMATION

Before presenting our solution, we first examine some characteristics of the demand bound function $\text{DBF}_i^{\text{frd}}(t, D_{i,1})$. This section will provide an important transformation of task τ_i to simplify the presentation of the following sections. Since all the step functions in Eqs. (1) and (2) have a period T_i, it is clear that $\text{DBF}_i^{\text{frd}}(t, D_{i,1})$ is in general periodic with at most four individual increasing points in a period of T_i.

Suppose that we are interested in $\ell T_i \leq t < (\ell + 1)T_i$ where ℓ is a non-negative integer. For $dbf_i^1(t)$, we have

- $dbf_i^1(t) = \ell(C_{i,1} + C_{i,2})$ when $\ell T_i \leq t < \ell T_i + D_{i,1}$;
- $dbf_i^1(t) = \ell(C_{i,1} + C_{i,2}) + C_{i,1}$ when $\ell T_i + D_{i,1} \leq t < (\ell + 1)T_i$.

For $dbf_i^2(t)$, we have

[4] The remaining proof is identical to the proof of Lemma 2 in [10]. Since our condition is stronger, we include the proof for completeness.

- $dbf_i^2(t) = \ell(C_{i,1} + C_{i,2})$ when $\ell T_i \le t < \ell T_i + (T_i - S_i - D_{i,1}) = \ell T_i + D_{i,2}$;
- $dbf_i^2(t) = \ell(C_{i,1} + C_{i,2}) + C_{i,2}$ when $\ell T_i + D_{i,2} \le t < \ell T_i + T_i - S_i$;
- $dbf_i^2(t) = (\ell+1)(C_{i,1} + C_{i,2})$ when $\ell T_i + T_i - S_i \le \ell T_i < (\ell+1)T_i$.

Therefore, $dbf_i^2(t) \ge dbf_i^1(t)$ if $(t \mod T_i) > T_i - S_i$. Moreover, we also have the following properties:

LEMMA 3. *If $C_{i,1} \le C_{i,2}$ and $D_{i,1} \ge (T_i - S_i)/2$, then*

$$\forall t \ge 0, \qquad \text{DBF}_i^{frd}(t, D_{i,1}) \ge \text{DBF}_i^{frd}(t, T_i - S_i - D_{i,1}).$$

LEMMA 4. *If $C_{i,1} \ge C_{i,2}$ and $D_{i,1} \le (T_i - S_i)/2$, then*

$$\forall t \ge 0, \qquad \text{DBF}_i^{frd}(t, D_{i,1}) \ge \text{DBF}_i^{frd}(t, T_i - S_i - D_{i,1}).$$

Proof. The proof of these two lemmas follow directly from the definitions. □

Therefore, the above lemmas suggest to assign a shorter relative deadline to the shorter computation segment with C_i^{min} for each task τ_i. However, it is notationally inconvenient to distinguish these two difference cases, depending on whether $C_{i,1}$ is smaller or not. Fortunately, the notational complication can be easily handled by swapping $C_{i,1}$ and $C_{i,2}$ if $C_{i,1} > C_{i,2}$, due to the following lemma.

LEMMA 5. *Suppose that $C_{i,1} > C_{i,2}$ for a task τ_i. We can create a corresponding task τ_i^* with the same parameters as τ_i but $C_{i,1}$ and $C_{i,2}$ are swapped in task τ_i^*. If $D_{i,1} \ge (T_i - S_i)/2$, then*

$$\forall t \ge 0, \qquad \text{DBF}_i^{frd}(t, D_{i,1}) = \text{DBF}_{i^*}^{frd}(t, T_i - S_i - D_{i,1}),$$

where $\text{DBF}_{i^*}^{frd}(t, T_i - S_i - D_{i,1})$ is the demand bound function of task τ_i^* by setting the relative deadline of the first computation segment in task τ_i^* (i.e., execution time $C_{i,2}$) to $T_i - S_i - D_{i,1}$.

Proof. This can be proved by inspecting the corresponding demand bound functions, as they are identical. □

By Lemma 5, *for the rest of this paper, we will implicitly consider that $C_{i,1} \le C_{i,2}$*. If $C_{i,2} < C_{i,1}$, we should simply reorder them before proceeding to the relative deadline assignment of task τ_i and swap them, together with the assigned deadlines, back after the assignment. By Lemma 5 and the discussions earlier, this does not give any additional restriction, but make our presentation flow much easier.

5. OUR GREEDY APPROACH

Our proposed algorithm, Shortest Execution Interval First Deadline Assignment (SEIFDA), works as follows: First, we re-index (sort) the given n tasks such that $T_i - S_i \le T_j - S_j$ for $i < j$. Then, we iteratively assign their relative deadlines under FRD scheduling, starting from task τ_1 to task τ_n. Suppose that the relative deadlines $D_{i,1}$ and $D_{i,2}$ of all tasks $\tau_i \in \{\tau_1, \tau_2, \ldots, \tau_{k-1}\}$ have been already assigned.

Note that, by the transformation in Section 4, we only have to consider $C_{k,1} \le C_{k,2}$ for the deadline assignment. If $C_{k,1} > C_{k,2}$ we swap $C_{k,1}$ and $C_{k,2}$ before the deadline assignment, swap them back after the assignment, and swap the respective deadlines as well. As shown in Lemma 3, if a feasible FRD assignment exists we can always assign the deadline of $C_{k,1}$ to a $D_{k,1}$ with $D_{k,1} \le (T_k - S_k)/2$. To be more precise, if there exists a certain x in the range of $(C_{k,1}, (T_k - S_k)/2]$ such that

$$\text{DBF}_k^{\text{frd}}(t, x) + \sum_{i=1}^{k-1} \text{DBF}_i^{\text{frd}}(t, D_{i,1}) \le t, \ \ \forall t \ge 0, \qquad (7)$$

then, we will greedily assign $D_{k,1}$ to one of such an x value. The pseudocode of Algorithm SEIFDA is presented in Algorithm 1.

Algorithm 1 Shortest Execution Interval First Deadline Assignment (SEIFDA)

Input: set $\mathbf{T}$ of n one-segment self-suspension sporadic real-time tasks with implicit deadlines;
1: re-index (sort) tasks such that $T_i - S_i \le T_j - S_j$ for $i < j$;
2: **for** $k = 1$ to n **do**
3: **if** $\exists x \in \left(C_{k,1}, \frac{T_k - S_k}{2}\right]$ such that the condition in Eq. (7) holds **then**
4: let x^* be one of such values

$$\text{DBF}_k^{\text{frd}}(t, x^*) + \sum_{i=1}^{k-1} \text{DBF}_i^{\text{frd}}(t, D_{i,1}) \le t, \ \ \forall t \ge 0;$$

5: set $D_{k,1} \leftarrow x^*$, and $D_{k,2} \leftarrow T_k - S_k - x^*$;
6: **else**
7: return "no feasible FRD schedule is found";
8: **end if**
9: **end for**
10: return the relative deadline assignment for each task τ_i in $\mathbf{T}$;

5.1 Selection of Relative Deadlines for Task τ_k

Algorithm 1 provides a framework to assign the relative deadlines for FRD scheduling. However, it also leaves an open design option. If there are multiple values of x such that the condition in Eq. (7) holds, which one should be chosen? Due to the greedy strategy, after the relative deadlines are assigned, they will not be changed later. Suppose that x^* is the chosen value of x when considering task τ_k. We are not able to provide the best strategy to choose x^*, but there are several strategies that can be applied:

- Minimum x (denoted by minD): The selection of x^* is to use the minimum x such that Eq. (7) holds.
- Maximum x (denoted by maxD): The selection of x^* is to use the maximum x such that Eq. (7) holds.
- Proportionally-Bounded-Min x (denoted by PBminD): The selection of x^* is to use the minimum x such that both $x \ge \frac{C_{i,1}}{C_{i,1}+C_{i,2}}(T_k - S_k)$ and Eq. (7) hold.

By the above discussions, depending on how we assign $D_{k,1}$ and $D_{k,2}$ in Algorithm SEIFDA, the resulting solutions are different. By the following theorem, EDA is a special case of SEIFDA-maxD and dominated by SEIFDA-maxD.

THEOREM 2. *If a task set $\mathbf{T}$ is schedulable by Algorithm EDA, the task set $\mathbf{T}$ is also schedulable by Algorithm SEIFDA-maxD.*

Proof. EDA assigns $D_{i,1} = D_{i,2} = (T_i - S_i)/2 \ \forall \tau_i \in \mathbf{T}$. If $\mathbf{T}$ is schedulable by Algorithm EDA, then

$$\forall t \ge 0, \sum_{\tau_i \in \mathbf{T}} \text{DBF}_i^{\text{frd}}(t, (T_i - S_i)/2) \le t.$$

when assigning the relative deadlines for task τ_k, Algorithm SEIFDA-maxD assigns the maximum $x \in \left(C_{k,1}, \frac{T_k - S_k}{2}\right]$ that

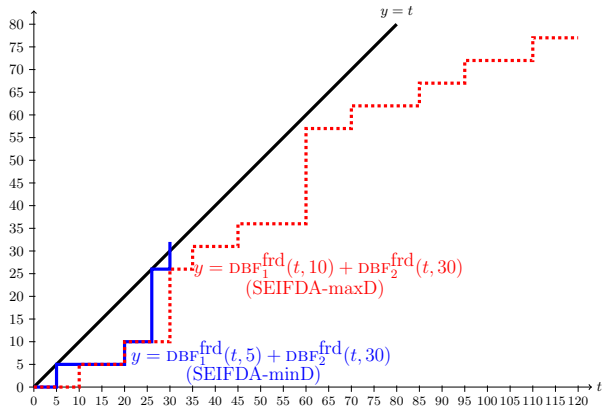

Figure 2: Schedulability test for Algorithms SEIFDA-maxD (red) and SEIFDA-minD (blue) for the task set in Table 1, $\varepsilon = 1$.

					SEIFDA-minD		SEIFDA-maxD	
Task	$C_{i,1}$	$C_{i,2}$	S_i	T_i	$D_{i,1}$	$D_{i,2}$	$D_{i,1}$	$D_{i,2}$
τ_1	5	5	5	25	5	15	10	10
τ_2	15+ε	15+ε	940	1000	↯	↯	30	30

Table 1: An example for comparing SEIFDA-maxD and SEIFDA-minD, where $0 < \varepsilon \leq 1$. ↯ denotes that SEIFDA-minD does not find a feasible value for $D_{2,1}$ and thus $D_{2,2}$ is not assigned either.

satisfies Eq. (7). Therefore, the algorithm always assigns $D_{k,1} = (T_k - S_k)/2 \ \forall \tau_k$. Hence, the deadline assignment by Algorithm SEIFDA-maxD is the same as by EDA. □

5.2 SEIFDA-maxD and SEIFDA-minD

In the previous subsection we showed that our Algorithm SEIFDA-maxD dominates EDA. It would be also interesting to have such a relation between SEIFDA-minD and EDA or between SEIFDA-maxD and SEIFDA-minD. Here we show that such a relation does not exists by creating one task set that is schedulable by SEIFDA-maxD but not by SEIFDA-minD (Table 1, Figure 2) and another one that is schedulable by SEIFDA-minD but not by SEIFDA-maxD (Table 2, Figure 3).

For the task set listed in Table 1, SEIFDA-minD assigns $D_{1,1} = 5 \Rightarrow D_{1,2} = 15$, resulting in steps at 5 and 20 for $\text{DBF}_1^{\text{frd}}(t, 5)$, periodically repeated with period 25. This leads to $D_{2,1} \in [25 + \varepsilon; 30]$ as possible values. No matter which value is assigned (in Figure 2 we assume 26) this results in a deadline miss for the second job of $C_{1,1}$ at $t = 30$ as the total workload is $2 \cdot C_{1,1} + C_{1,2} + C_{2,1} = 30 + \varepsilon > 30$. However, the EDA is feasible as $D_{1,1} = 10 \Rightarrow D_{1,2} = 10$ and the second release of $C_{i,1}$ is feasible with absolute deadline 35.

For the task set listed in Table 2 SEIFDA-minD assigns $D_{1,1} = \varepsilon \Rightarrow D_{1,2} = 20 + \varepsilon$. For $\text{DBF}_1^{\text{frd}}(t, \varepsilon)$ the steps are at ε, 20+ε, and 20+2ε. With $D_{2,1} = 10 + 2\varepsilon$ this leads to a schedulable task set as shown by the DBF in Figure 3. If SEIFDA-maxD is used $D_{1,1} = D_{1,2} = 10 + \varepsilon$. This leads to a deadline miss for $C_{2,1}$ no matter which deadline is assigned (in Figure 2 we assume $C_{2,1} = 20$) as the total workload in the interval $[0, 20]$ is $20 + 2\varepsilon$ if $C_{1,2}$ and at $C_{2,1}$ are both released at 0. This directly leads to the following theorem:

THEOREM 3. *SEIFDA-minD does not dominate SEIFDA-maxD and SEIFDA-maxD does not dominate SEIFDA-minD.*

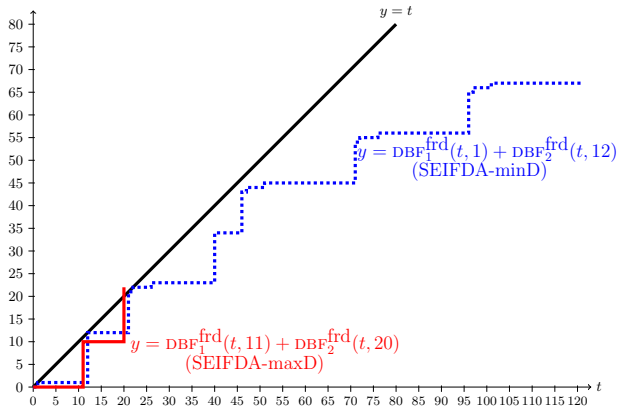

Figure 3: Schedulability test for Algorithms SEIFDA-maxD (red) and SEIFDA-minD (blue) for the task set in Table 2, $\varepsilon = 1$.

					SEIFDA-minD		SEIFDA-maxD	
Task	$C_{i,1}$	$C_{i,2}$	S_i	T_i	$D_{i,1}$	$D_{i,2}$	$D_{i,1}$	$D_{i,2}$
τ_1	ε	10	5-2ε	25	ε	20+ε	10+ε	10+ε
τ_2	10+ε	10+ε	960	1000	10+2ε	30-2ε	↯	↯

Table 2: An example for comparing SEIFDA-maxD and SEIFDA-minD, where $0 < \varepsilon \leq 1$. ↯ denotes that SEIFDA-maxD does not find a feasible value for $D_{2,1}$ and thus $D_{2,2}$ is not assigned either.

6. SPEEDUP FACTOR OF SEIFDA

Based on the assumption that $C_{i,1} \leq C_{i,2}$, the following lemma gives the inequalities between $\text{DBF}_i^{\text{frd}}(t, D_{i,1})$ and the necessary conditions when $t \geq (T_i - S_i)/2$.

LEMMA 6. *Suppose that* $0 < D_{i,1} \leq (T_i - S_i)/2$. *For any* $t \geq (T_i - S_i)/2$, *we have*

$$\text{DBF}_i^{frd}(t, D_{i,1}) \leq 2\text{DBF}_i^{nece}(t) \quad \textit{if } T_i - S_i \leq t < T_i + D_{i,1} \tag{8}$$

$$\text{DBF}_i^{frd}(t, D_{i,1}) \leq \text{DBF}_i^{nece}(2t) \quad \textit{otherwise} \tag{9}$$

Proof. We consider all the cases when $t \geq (T_i - S_i)/2$:

- If $(T_i - S_i)/2 \leq t < T_i - S_i$, we have $\text{DBF}_i^{\text{frd}}(t, D_{i,1}) \leq C_{i,2} = \text{DBF}_i^{\text{nece}}(T_i - S_i) \leq \text{DBF}_i^{\text{nece}}(2t)$.
- If $T_i - S_i \leq t < T_i + D_{i,1}$, we have $\text{DBF}_i^{\text{frd}}(t, D_{i,1}) = C_{i,1} + C_{i,2} \leq 2\text{DBF}_i^{\text{nece}}(t)$.
- If $T_i + D_{i,1} \leq t \leq (3T_i - S_i)/2$, we have $\text{DBF}_i^{\text{frd}}(t, D_{i,1}) \leq C_{i,1} + 2C_{i,2} = \text{DBF}_i^{\text{nece}}(2T_i - S_i) \leq \text{DBF}_i^{\text{nece}}(2t)$.
- If $(3T_i - S_i)/2 < t < 2T_i + D_{i,1}$, we have $\text{DBF}_i^{\text{frd}}(t, D_{i,1}) \leq 2\,(C_{i,1} + C_{i,2}) \leq 2C_{i,1} + 3C_{i,2} = \text{DBF}_i^{\text{nece}}(3T_i - S_i) \leq \text{DBF}_i^{\text{nece}}(2t)$.
- If $2T_i + D_{i,1} \leq t$, we have $\text{DBF}_i^{\text{frd}}(t, D_{i,1}) \leq \left(\left\lfloor \frac{t}{T_i} \right\rfloor + 1\right)(C_{i,1} + C_{i,2}) \leq \text{DBF}_i^{\text{nece}}(t + 2T_i) \leq \text{DBF}_i^{\text{nece}}(2t)$.

□

THEOREM 4. *The arbitrary speedup factor of SEIFDA by adopting the schedulability test in Theorem 1 is* 3.

Proof. Suppose that the task set $\mathbf{T}$ cannot be feasibly scheduled by SEIFDA. We will show that this task set is not schedulable by any algorithm at speed $\frac{1}{3}$. Recall that the tasks are indexed such that $T_i - S_i \leq T_j - S_j$ if $i \leq j$. Let $\mathbf{T}' = \{\tau_1, \tau_2, \ldots, \tau_k\}$ be the subset of $\mathbf{T}$ such that task set $\mathbf{T}'$ cannot be feasibly scheduled by SEIFDA, and $\mathbf{T}' \setminus \{\tau_k\}$ can be feasibly schedule by SEIFDA, using the schedulability test in Theorem 1.

If k is 1, it is due to $C_{i,1} + C_{i,2} > T_i - S_i$. For such a case, the arbitrary speedup factor is 1 since the task set is by definition not schedulable by any algorithm at the original system speed. We only focus on the other cases when $k \geq 2$. By the assumption that $\mathbf{T}' \setminus \{\tau_k\}$ can be feasibly scheduled by SEIFDA under the schedulability test in Theorem 1, we have

$$\forall t \geq 0, \qquad \sum_{i=1}^{k-1} \text{DBF}_i^{\text{frd}}(t, D_{i,1}) \leq t, \tag{10}$$

where $D_{i,1}$ is the relative deadline for $C_{i,1}$ under SEIFDA.

The infeasibility of SEIFDA for $\mathbf{T}'$ under the schedulability test in Theorem 1 when we intend to assign the relative deadlines for task τ_k implies that

$$\exists t \geq 0, \qquad \text{DBF}_k^{\text{frd}}\left(t, \frac{T_k - S_k}{2}\right) + \sum_{i=1}^{k-1} \text{DBF}_i^{\text{frd}}(t, D_{i,1}) > t. \tag{11}$$

That is, at least setting $D_{k,1}$ to $(T_k - S_k)/2$ cannot successfully pass the schedulability test in Theorem 1. For notational brevity, we set $D_{k,1}$ to $(T_k - S_k)/2$ for the rest of the proof. This indicates that SEIFDA fails to derive a feasible FRD schedule when assigning $D_{k,1}$ to $(T_k - S_k)/2$.

Suppose that t^* is a certain t such that the above condition in Eq. (11) holds. By the definition that $\text{DBF}_k^{\text{frd}}(t, D_{k,1}) = 0$ when $t < D_{k,1}$ and the assumption that the FRD schedule of $\mathbf{T}' \setminus \{\tau_k\}$ is feasible defined in Eq. (10), we know that t^* must be no less than $D_{k,1}$, defined as $(T_k - S_k)/2$. Since $T_i - S_i \leq T_k - S_k$ for $i = 1, 2, \ldots, k$, we also know that $t^* \geq (T_i - S_i)/2$, i.e., the conditions in Lemma 6 are applicable. We further classify the task set $\mathbf{T}'$ into two subsets:

- $\mathbf{T}'_1 =^{\text{def}} \{\tau_i \in \mathbf{T}' \mid T_i - S_i \leq t^* < T_i + D_{i,1}\}$, and
- $\mathbf{T}'_2 =^{\text{def}} \mathbf{T}' \setminus \mathbf{T}'_1$.

That is, for task τ_i in $\mathbf{T}'_1$, we can use the condition in Eq. (8) by Lemma 6; for task τ_i in $\mathbf{T}'_2$, we can use the condition in Eq. (9) by Lemma 6. By the above discussions, we have

$$\begin{aligned} t^* &< \sum_{\tau_i \in \mathbf{T}'_1} \text{DBF}_i^{\text{frd}}(t^*, D_{i,1}) + \sum_{\tau_i \in \mathbf{T}'_2} \text{DBF}_i^{\text{frd}}(t^*, D_{i,1}) \\ &\leq \sum_{\tau_i \in \mathbf{T}'_1} 2\text{DBF}_i^{\text{nece}}(t^*) + \sum_{\tau_i \in \mathbf{T}'_2} \text{DBF}_i^{\text{nece}}(2t^*) \end{aligned} \tag{12}$$

By dividing both sides by t^*, we get

$$1 < 2 \sum_{\tau_i \in \mathbf{T}'_1} \frac{\text{DBF}_i^{\text{nece}}(t^*)}{t^*} + 2 \sum_{\tau_i \in \mathbf{T}'_2} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*}. \tag{13}$$

Since $\mathbf{T}'_1 \cup \mathbf{T}'_2$ is $\mathbf{T}'$ and $\mathbf{T}'_1 \cap \mathbf{T}'_2$ is $\emptyset$, we have

$$y =^{\text{def}} \sum_{\tau_i \in \mathbf{T}'_1} \frac{\text{DBF}_i^{\text{nece}}(t^*)}{t^*} \leq \sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(t^*)}{t^*}. \tag{14}$$

$$\begin{aligned} z &=^{\text{def}} \sum_{\tau_i \in \mathbf{T}'_2} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} \\ &= \sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} - \sum_{\tau_i \in \mathbf{T}'_1} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} \\ &\leq \sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} - \sum_{\tau_i \in \mathbf{T}'_1} \frac{\text{DBF}_i^{\text{nece}}(t^*)}{2t^*} \\ &= \sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} - y/2 \end{aligned} \tag{15}$$

Therefore, we have that $\sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} \geq z + y/2$ and $\sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(t^*)}{t^*} \geq y$. By the fact $1 < 2y + 2z$ in Eq. (13), we can reach the conclusion of the proof, i.e., either $\sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(t^*)}{t^*} > 1/3$ or $\sum_{\tau_i \in \mathbf{T}'} \frac{\text{DBF}_i^{\text{nece}}(2t^*)}{2t^*} > 1/3$.[5] Therefore, the arbitrary speedup factor is 3. □

7. APPROXIMATED TEST AND TIME COMPLEXITY

The schedulability test in Theorem 1 is a necessary and sufficient test that requires exponential time complexity. To make the test faster, we do not have to test for all $t \geq 0$. Instead, we only have to test at the t values where the demand bound function $\text{DBF}_i^{\text{frd}}(t)$ changes. This means, the test in Theorem 1 is equivalent to

$$\forall \tau_i \in \mathbf{T}, \quad \forall t \in \mathbf{\Psi}_i, \quad \sum_{\tau_i \in \mathbf{T}} \text{DBF}_i^{\text{frd}}(t) \leq t \tag{16}$$

$$\mathbf{\Psi}_i = \{D_{i,1} + \ell T_i, T_i - S_i - D_{i,1} + \ell T_i, T_i - S_i + \ell T_i | \ell \in \mathbb{N}^0\} \tag{17}$$

where $\mathbb{N}^0$ is the set of non-negative integers. One may further constrain ℓ to be at most $LCM(\mathbf{T})/T_i$, where $LCM(\mathbf{T})$ is the least common multiple of the periods of the tasks in $\mathbf{T}$. However, the time complexity remains exponential.

To reduce the time-complexity, we can use the approximated demand bound functions, as used in [6, 9]. Our general approach is to use the exact demand bound function for g periods of a task, where g is a user-defined (positive) integer, and use a linear approximation to upper bound the DBF after the given number of periods. Similar to the construction of the exact DBFs we will use one approximated DBF for the case where $C_{i,1}$ is released at $t = 0$ in Eq. (18a), one for the case where $C_{i,2}$ is released at $t = 0$ in Eq. (18b), and take the maximum of both values in Eq. (19).

$$\widehat{dbf}_i^1(t, D_{i,1}) = \begin{cases} dbf_i^1(t, D_{i,1}) & \text{if } t < gT_i \\ U_i t - D_{i,1}U_{i,1} + C_{i,1} & \text{otherwise.} \end{cases} \tag{18a}$$

$$\widehat{dbf}_i^2(t, D_{i,1}) = \begin{cases} dbf_i^2(t, D_{i,1}) & \text{if } t < gT_i - S_i \\ U_i(t + S_i) + C_{i,2}\frac{D_{i,1}}{T_i} & \text{otherwise.} \end{cases} \tag{18b}$$

$$\widehat{\text{DBF}_i^{\text{frd}}}(t, D_{i,1}) = \max(\widehat{dbf}_i^1(t, D_{i,1}), \widehat{dbf}_i^2(t, D_{i,1})) \tag{19}$$

As the proofs in this section are rather technical and straight forward we just provide the ideas of the proofs here. The complete proofs can be found in the extended report [25].

THEOREM 5. *The function* $\widehat{\text{DBF}^{frd}}_i(t, D_{i,1})$ *in Eq.* (19) *is a safe upper bound of* $\text{DBF}^{frd}(t, D_{i,1})$ *for any* $t \geq 0$ *and a specified* $D_{i,1} \leq (T_i - S_i)/2$. *Therefore, if* $\sum_{\tau_i \in \mathbf{T}} U_i \leq 1$ *and*

$$\forall t \geq 0, \qquad \sum_{\tau_i \in \mathbf{T}} \widehat{\text{DBF}^{frd}}_i(t, D_{i,1}) \leq t,$$

then the resulting FRD schedule is feasible. Moreover, this schedulability test can be done in $O(g|\mathbf{T}|^2)$ *time complexity.*

[5] Either $y > 1/3$ or $z + y/2 > 1/3$ holds. They can be calculated by using the intersection $z + y/2 = y$, i.e., $z = y/2$, and $1 < 2y + 2z = 3y$.

Proof. The first part of the proof, to show that Eq. (18a) is an over approximation of Eq. (1) and that Eq. (18b) is an over approximation of Eq. (2), can be done by inspecting the corresponding values at the non-linear points of Eq. (1) and Eq. (2), respectively, for $t > gT_i - S_i$. This directly leads to the conclusion that Eq. (19) is an over approximation of Eq. (3). The details are in the appendix of the technical report [25].

For analyzing the time complexity we only have to perform the schedulability tests at the points in time where $\sum_{\tau_i \in \mathbf{T}} \widehat{\text{DBF}_i^{\text{frd}}}(t, D_{i,1})$ changes discontinuously. Each task τ_i has exactly 3 jump points in each of the g periods when $\widehat{\text{DBF}_i^{\text{frd}}}(t, D_{i,1})$ (Eq. (19)) is used which leads to $3g$ discrete jump points at $\ell T_i + D_{i,1}$, $\ell T_i + T_i - S_i - D_{i,1}$, and $t = \ell T_i + T_i - S_i$ with $\ell = 0, 1, 2, \ldots, g-1$ for each $\tau_i \in \mathbf{T}$.[6] Let $\mathbf{P}$ be the set of all these $3g|\mathbf{T}|$ jump points of all $\tau_i \in \mathbf{T}$ and let t^* be the maximum of the points in $\mathbf{P}$. It is easy to see that $\sum_{\tau_i \in \mathbf{T}} \widehat{\text{DBF}_i^{\text{frd}}}(t, D_{i,1})$ is a linear function for $t > t^*$. Due to the condition $\sum_{i=1}^{n} U_i \leq 1$, this means that we have $\sum_{\tau_i \in \mathbf{T}} \widehat{\text{DBF}_i^{\text{frd}}}(t, D_{i,1}) \leq t$ for all $t > t^*$. In addition to testing $\sum_{i=1}^{n} U_i \leq 1$ we have to check all the time points where $\sum_{\tau_i \in \mathbf{T}} \widehat{\text{DBF}_i^{\text{frd}}}(t, D_{i,1})$ is not linear, i.e., all points in $\mathbf{P}$ which are $3g|\mathbf{T}|$ points in total. As each test has to calculate the workload up to the tested point for each of the $|\mathbf{T}|$ tasks this leads to $O(g|\mathbf{T}|^2)$ time complexity. □

In Theorem 5 we proved that a linear approximation of the demand bound functions in Eq. (3) can be calculated in $O(g|\mathbf{T}|^2)$ where $g \in \mathbb{N}^0$ is given and $|\mathbf{T}|$ is the number of tasks in the set. For a good approximation algorithm we need to give some information about the quality of the approximation with relation to the given g, i.e., an upper bound on ratio between over approximation and exact value.

THEOREM 6. *For a given integer $g \geq 1$*

$$\forall t \geq 0, \qquad \widehat{\text{DBF}_i^{frd}}(t, D_{i,1}) \leq \left(1 + \frac{1}{g}\right) \text{DBF}_i^{frd}(t, D_{i,1})$$

Proof. We know that both the exact DBFs in Eq. (1) and Eq. (2) are step functions with two steps per period, resulting in two intervals with the same value. We have to compare the value they have over this interval to the maximum value the approximated DBF takes over this value. For example, we have to compare the values of $dbf_i^1(gT_i, D_{i,1})$ with $\widehat{dbf}_i^1(gT_i + D_{i,1}, D_{i,1})$ and $dbf_i^1(gT_i + D_{i,1}, D_{i,1})$ with $\widehat{dbf}_i^1((g+1)T_i, D_{i,1})$ to conclude for Eq. (1) compared to Eq. (18a). The details can be found in [25]. □

This shows that we can use Eq. (19) to formulate Algorithm 1 as an approximation scheme for finding FRD solutions. The needed quality guarantee of $1 + \frac{1}{g}$ (in the schedulability test) follows directly from Theorem 6.

8. MIXED INTEGER LINEAR PROGRAMMING

In this section, we present a programming under logical conditions to assign the relative deadlines of the computation segments. This can be rephrased as a mixed integer linear programming (MILP). In this section, we will use the schedulability test in Theorem 5 by assuming that $g \geq 1$ is given as an integer. Moreover, let $\mathbf{L}$ be $\{0, 1, 2, \ldots, g-1\}$ for notational brevity. We can formulate the studied problem as the following programming under logical constraints:

$$\text{find a feasible solution} \tag{20a}$$

s.t.

$$0 \leq D_{i,1} \leq \frac{T_i - S_i}{2}, \qquad \forall \tau_i \in \mathbf{T} \tag{20b}$$

$$b_{i,j}^{3\ell+1} = \widehat{\text{DBF}_i^{\text{frd}}}(\ell T_i + D_{i,1}, D_{j,1}), \tag{20c}$$

$$b_{i,j}^{3\ell+2} = \widehat{\text{DBF}_i^{\text{frd}}}((\ell+1)T_i - S_i - D_{i,1}, D_{j,1}), \tag{20d}$$

$$b_{i,j}^{3\ell+3} = \widehat{\text{DBF}_i^{\text{frd}}}((\ell+1)T_i - S_i, D_{j,1}), \tag{20e}$$

$$\text{(Eqs. (20c), (20d), (20e) } \forall \tau_i \in \mathbf{T}, , \tau_j \in \mathbf{T}, \ell \in \{\mathbf{L}\})$$

$$\sum_{\tau_j \in \mathbf{T}} b_{i,j}^{3\ell+1} \leq \ell T_i + D_{i,1}, \tag{20f}$$

$$\sum_{\tau_j \in \mathbf{T}} b_{i,j}^{3\ell+2} \leq (\ell+1)T_i - S_i - D_{i,1} \tag{20g}$$

$$\sum_{\tau_j \in \mathbf{T}} b_{i,j}^{3\ell+3} \leq (\ell+1)T_i - S_i \tag{20h}$$

$$\text{(Eqs. (20f) (20g) (20h) } \forall \tau_i \in \mathbf{T}, \ell \in \{\mathbf{L}\})$$

$D_{i,1}$, $b_{i,j}^h$ are variables that can be assigned to real numbers. The variable $b_{i,j}^{3\ell+1}$ is the approximate demand bound function $\widehat{\text{DBF}_i^{\text{frd}}}(t, D_{j,1})$ of task τ_j when $t = \ell T_i + D_{i,1}$. Similarly, the variable $b_{i,j}^{3\ell+2}$ is the approximate demand bound function $\widehat{\text{DBF}_i^{\text{frd}}}(t, D_{j,1})$ of task τ_j when $t = \ell T_i + D_{i,2} = (\ell+1)T_i - S_i - D_{i,1}$. The variable $b_{i,j}^{3\ell+3}$ is the approximate demand bound function $\widehat{\text{DBF}_i^{\text{frd}}}(t, D_{j,1})$ of task τ_j when $t = \ell T_i + T_i - S_i = (\ell+1)T_i - S_i$. Therefore, the condition in Eqs. (20f), (20g), and (20h) is identical to the inequality $\sum_{\tau_j \in \mathbf{T}} \widehat{\text{DBF}_i^{\text{frd}}}(t, D_{j,1}) \leq t$ when t is $\ell T_i + D_{i,1}$, $(\ell+1)T_i - S_i - D_{i,1}$, and $(\ell+1)T_i - S_i$ for every task τ_i in $\mathbf{T}$ and $\ell = 0, 1, 2, \ldots, g-1$.

Therefore, by Theorem 5, the above programming can be used to search a feasible relative deadline assignment $D_{i,1}$ for $\tau_i \in \mathbf{T}$. The constraints except Eqs. (20c), (20d), and (20e) (due to the logical conditions inherited from Eq. (18a) and Eq. (18b)), are linear functions with respect to the variables. By adopting the well-known Big-M Method, each of the logical conditions in Eqs. (20c), (20d), and (20e) can be expressed by several linear constraints and several binary variables. Thus the above programming can be implemented as an MILP.

Please note, that the presented MILP is a spacial case of the MILP developed in parallel by Peng and Fisher in [21][7], published in RTCSA 2016.

9. EXPERIMENTAL RESULTS

We conducted experiments using synthesized task sets to evaluate the proposed approaches compared with other approaches. The metric to compare the results is measuring the *acceptance ratio* of these approaches with respect to the task set utilization. We generated 100 task sets with a cardinality of 10 tasks for each of the analyzed utilization levels that ranged from 0% to 100% with steps of 5%. The acceptance ratio of a level is the percentage of accepted task sets.

[6] The jump of Eq. (18a) at $(l+1)T_i$ is to the same value as the jump of Eq. (18b) at $t = \ell T_i + T_i - S_i$.

[7] The work in [21] was published after this paper was submitted.

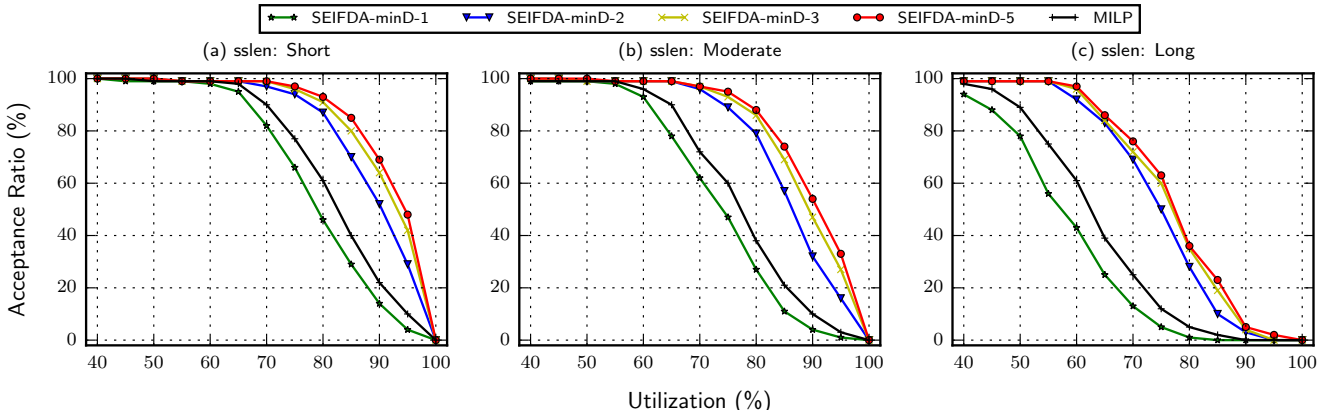

Figure 4: Impact of the g value for SEIFDA-minD under different suspension lengths (sslen) compared to MILP in Eq. (20) with $g = 1$.

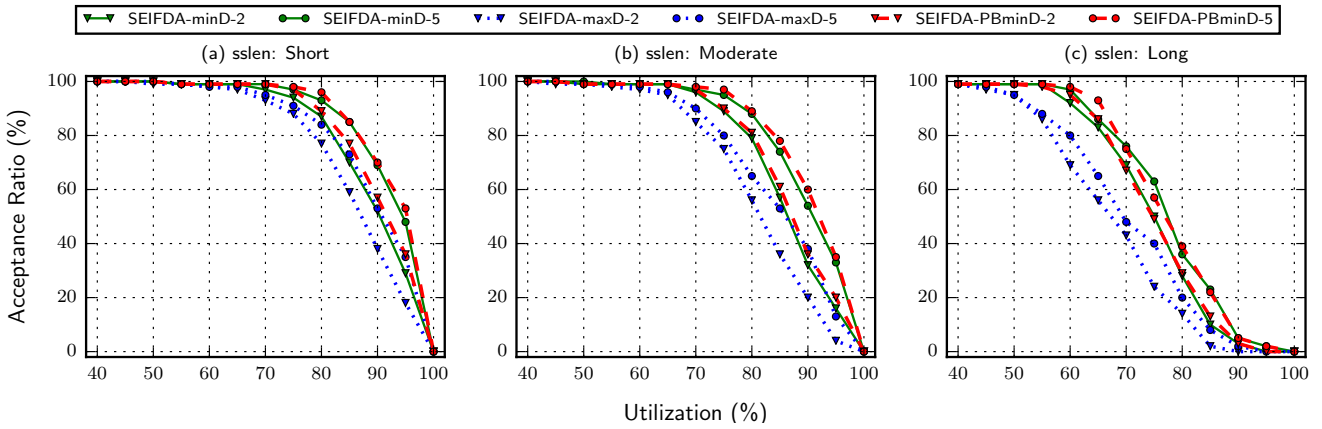

Figure 5: Comparison of the three presented approaches for SEIFDA: minD, maxD, and PBminD for g-values 2 and 5 under different suspension lengths (sslen).

For each task set we first generated a set of sporadic tasks with cardinality 10 where the UUniFast method [3] was adopted to generate a set of utilization values with the given goal. We used the approach suggested by Emberson et al. [13] to generate the task periods according to a log-uniform distribution with two orders of magnitude, i.e., $[1ms-100ms]$. Specifically, $\log_{10} T_i$ is a uniform distribution in the defined range. The execution time was set accordingly to $C_i = T_i U_i$ and the relative deadline was set to the task periods, i.e., $D_i = T_i$. We converted them to self-suspending tasks where the suspension lengths of the tasks were generated according to a uniform distribution, in either of three ranges depending on the self-suspension length (sslen):

- short suspension (sslen=Short): $[0.01(T_i - C_i), 0.1(T_i - C_i)]$
- moderate susp. (sslen=Moderate): $[0.1(T_i - C_i), 0.3(T_i - C_i)]$
- long suspension (sslen=Long): $[0.3(T_i - C_i), 0.6(T_i - C_i)]$

We then generated $C_{i,1}$ as a percentage of C_i, according to a uniform distribution, and set $C_{i,2}$ accordingly.

First, we analyzed the acceptance rate of SEIFDA with the approaches minD, maxD, and PBminD for $g \in \{1, 2, 3, 5\}$ and compared it to the MILP approach in Eq. (20) with $g = 1$ under all three types of suspension lengths. In Figure 4 these results are displayed for SEIFDA-minD. In all three subfigures we can see that SEIFDA-minD-1 already does not loose much compared to the MILP with $g = 1$ while SEIFDA-minD-2, SEIFDA-minD-3, and SEIFDA-minD-5 deliver far better results. Also the gap between SEIFDA-minD-2 and SEIFDA-minD-5 is pretty small due to our approximation scheme.[8]

Next, we compared SEIFDA-minD, SEIFDA-maxD, and SEIFDA-PBminD with each other, using $g = 2$ and $g = 5$, to evaluate the performance of the different approaches as shown in Figure 5. It can be seen that SEIFDA-minD and SEIFDA-PBminD are close to each other. SEIFDA-PBminD has better performance than SEIFDA-minD in most cases. Only for some values with long suspension length SEIFDA-minD performs slightly better. SEIFDA-minD and SEIFDA-PBminD both perform clearly better than SEIFDA-maxD. Even SEIFDA-minD-2 and SEIFDA-PBminD-2 are better than SEIFDA-maxD-5 most of the time.

After that SEIFDA-maxD-5 and SEIFDA-PBminD-5 were compared with the following scheduling approaches:

- *SCEDF*: the suspension-oblivious approach by converting suspension time into computation time.
- EDA: The state-of-the-art approach, Equal-Deadline Assignment (EDA), under linear demand bound approximations in Theorem 8 in [10].
- MILP: The proposed approach in Section 8 in this paper. Gurobi [1], a state-of-the-art MILP solver, is used to solve Eq. (20) with our manual Big-M Method.
- NC: The necessary condition in Lemma 2. We compared to the necessary condition to know how much

[8] While MILP with $g = 1$ is better than SEIFDA-minD-1, the number of variables and constraints grows quadratically with respect to g in our MILP implementation in Gurobi by using the Big-M Method while SEIFDA is linear with respect to g.

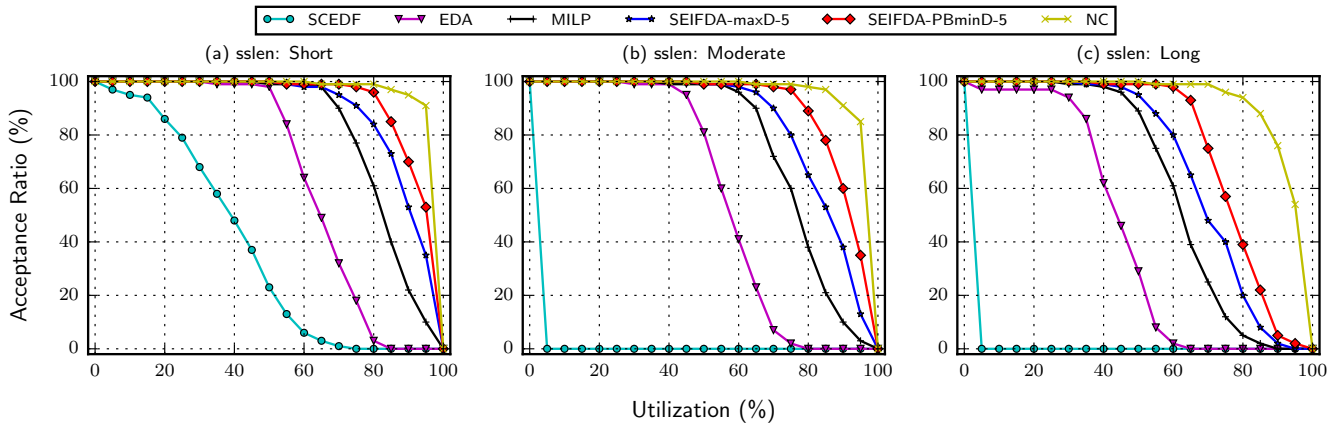

Figure 6: Comparison of SEIFDA-maxD-5 and SEIFDA-PBminD-5 with suspension-oblivious EDF (SCEDF), EDA, the MILP in Eq. (20) with $g = 1$, and the necessary condition (NC) for arbitrary algorithms (Lemma 2) under different suspension lengths (sslen).

we may lose to a theoretical optimal algorithm in the worst case.

We chose SEIFDA-PBminD-5 and SEIFDA-maxD-5 to have an idea about the performance range of SEIFDA-Algorithm. The results are shown in Figure 6. EDA is clearly outperformed by the MILP (g=1), SEIFDA-PBminD-5, and SEIFDA-maxD-5. While NC does not decrease much when the suspension length is increasing the gap of SEIFDA-PBminD-5 to NC gets larger with increasing suspension length.

10. CONCLUSION AND FUTURE WORK

In this paper, we investigate uniprocessor scheduling for hard real-time self-suspending task systems where each task may contain a single self-suspension interval. We improve the state-of-the-art by designing new FRD scheduling algorithms that yield significantly better performance than existing approaches, as shown by both analysis and experiments. As we only consider preemptive scheduling for tasks with one suspension interval on a uniprocessor system we plan to explore multiprocessor scheduling, non-preemptive scheduling and tasks with multiple suspension intervals.

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