Reservation-Based Federated Scheduling for Parallel Real-Time Tasks

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***Abstract*—Multicore systems are increasingly utilized in real-time systems in order to address the high computational demands. To fully exploit the advantages of multicore processing, possible intra-task parallelism modeled as a directed acyclic graph (DAG) must be utilized efficiently. This paper considers the scheduling problem for parallel real-time tasks with constrained and arbitrary deadlines. In contrast to prior work in this area, it generalizes federated scheduling and proposes a novel reservation-based approach. Namely, we propose a reservation-based federated scheduling strategy that reduces the problem of scheduling arbitrary-deadline DAG task sets to the problem of scheduling arbitrary-deadline sequential task sets by allocating reservation servers. We provide the general reservation design for sporadic parallel tasks, such that any scheduling algorithm and analysis for sequential tasks with arbitrary deadlines can be used to execute the allocated reservation servers of parallel tasks. Moreover, the proposed reservation-based federated scheduling algorithms provide constant speedup factors with respect to any optimal scheduler for arbitrary-deadline DAG task sets. We demonstrate via numerical and empirical experiments that our algorithms are competitive with the state of the art.**

***Index Terms*—Parallel Real-Time Tasks, DAG, Federated Scheduling, Partitioned Scheduling, Servers**

I. Introduction

Modern real-time systems increasingly facilitate multicore systems to account for the growing computational demands of real-time applications. In uniprocessor platforms, the execution-demand of sequential real-time tasks is solely modeled by their worst-case execution times, since the processor executes only one job at each point in time and thus there is no need to express potential parallel execution paths. In contrast, multicore platforms allow *inter-task parallelism*, i.e., to execute sequential programs concurrently, and *intra-task parallelism*, i.e., to execute a job of a parallelized task on multiple processors (cores) at the same time. To enable intra-task parallelism, potentially parallel execution of an application must be considered at design time. As demonstrated by Serrano et al. [38], the asynchronous parallel task model, represented as a directed acyclic graph (DAG) where each thread corresponds to a node and the edges denote precedence constraints, can be realized using the untied task model of OpenMP. Due to this fact and the increasing OpenMP support by newly developed multicore processors [38], the DAG task model is a reasonable parallel real-time task model to be used for the design of scheduling algorithms.

A recent approach for scheduling parallel real-time DAG tasks is *federated scheduling* by Li et al. [31], where *heavy* tasks, i.e., tasks that need to execute on more than one processor simultaneously in order to meet their relative deadlines, are assigned a set of processors exclusively, whilst the *light* tasks are sequentialized and scheduled on the remaining processors. The exclusive assignment allows a simple response-time analysis due to the absence of inter-task interference and contiguous service provided to the DAG task by the assigned processors. Additionally, limiting the number of processors that a DAG task can execute on results in lower synchronization and scheduling overheads, and the non-preemption of the heavy tasks has potential cache advantages. On the other hand, the exclusive granting of processors to a heavy task can potentially waste many system resources when a job of the task completes before the release of the next one, especially for constrained- and arbitrary-deadline tasks (an example is provided in [16]).

To address these limitations of federated scheduling, whilst maintaining the favorable analytical properties of absent inter-task interference, we provide the following results:

- We propose a novel **reservation-based federated scheduling** approach for scheduling sporadic DAG task sets with arbitrary deadlines in Section IV. That is, each DAG task is assigned a set of dedicated reservation servers that inherit the timing models of the associated DAG task. These reservation servers can then be scheduled as sequential tasks by any multiprocessor scheduling algorithm that supports such timing models.
- We provide constraints and design rules for the dimensioning and assignment of the reservation servers, such that each DAG task is schedulable if all its reservation servers are schedulable under any multiprocessor scheduling algorithm that has a corresponding analysis guaranteeing the schedulability of the aforementioned servers. Specifically, in Section V we present provably sufficient reservations and different reservation assignment algorithms. One of them, to the best of our knowledge, is the first algorithm for scheduling DAG task sets with arbitrary deadlines that has a constant speedup factor.
- Additionally, we resolve the problem of non-constant speedup factors for arbitrary-deadline DAG task sets under federated scheduling with respect to any optimal DAG task set scheduling algorithm as pointed out by Chen [16]. We show that the speedup factor of our approach is at most $3 + 2\sqrt{2}$ by the design of a specific set of reservation servers that are scheduled under partitioned

and global multiprocessor scheduling in Sections VI and VII, respectively. We further improve the reservation assignment algorithm in Section VIII. Note that our treatment for arbitrary-deadline task systems may result in different jobs from a task being executed at the same time since our reservation-based strategies provide the reservation immediately when a DAG job arrives.
- Finally, we conduct various numerical and empirical experiments in Section IX and show that our algorithms are comparable to the state of the art for implicit-deadline tasks. In addition, our algorithms have the capability of handling arbitrary-deadline tasks and scheduling constrained-deadlines tasks with stringent timing requirement efficiently.

II. Related Work

In this section, we review the prior work that is most related to this research. The problem of scheduling parallel real-time DAG tasks has been broadly researched. To the best of our knowledge, three types of scheduling approaches exist.

Decomposition-based strategies: A DAG task is decomposed into a set of sequential subtasks with specified relative deadlines and offsets of their release times, where the internal dependencies are maintained by the decomposition. These sequential subtasks are then scheduled accordingly without considering the DAG structure anymore, e.g., [27]–[29], [36], [37]. Decomposition-based strategies must utilize the DAG structure off-line in order to apply the decomposition.

Scheduling without any treatment: None of the parameters of a task nor its DAG structure is used at all when making the scheduling decisions. The subtasks of a job have the same static or dynamic priority as the job. Whenever a subtask is ready to be executed, the standard global or partitioned multiprocessor scheduling is used to schedule the subtasks, e.g., [2], [14], [30], [31]. Moreover, when the DAG task is further modelled with conditional branches, global scheduling has been also studied in [6], [34], [35].

Federated scheduling: The task set is partitioned into light and heavy tasks based on their utilizations. Light tasks are those that can be completely sequentialized and still meet their deadlines on one processor. Heavy tasks are those that need more than one processor to meet their deadlines. In the original design of federated scheduling for implicit-deadline task systems proposed by Li et al. [31], a light task is solely executed *sequentially* without exploiting the parallelized structure, and a heavy task is assigned to its designated processors that *exclusively* execute only this heavy task. Jiang et al. [26] extended this approach to semi-federated scheduling, in which one or two processors, used by a heavy task, can be shared with other tasks. Baruah [3]–[5] adopted the concept of federated scheduling for scheduling constrained- and arbitrary-deadline task systems. However, Chen [16] later showed that federated scheduling does not have any constant speedup factor with respect to the optimal scheduling algorithm for constrained- and arbitrary-deadline tasks. As discussed earlier, despite the advantages of federated and semi-federated scheduling, it cannot fully utilize the processors when tasks have constrained or arbitrary deadlines due to the dedicated processors assigned to heavy tasks. Chen in [16] provided a concrete task set to further explain this issue.

A related but different problem that has been extensively studied is the hierarchical scheduling for real-time tasks in virtual machines on multiprocessors [12], [15], [33], [39], [41]. Most of these works model this problem using the compositional analysis framework where virtual processors of a virtual machine are abstracted using interfaces with certain guaranteed capacity, and the schedulability of tasks running within each component can be analyzed independently. Although some of these interfaces are also named as resource reservations or servers, their focus is on first designing an appropriate interface that can accurately describe the virtual resources yet is simple to use and then providing schedulability tests to tasks over that interface. In addition, none of these works considers parallel real-time tasks.

III. Parallel Real-Time Task Model

This work considers the real-time scheduling problem on a multiprocessor system comprised of M homogeneous (identical) processors. We adopt the widely used sporadic task model to describe the real-time tasks, where a task τ_i is characterized by its relative deadline D_i, its period (minimum inter-arrival time) T_i, and its worst-case execution time (WCET) C_i. A sporadic task is an infinite sequence of task instances, referred to as *jobs*, where the arrival of two consecutive jobs of a task is separated at least by its minimum inter-arrival time, i.e., the arrival rate is limited. To fulfill its timing requirement, a job of task τ_i must finish at most C_i units of computation between the arrival of a job at t_a and that jobs absolute deadline at $t_a + D_i$. A sporadic task system $\mathbf{T}$ is called an ***implicit-deadline*** system if $D_i = T_i$ holds for each τ_i in $\mathbf{T}$ and is called a ***constrained-deadline*** system if $D_i \leq T_i$ holds for each τ_i in $\mathbf{T}$. Otherwise, $\mathbf{T}$ is an ***arbitrary-deadline*** task system.

Through out this paper, we focus on how to schedule parallel real-time tasks that have internal parallelism and allow subtasks to be executed simultaneously on multiple processors. An established model for parallel tasks is the Directed-Acyclic-Graph (DAG) model, where the execution of task τ_i is divided into subtasks and the precedence constraints of these subtasks are defined by a DAG structure. Each node of a DAG represents a subtask that can be executed whenever all precedence constraints are met, i.e., all directly incident jobs finished their executions.

Two parameters are used to characterize a DAG task τ_i:

- ***Total execution time (or work)*** C_i: the summation of the worst-case execution times of all the subtasks of task τ_i.
- ***Critical-path length*** L_i: the length of the critical path in the given DAG, i.e., the worst case execution time of the task on an infinite number of processors.

By definition, $C_i \geq L_i > 0$ for every task τ_i. Furthermore, the *utilization* of τ_i is denoted by $U_i = \frac{C_i}{T_i}$.

The abstraction of using work and critical-path length to describe the DAG has the advantage that it is completely agnostic of the internal parallelization structure, i.e., how many subtasks exist and how the precedence constraints amongst them are. Scheduling algorithms that can feasibly schedule DAG task sets solely based on these two parameters also allow a task to change the DAG structure during runtime as long as those parameters persist. The apparent downside of this abstraction is the pessimism since the worst possible structure has to be considered regardless of the actual structure, i.e., the scheduling algorithms have to suffice the tasks deadline for all possible structures under given parameter constraints.

We assume that the worst-case execution time already covers the worst-case combination of the interference in the memory and bus contention, similar to all the approaches in this research line to schedule DAG tasks, e.g., [26], [31], [35]. We note that such a safe bound may introduce pessimism into the analysis.

IV. Reservation-Based Federated Scheduling

In this paper, we propose to use reservation-based allocation instead of exclusive allocation for parallel tasks with constrained or arbitrary deadlines. That is, instead of dedicating a few processors to a heavy task, we assign a few reservation servers to a heavy task. The timing properties of a DAG task will be guaranteed as long as the corresponding reservations can be guaranteed to be feasibly provided. We now describe the basic ideas behind reservation-based federated scheduling for DAG tasks and some constraints on the reservation design.

A. Reservation Server Design

The reservation-based federated scheduling is a natural generalization of the federated scheduling approach, where sufficient exclusive processor time budgets (but not necessarily entire processors) are reserved for heavy tasks to execute potentially in parallel and to meet their deadlines. Therefore, it is a *hierarchical scheduling* strategy: scheduling a DAG task inside the reserved resources, namely reservation servers, and scheduling the reservation servers on the multiprocessors.

In Federated Scheduling, heavy tasks are allocated to processors exclusively based on the DAG task's density, which may result in very low utilization of the processors in constrained-deadline task systems due to DAG tasks with high density and low utilization, i.e., the density C_i/D_i is large compared to the utilization C_i/T_i. Thus, the provided 100% utilization of each allocated processor can not be benefited from.

In consequence, speeding up processors and thus decreasing the DAG task's execution-time does not improve schedulability if the system can not provide a sufficient number of processors. This issue is further explained in [16]. Contrary, in our reservation-based approach, we relate the number of in-parallel required service and minimal-sequential service, and are thus able to decouple reservation-server utilization (with a constant inflation factor γ) in trade for more in-parallel service. By enforcing this constant inflation factor for each heavy task, we can relate the reservation demands to constant speedup factors.

In reservation-based scheduling, each DAG task τ_i gets m_i sequential ***reservation servers*** with ***budgets*** (service provisioning) $E_{i,1},..,E_{i,\ell},..,E_{i,m_i}$. We enforce that the reservation servers are synchronous with the release of a DAG task's job and the reservation servers corresponding to a DAG task τ_i have the same relative deadline and inter-arrival time as τ_i. This means that the release pattern of a reservation server is inherited from the DAG task and whenever a DAG task releases a job at t_r, the associated service is provided in the release- and deadline-interval $[t_r, t_r + D_i)$.

We now formally define the reservation server as follows:

Definition 1: The ℓ-th reservation server $\tau_{i,\ell}$ for serving a DAG task τ_i is defined by the tuple $(E_{i,\ell}, D_i, T_i)$, such that $E_{i,\ell}$ amount of service (computation time) is provided to the DAG task τ_i over the interval $[t_r, t_r + D_i)$ with a minimum inter-arrival time of T_i. $\square$

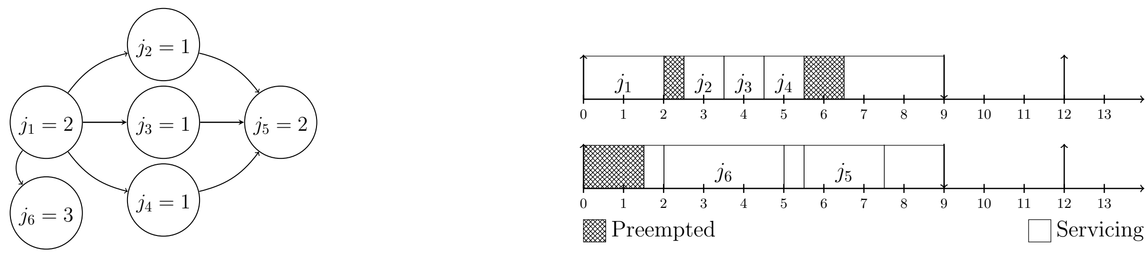
In order to analytically treat a reservation server as if it is a sporadic sequential task, we enforce the reservation to be active, i.e., eligible for scheduling, until the whole runtime budget is depleted. This also allows us to schedule multiple reservation servers in parallel instead of sequencing multiple pending servers of the same tasks in a first-in first-out manner. However, the reservation servers that are released at time t_r by a task τ_i are used to serve the DAG job of task τ_i that arrived at time t_r exclusively. This implies that multiple consecutive DAG jobs may be executing in parallel. Note that we do not restrict any reservation server to service certain subtasks of a task (nodes in the DAG) exclusively. Instead, reservation servers of the same task are eligible to service any subtask as long as they belong to the job that initiated the activation of the reservations. Each DAG job is serviced by *list scheduling* — when a reservation of a job is active, it can execute any ready node of the DAG job. List scheduling is work-conserving — namely, at every point in time in which the DAG job has pending workload and the system provides service, some node is executing.

As long as the assigned server budgets are sufficient for DAG tasks to meet their deadlines, the system can use any scheduling algorithm and schedulability test to schedule these sequential servers on a homogeneous multiprocessor system with M processors. The exact time of service, i.e., the schedule of the reservation servers, is determined by the applied multiprocessor scheduling algorithm. Therefore, under reservation-based federated scheduling the problem of scheduling DAG task sets and the analysis thereof is divided into the following two problems:

1) Scheduling of sporadic constrained- or arbitrary-deadline reservation servers to satisfy the budget requirement.
2) Assigning provably sufficient budget to service an arbitrary DAG task.

The reservation concept is illustrated in Figure 1b, where two reservations are partitioned on two different processors and scheduled according to an arbitrary scheduling algorithm. The two reservation servers provide 7.5 time-units of runtime budget (as computed by the *R-MIN* algorithm, explained later) over the interval $[0, 9)$ in parallel to service the DAG task shown in Figure 1a. To improve readability, higher priority tasks or reservation servers in the system are not included in Figure 1b. Instead, the resulting preemptions and the servicing of the reservations are denoted by the different hatchings.

If a reservation is scheduled to execute but no subjob of the



(a) An example DAG

(b) Arbitrary schedule with two reservations

Fig. 1: **(a)** A sporadic, constrained-deadline DAG task τ_i with $C_i = 10$, $L_i = 5, D_i = 9, T_i = 12$.
(b) An arbitrary schedule of two equal reservations, as computed by the *R-MIN* algorithm. The DAG task shown in Figure 1a is serviced according to the *list-scheduling* algorithm by any reservation that does not service an unfinished job at that time. Both reservations provide 7.5 units of service over the interval $[0, 9)$ on two processors in parallel. The white areas denote that the reservation is scheduled, i.e., serving a job of the corresponding DAG task whereas the hatching denotes the preemption of the reservation. The reservation server may be servicing, i.e., executed by the scheduler, without serving any DAG subjob due to unmet precedence constraints as is indicated by the empty slots during the service intervals.

served DAG task is eligible, the reservation spins. For instance at time $t = 5.5$ on the second processor, no subjob is eligible to be serviced, due to unmet precedence constraints. In this case, the associated scheduled reservation spins. Note that the total reservation of 15 necessarily exceeds the work of the DAG job 10 since the reservation must guarantee that the job completes regardless of how the reservation servers are scheduled by the underlying scheduling algorithm.

B. Sufficient Conditions for Reservation Server Design

To guarantee the schedulability of a DAG task within its reservation servers, we must quantify the processor time budgets that are always sufficient to schedule a DAG task over an interval. Here, we derive conditions for designing reservation servers for DAG tasks. In particular, we will show that any design that satisfies the two conditions in the following theorem guarantees that the DAG job is completed by its deadline.

Theorem 1: Suppose that m_i sequential reservation servers with budgets $E_{i,1}, E_{i,2}, .., E_{i,m_i}$ are assigned to serve a DAG task τ_i. The job of task τ_i arrived at time t_0 can be finished no later than its absolute deadline $t_0 + D_i$ if the two following conditions are satisfied:

- **[Schedulability Condition]**: the m_i reservation servers can be guaranteed to finish no later than their absolute deadline at $t_0 + D_i$;
- **[Reservation Condition]**: $C_i + L_i \cdot (m_i - 1) \leq \sum_{j=1}^{m_i} E_{i,j}$.

Proof: We consider an arbitrary execution schedule S of the m_i servers to complete their budgets from t_0 to $t_0 + D_i$. Suppose, *for contradiction*, that the schedulability and reservation conditions hold but that the DAG job of task τ_i released at time t_0 misses its deadline at time $t_0 + D_i$ in the schedule S.

Since the list scheduling algorithm is applied, the DAG job is executed within the reservation servers in a work-conserving manner — at each time step in S each executing server will process one ready node of the DAG job. Therefore, if there is some server idling at a time step, all the ready nodes must be processed by some of the executing servers.

During S, we define a time step as a *complete step* if the number of servers executing is no more than the number of ready nodes. Hence, in these complete steps all the executing servers are processing useful work of the job. In contrast, we define a time step is *incomplete* if the number of executing servers is more than the number of ready nodes, i.e., there is at least one executing server idling. We denote the number of incomplete steps during the interval from t_0 to $t_0 + D_i$ as x.

Since the job is not finished at $t_0 + D_i$, there is at least one ready node at any time step. By the definition of incomplete time steps and critical-path length, if $x > L_i$, in these incomplete steps the job must have finished all of its nodes, which leads to contradiction. Thus, it must be the case that $x \leq L_i$. In the worst case, i.e., solely sequential execution, in each of the incomplete step all the m_i servers are executing and $m_i - 1$ servers are idling. Since $x \leq L_i$, we know that the amount of server idling time is at most $(m_i - 1)x \leq (m_i - 1)L_i$.

Note that by the schedulability condition, the total amount of execution budgets the reservation servers get in the schedule S is $\sum_{j=1}^{m_i} E_{i,j}$. Among them, only at most $(m_i - 1)L_i$ are idle. Therefore, the amount of useful work these servers complete is at least $\sum_{j=1}^{m_i} E_{i,j} - (m_i - 1)L_i$.

By the reservation condition, we know that $\sum_{j=1}^{m_i} E_{i,j} - (m_i - 1)L_i \geq C_i$. This means that the reservation servers process more work than the worst-case execution time of the job, which leads to contradiction. Therefore, the job must have completed by its deadline. ■

Note that in the above theorem the reservation server design is independent of the actual structure of the DAG job — it depends only on the worst-case work C_i and critical-path length L_i of the job. The following lemma argues that providing a reservation server with $E_{i,j*} < L_i$ is never useful.

Lemma 2: Let $E_{i,1}, E_{i,2}, \cdots, E_{i,m_i}$ denote a set of sufficient reservation budgets for a DAG task τ_i, i.e., $\sum_{j=1}^{m_i} E_{i,j} \geq C_i + L_i(m_i - 1)$. If there exists a reservation server with budget $E_{i,j^*} < L_i$, then removing this reservation already provides sufficient budget with one less reservation server and less cumulative budget.

Proof: By assumption, we know that $\sum_{j=1}^{m_i} E_{i,j} - E_{i,j^*} + E_{i,j^*} \geq C_i + L_i(m_i - 1)$. Simple arithmetic yields $\sum_{j=1}^{m_i} E_{i,j} - E_{i,j^*} + \geq C_i + L_i(m_i - 1) - E_{i,j^*}$. By the property that $E_{i,j^*} < L_i$, it must be that $\sum_{j=1}^{m_i} E_{i,j} - E_{i,j^*} + \geq C_i + L_i(m_i - 1) - E_{i,j^*} > C_i + L_i(m_i - 2)$. ∎

From now on, we will implicitly assume that $E_{i,j} \geq L_i \ \forall j$ due to Lemma 2. In particular, we define the ***strech ratio*** $\gamma_{i,j}$ for each reservation server and let $E_{i,j} \stackrel{\text{def}}{=} \gamma_{i,j} L_i$ with $1 < \gamma_{i,j} \leq \frac{D_i}{L_i}$. Subsequently, we have the following corollary.

Corollary 3: Suppose that m_i sequential reservation servers with budgets $\{\gamma_{i,1}L_i, \gamma_{i,2}L_i, .., \gamma_{i,m_i}L_i\}$ are assigned to serve a DAG task τ_i. Task τ_i is feasible if the budget assignment satisfies:

$$L_i \cdot (m_i - 1) + C_i \leq \sum_{j=1}^{m_i} \gamma_{i,j} \cdot L_i \tag{1a}$$

$$\gamma_{i,j} \cdot L_i \leq D_i \ \forall 1 \leq j \leq m_i \tag{1b}$$

$$\gamma_{i,j} > 1 \ \forall 1 \leq j \leq m_i \tag{1c}$$

Eq. (1) has the following properties. First, the equation depends only on the task parameters C_i, L_i, D_i as well as the number of reservation servers m_i. Second, as m_i increases, the sum of execution requirements $\sum_{j=1}^{m_i} \gamma_{i,j} \cdot L_i$ also increases. Therefore, there are many possible feasible designs of reservation servers for the same DAG task. The best reservation assignment depends on the scheduling algorithm for the reservation servers and the corresponding analysis used to verify schedulability of the servers.

V. Reservation Server Assignment Algorithms

In this section, we describe two simple algorithms for calculating reservation budgets for DAG tasks which both have advantages and disadvantages. The first algorithm is the direct generalization of federated scheduling core allocation. The second algorithm is designed so that we can easily prove the speedup bounds for the reservation-based federated scheduling in Sections VI and VII.

In principle, from Theorem 1 and Corollary 3 we know that the reservation servers of the same task τ_i could potentially have different budgets $E_{i,j}$ and stretch ratio $\gamma_{i,j}$. However, in the two simple server assignment algorithms, we compute the reservation budgets by enforcing ***equal-reservation*** E_i and ***equal stretch ratio*** γ_i for task τ_i. Therefore, the conditions for feasible reservation servers can be solved analytically to $L_i \cdot (m_i - 1) + C_i \leq \gamma_i \cdot m_i \cdot L_i$, which yields $\frac{C_i - L_i}{L_i \cdot (\gamma_i - 1)} \leq m_i$. Note that the notation of $\gamma_{i,j}$ from now on changed to γ_i due to the equality of all m_i reservation servers.

Since the number of reservation servers must be a natural number, we know that the smallest number of reservation servers m_i required under the equal-reservation constraint given a stretch ratio γ_i is

$$m_i \stackrel{\text{def}}{=} \left\lceil \frac{C_i - L_i}{L_i \cdot (\gamma_i - 1)} \right\rceil \tag{2}$$

A. R-MIN: *Minimum Reservation Budgets*

The intuition behind the first algorithm, namely *R-MIN*, is similar to the original federated scheduling core allocation, which is to assign the minimum reservation budgets that can still guaranteed the schedulability of tasks. Specifically, *R-MIN* classifies tasks into light and heavy tasks based on whether a task requires to be serviced by more than one reservation server or not. Based on the classification, the algorithm assigns each light task one reservation server with a budget identical to the work of the task. It assigns each heavy task the minimum number of reservation servers $m_i = \left\lceil \frac{C_i - L_i}{D_i - L_i} \right\rceil$ using the equal-reservation constraint. The servers of the same task is assigned the minimum stretch ratio $\gamma_i = 1 + \frac{C_i - L_i}{m_i L_i}$, i.e., the minimum budget, while guaranteeing the schedulability of the task.

Therefore, the *R-MIN* has the following properties:

Theorem 4: The *R-MIN* algorithm generates a minimal number of sporadic equal-reservation servers with minimum budgets for a given sporadic constrained- or arbitrary-deadline DAG task set that provide sufficient resources to service their respective DAG tasks.

Proof: First, the minimum m_i gives the minimum total execution requirements $\sum_{j=1}^{m_i} \gamma_i \cdot L_i$ in inequality (1a). In equal-reservations, the left-hand side of the above equation (2) is minimized if γ_i is maximized, i.e., $m_i = \left\lceil \frac{C_i - L_i}{D_i - L_i} \right\rceil$, and the corresponding smallest γ_i that achieves an equally minimal number of reservations is given by $1 + \frac{C_i - L_i}{m_i L_i}$. ∎

The R-MIN strategy is a generalized approach of federated scheduling in the sense that an instance of R-MIN can be transformed into an instance of federated scheduling by inflating the assigned reservation budgets E_i to T_i (for implicit-deadline systems) such that each heavy DAG task is reserved 100% processor utilization exclusively. One can consider R-MIN as a resource optimized version of federated scheduling.

The R-MIN strategy assigns the minimum total budgets to all the tasks, leaving the minimum idling time inside the reservation servers. On the flip side, this strategy is the closest to federated scheduling and inherits its disadvantages. First, each task's reservation is calculated completely independently from all the other jobs in the system. Second, each reservation may be "tight", i.e., $E_{i,j}$ may be very close to D_i. Both these properties may make it difficult to ensure that the assigned servers of a task set are schedulable even though individual tasks require their minimum budgets. This strategy is unlikely to provide a constant speedup bound for the same reason that federated scheduling does not admit a constant speedup bound for constrained-deadline tasks. Therefore, we will now look at a different strategy which potentially increases the number of reservation servers for each task, but each reservation server may have a smaller execution requirement compared to *R-MIN*.

B. R-EQUAL: *Equal Stretch Ratio for All Tasks*

As mentioned above, one of the reasons *R-MIN* may generate unschedulable reservation servers is that each job calculates its own stretch ratio which can be as large as possible for that job. Jobs that have large stretch ratio γ_i have little slack, i.e., their reservation $E_i = \gamma_i L_i$ is very close to D_i. We now present an algorithm *R-EQUAL* that uses a single common stretch ratio γ across all tasks in Algorithm 1.

In algorithm *R-EQUAL*, all DAG tasks are classified into heavy and light tasks based on whether $C_i > \gamma L_i$ for a single common stretch ratio γ , i.e., whether they require more than one reservation (as generated by the algorithm) to be feasibly serviceable or not based on γ. Since γ must be valid for all DAG tasks, i.e., $L_i < \gamma_i L_i \leq D_i$, we know that $1 < \gamma \leq \min_{\tau_i}\{\frac{D_i}{L_i}\}$. Based on the classification, the algorithm assigns each light task one reservation server with the same budget as the work of the task. It assigns each heavy task $m_i = \left\lceil \frac{C_i - L_i}{L_i(\gamma-1)} \right\rceil$ reservation servers given the same stretch ratio γ.

It turns out that *R-EQUAL* provides good analytical results and allows us to prove speedup bounds. It does not, however, lead to heuristically *good* reservations as will be seen in the evaluations. Thus, in Section VIII we present an improved server assignment algorithm that can exploit the properties of the scheduling algorithm applied to the servers.

In the following theorem, we prove an important property of R-EQUAL servers, which is used to prove the speedup bounds for reservation-based federated scheduling.

Theorem 5: Suppose that $\gamma > 1$ is given and there are exactly m_i reservation servers for task τ_i where $m_i \stackrel{\text{def}}{=} \left\lceil \frac{C_i - L_i}{L_i(\gamma-1)} \right\rceil$ with $m_i \geq 2$. If $C_i' = \sum_{j=1}^{m_i} E_{i,j} = C_i + (m_i - 1)\cdot L_i$ and $\gamma = \frac{\sum_{j=1}^{m_i} E_{i,j}/L_i}{m_i}$, then $C_i' \leq (1 + \frac{1}{\gamma-1})\cdot C_i$.

Proof: By the assumption $L_i > 0$ and $\gamma > 1$, the setting of $m_i = \left\lceil \frac{C_i - L_i}{L_i(\gamma-1)} \right\rceil$ implies that

$$m_i - 1 < \frac{C_i - L_i}{L_i(\gamma - 1)} \leq m_i \tag{3}$$

$$\Rightarrow \quad (m_i - 1)(\gamma - 1)L_i < C_i - L_i \leq m_i L_i(\gamma - 1) \tag{4}$$

$$\Rightarrow \quad C_i + (m_i - 1)L_i \leq m_i\gamma L_i < C_i + (m_i + \gamma - 2)L_i \tag{5}$$

Because $\gamma > 0$, Eq. (5) implies $(m_i - 1)L_i < \frac{C_i + (m_i-2)L_i}{\gamma}$. Since $C_i' = C_i + (m_i - 1)L_i$ by definition, we know

$$\begin{aligned} C_i' < \;& C_i + \frac{C_i + (m_i - 2)L_i}{\gamma} \\ <_1 \;& \frac{C_i(\gamma+1)}{\gamma} + \frac{(m_i-2)}{\gamma}\left(\frac{C_i}{(m_i-1)(\gamma-1)}\right) \\ \leq_2 \;& C_i\left(\frac{\gamma+1}{\gamma} + \frac{1}{\gamma^2 - \gamma}\right) \\ = \;& \left(1 + \frac{1}{\gamma - 1}\right)\cdot C_i \end{aligned}$$

where $<_1$ is due to $L_i < \frac{C_i}{(m_i-1)(\gamma-1)+1} < \frac{C_i}{(m_i-1)(\gamma-1)}$ by reorganizing the condition in Eq. (4) and $\leq_2$ is due to $m_i \geq 2$ and $\frac{m_i-2}{m_i-1} \leq 1$. ■

VI. Partitioned Scheduling for Reservation Servers

This section analyzes the theoretical properties of reservation-based federated scheduling when the reservation servers are scheduled under multiprocessor partitioned scheduling. Using the *R-EQUAL* algorithm and Theorem 5, a worst-case setting can be derived to prove a constant speedup factor of reservation-based federated scheduling with respect to an optimal DAG task scheduling algorithm.

Algorithm 1 R-EQUAL Algorithm

Require: Sporadic arbitrary-deadline DAG task set and γ-value
Ensure: Set of reservations, that can provably service all DAG tasks sufficiently

$\mathbf{T}_{\text{HEAVY}} \leftarrow \{\tau_i \in \mathbf{T} \mid C_i > \gamma \cdot L_i\}$
$\mathbf{T}_{\text{LIGHT}} \leftarrow \{\tau_i \in \mathbf{T} \mid C_i \leq \gamma \cdot L_i\}$
$\mathbf{T} \leftarrow \mathbf{T}_{\text{LIGHT}}$
for each task τ_i in $\mathbf{T}_{\text{HEAVY}}$ **do**
  $m_i \leftarrow \left\lceil \frac{C_i - L_i}{L_i(\gamma-1)} \right\rceil$
  for $\ell \leftarrow 1$ to m_i **do**
    $\tau_{i,\ell} \leftarrow (\gamma L_i, D_i, T_i)$
    $\mathbf{T} \leftarrow \mathbf{T} \cup \{\tau_{i,\ell}\}$
  end for
end for
return $\mathbf{T}$

There have been analyses for the speedup bounds of different scheduling algorithms for ordinary sequential real-time tasks under partitioned scheduling. In particular, Baruah and Fisher developed a greedy heuristic in [9].

Definition 2: Under *Deadline-Monotonic Partitioning (DMP)*, sequential tasks are considered in a non-decreasing order of their relative deadlines. When a task τ_k is considered in this order, if task τ_k and the other *previously assigned* tasks on a processor can be feasibly scheduled under the specified scheduling strategy, then task τ_k is assigned to one of such processors (if there are more than one). Otherwise, task τ_k is assigned to a newly allocated processor. □

This strategy is further analyzed by Chen et al. in [17], [18] and proved to have a speedup bound of 3. Note that DMP only specifies the order of the tasks to be considered and assigned. The underlying uniprocessor scheduling strategy after task partitioning can be arbitrary, e.g., EDF or deadline-monotonic fixed-priority scheduling.

Since we consider DMP in this section, we index the tasks such that $D_i \leq D_j$ if $i \leq j$. Before we prove the competitiveness of reservation-based federated scheduling under DMP, we first briefly restate the schedulability tests used for the partitioned scheduling of arbitrary-deadline task sets, that are used in this paper for scheduling server tasks.

Suppose that the partitioning algorithm attempts to assign a server task τ_k with budget E_k to a processor m and $\mathbf{T}_m$ is the set of higher-priority server tasks that are already assigned on processor m. When considering arbitrary-deadline tasks on uniprocessor, Fisher, Baruah, and Baker [22] provided the following approximated schedulability test:

$$E_k + \sum_{\tau_i \in \mathbf{T}_m}\left(1 + \frac{D_k}{T_i}\right)E_i \leq D_k \quad \text{and} \tag{6a}$$

$$U_k + \sum_{\tau_i \in \mathbf{T}_m} U_i \leq 1 \tag{6b}$$

Bini et al. [13] provided a tighter schedulability test as follows:

$$E_k + D_k(\sum_{\tau_i \in \mathbf{T}_m} U_i) + \sum_{\tau_i \in \mathbf{T}_m} E_i - \sum_{\tau_i \in \mathbf{T}_m} U_i E_i \leq D_k \tag{7a}$$

$$U_k + \sum_{\tau_i \in \mathbf{T}_m} U_i \leq 1 \tag{7b}$$

With these analyses, we now prove that DMP and reservation-based federated scheduling can yield constant speedup bounds. To prove the speedup bound, we incorporate the over-provisioning of the generated reservation servers with respect to the worst-case execution time of the original DAG tasks.

Theorem 6: A system of arbitrary-deadline DAG tasks scheduled by ***reservation-based federated scheduling under DMP***, in which each processor uses deadline-monotonic fixed-priority scheduling for the reservation servers, has a constant speedup factor of $3+2\sqrt{2}$ with respect to any optimal scheduler by setting γ to $1+\sqrt{2}$.

Proof: We prove this by adopting *R-EQUAL* (c.f. Algorithm 1) with a setting of $\gamma = 1+\sqrt{2}$. Suppose, *for contradiction*, that reservation-based federated scheduling under DMP is given a speedup of $\alpha > 3+2\sqrt{2}$ compared to the optimal scheduler but still fails to schedule the DAG tasks. By the definition of R-EQUAL and Corollary 3 we know that it fails to schedule the DAG tasks if and only if DMP cannot partition all the assigned reservation servers.

Since we apply DMP, we index the tasks such that $D_i \leq D_j$ if $i \leq j$. Suppose that $\tau_{k,\ell}$ is a reservation server assigned to the DAG task τ_k that is not able to be partitioned to any of the given M processors, where $1 \leq \ell \leq m_k$. Let $\mathbf{M}_1$ be the set of processors in which Eq. (6a) fails. Let $\mathbf{M}_2$ be the set of processors in which Eq. (6a) succeeds but Eq. (6b) fails. Since $\tau_{k,\ell}$ cannot be assigned on any of the M processors, we have $|\mathbf{M}_1| + |\mathbf{M}_2| = M$.

By the violation of Eq. (6a), we know that

$$
\begin{aligned}
&|\mathbf{M}_1| E_{k,\ell} + \sum_{m\in\mathbf{M}_1} \sum_{\tau_{i,j}\in\mathbf{T}_m} \left(1+\frac{D_k}{T_i}\right) E_{i,j} > |\mathbf{M}_1| D_k \\
\Rightarrow &|\mathbf{M}_1| \frac{E_{k,\ell}}{D_k} + \sum_{m\in\mathbf{M}_1} \sum_{\tau_{i,j}\in\mathbf{T}_m} \left(\frac{E_{i,j}}{D_k}+\frac{E_{i,j}}{T_i}\right) > |\mathbf{M}_1| \quad (8)
\end{aligned}
$$

By the violation of Eq. (6b), we know that

$$
|\mathbf{M}_2| \frac{E_{k,\ell}}{T_k} + \sum_{m\in\mathbf{M}_2} \sum_{\tau_{i,j}\in\mathbf{T}_m} \frac{E_{i,j}}{T_{i,j}} > |\mathbf{M}_2| \quad (9)
$$

Due to the definition $\sum_{j=1}^{m_i} E_{i,j} = C'_i$, and the fact that $\tau_{i,j}$ is assigned either on a processor of $\mathbf{M}_1$ or on a processor of $\mathbf{M}_2$ if $\tau_{i,j}$ is assigned successfully prior to $\tau_{k,\ell}$, by taking the summation of Eqs. (8) and (9) we know that

$$
M \frac{E_{k,\ell}}{\min\{T_k, D_k\}} + \sum_{i=1}^{k} \left(\frac{C'_i}{T_i} + \frac{C'_i}{D_k}\right) > M \quad (10)
$$

By the reservation-budget setting $E_{k,\ell} = \gamma L_k$ and by Theorem 5, the above inequality can be upper bounded by

$$
\frac{M\gamma L_k}{\min\{T_k, D_k\}} + \sum_{i=1}^{k} \left(\frac{C_i}{T_i} + \frac{C_i}{D_k}\right)\left(1+\frac{1}{\gamma-1}\right) > M \quad (11)
$$

By assumption, the reservation-based federated scheduling under DMP is given a speedup of $\alpha > 3+2\sqrt{2}$ compared to the optimal scheduler. Since the DAG task set is schedulable under the optimal scheduler on unit speed processors, on α-speed processors we have $\alpha L_i \leq \min\{T_k, D_k\}$, $\alpha \sum_{i=1}^{k} \frac{C_i}{T_i} \leq M$ and $\alpha \sum_{i=1}^{k} \frac{C_i}{D_i} \leq M$. Therefore, we have

$$
\begin{aligned}
&\frac{1}{\alpha} \geq \max\left\{\frac{L_k}{\min\{T_k, D_k\}}, \sum_{i=1}^{k} \frac{C_i}{MT_i}, \sum_{i=1}^{k} \frac{C_i}{MD_k}\right\} \quad (12)\\
\Rightarrow \quad &\gamma\frac{1}{\alpha} + 2\left(1+\frac{1}{\gamma-1}\right)\frac{1}{\alpha} > 1 \quad (13)\\
\Rightarrow \quad &\frac{1}{\alpha} > \frac{1}{\gamma+\frac{2\gamma}{\gamma-1}} = \frac{\gamma-1}{\gamma^2+\gamma} = \frac{\sqrt{2}}{4+3\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \quad (14)
\end{aligned}
$$

Hence, $\alpha < 3+2\sqrt{2}$ leads to contradiction. Therefore, the speedup factor is at most $3+2\sqrt{2}$. Note that the setting of γ as $1+\sqrt{2}$ is in fact to maximize $\frac{\gamma-1}{\gamma^2+\gamma}$. ■

Corollary 7: Based on Theorem 6, the speedup factor of reservation-based federated scheduling under DMP under a parameter $\gamma > 1$ in the *R-EQUAL* algorithm is $\frac{\gamma^2+\gamma}{\gamma-1}$.

Proof: This follows from the proof in Theorem 6. ■

Theorem 8: A system of arbitrary-deadline DAG tasks scheduled by reservation-based federated scheduling under DMP, in which each processor uses EDF for the reservation servers, admits a constant speedup factor of $3+2\sqrt{2}$ with respect to any optimal scheduler by setting γ to $1+\sqrt{2}$.

Proof: Since EDF is optimal on uniprocessor and is no worse than deadline monotonic, the task partitioning algorithm and analysis used in Theorem 6 yields the result directly. ■

Note that the setting of $\alpha = 1+\sqrt{2}$ is to achieve a bounded speedup factor in the analysis. Nevertheless, while ensuring the worst-case behaviour, such an enforcement is usually not beneficial for the average-case performance. Details regarding this matter can be found in a recent paper by Chen et al [19].

VII. Global Scheduling for Reservation Servers

In contrast to partitioned scheduling, global scheduling schemes employ a single ready queue and allow arbitrary task migrations. Analogously to the problem of computing reservation servers for partitioned scheduling, no single best server assignment algorithm exists for global scheduling. Instead, it depends on the analysis used to verify the schedulability of server tasks, since different schedulability analyses may be sensitive to different parameters. For example, if the schedulability test only considers the cumulative demands of the generated reservation servers, then the individual settings of the reservations become irrelevant and we should thus choose the one with minimal reservation demands as in *R-MIN*. Otherwise, optimization techniques that are suitable to optimize the parameters relevant for the schedulability analysis may be used, e.g., linear- or quadratic-programming in conjunction with the constraints for feasible reservation systems.

Global scheduling algorithms for constrained- and arbitrary-deadline sequential tasks have been extensively studied, e.g., [8], [10], [11], [23], [24], [40]. In general, any of these schedulability tests can be applied for validating the underlying global scheduling algorithm.

To prove that the speedup factor of global deadline-monotonic (DM) scheduling for arbitrary-deadline DAG task sets is the same as in partitioned deadline-monotonic scheduling, namely $3+2\sqrt{2}$, we adopt the following definitions.

The *load* of the server tasks $\mathbf{T}$ is defined as $\text{LOAD}(\mathbf{T}) \stackrel{\text{def}}{=} \max_{t>0}\left\{\sum_{\tau_k\in\mathbf{T}}\frac{\text{DBF}(\tau_k,t)}{t}\right\}$, where $\text{DBF}(\tau_k,t)$ is the demand-bound function of server task τ_k during interval length t as introduced by Baruah [7]. Further, the *maximum density* of the server tasks is defined as $\delta_{max}(\mathbf{T}) \stackrel{\text{def}}{=} \max_{\tau_k\in\mathbf{T}}\left\{\frac{E_k}{\min\{D_k,T_k\}}\right\}$.

We use the following sufficient (but more pessimistic) schedulability test derived by Chen et al. (Theorem 4.7 in [20]) for scheduling reservation servers under global DM scheduling.

Theorem 9: A sequential arbitrary-deadline task set $\mathbf{T}$ is schedulable by global deadline-monotonic scheduling on $M \geq 1$ processors if

$$2\cdot\text{LOAD}(\mathbf{T}) + (M-1)\cdot\delta_{max}(\mathbf{T}) \leq M \tag{15}$$

Proof: By Theorem 4.7 in Chen et al. [20], a task set of sporadic arbitrary-deadline tasks that is indexed according to global deadline-monotonic priority assignment is schedulable on M processors if for all tasks τ_k

$$\delta_k + \sum_{i=1}^{k-1}\left(\frac{C_i - C_iU_i}{D_k} + U_i\right) \leq M - (M-1)\cdot\max\{\max_{i=1}^{k-1} U_i, \delta_k\} \tag{16}$$

Further, by Corollary 5.1 [20]

$$\delta_k + \sum_{i=1}^{k-1}\left(\frac{C_i - C_iU_i}{D_k} + U_i\right) \leq 2\cdot\text{LOAD}(k) \tag{17}$$

and due to the fact that $\max\{\max_{i=1}^{k-1} U_i, \delta_k\} \leq \delta_{max}(k)$ (since $\delta_i \geq U_i$), we obtain the over-approximated sufficient schedulability test stated in Theorem 9. ■

This sufficient condition is used to prove the following theorem:

Theorem 10: A system of arbitrary-deadline DAG tasks scheduled by ***reservation-based federated scheduling using global deadline-monotonic scheduling*** to schedule the generated reservation servers has a constant speedup factor of $3+2\sqrt{2}$ with respect to any optimal scheduler.

Proof: We prove this by adopting the *R-EQUAL* algorithm (Alg. 1) with $\gamma = 1+\sqrt{2}$. Similar to the proof of Theorem 6, *for contradiction* we assume that the algorithm is given a speedup of $\alpha > 3+2\sqrt{2}$ compared to the optimal scheduler but still fails to schedule the DAG tasks, which is because global DM cannot schedule all the assigned reservation servers.

Let $\mathbf{T}$ denote the original set of arbitrary-deadline DAG tasks and let $\mathbf{T}'$ denote the set of associated reservation servers that are generated by *R-EQUAL*. Since $\mathbf{T}'$ fail to suffice theorem Thm. 10, we have

$$2\cdot\text{LOAD}(\mathbf{T}') + (M-1)\cdot\delta_{max}(\mathbf{T}') > M \Rightarrow \tag{18}$$

$$2\max_{t>0}\left\{\sum_{\tau_i\in\mathbf{T}'}\frac{\text{DBF}(\tau_i,t)}{Mt}\right\} + \frac{(M-1)}{M}\cdot\max_{\tau_i\in\mathbf{T}'}\left\{\frac{E_i}{\min\{D_i,T_i\}}\right\} > 1 \tag{19}$$

Since $E_i = \gamma L_i$ and by theorem Thm. 5, it follows that

$$2(1+\frac{1}{\gamma-1})\max_{t>0}\left\{\sum_{\tau_i\in\mathbf{T}}\frac{\text{DBF}(\tau_i,t)}{Mt}\right\} + \frac{(M-1)}{M}\cdot\gamma\cdot\max_{\tau_i\in\mathbf{T}}\left\{\frac{L_i}{\min\{D_i,T_i\}}\right\} > 1 \tag{20}$$

Again, by assumption the reservation-based federated scheduling under global DM is given a speedup of $\alpha > 3+2\sqrt{2}$ compared to the optimal scheduler. Since the DAG task set is schedulable under the optimal scheduler on unit speed processors, on α-speed processors we have $\alpha L_i \leq \min\{T_k, D_k\}$ and $\alpha\cdot\max_{t>0}\left\{\sum_{\tau_i\in\mathbf{T}}\frac{\text{DBF}(\tau_i,t)}{t}\right\} \leq M$. Therefore, we have

$$3+2\sqrt{2} \geq \frac{\gamma^2+\gamma}{\gamma-1} \geq 2(1+\frac{1}{\gamma-1}) + \gamma\cdot(1-\frac{1}{M}) > \alpha$$

Hence, $\alpha < 3+2\sqrt{2}$ leads to contradiction. As a result, the speedup factor is at most $3+2\sqrt{2}$. ■

VIII. Improved Algorithms for Reservation Servers

Algorithm 2 The SOF algorithm yields feasible partitions and associated reservations.

Require: An arbitrary-deadline DAG task set $\mathbf{T}$, processors M, boundaries $b_1, b_2, .., b_N$.
Ensure: Feasible partition and reservations, that can service $\mathbf{T}$ provably (if one could be found).

$\mathbf{T_H}, \mathbf{T_L} \leftarrow$ R-MIN($\mathbf{T}$) or R-EQUAL($\mathbf{T}$)
re-index $\mathbf{T_H} \cup \mathbf{T_L}$ such that $D_i \leq D_j$ for $i < j$
for each $\tau_{i,j}$ in $\mathbf{T_H} \cup \mathbf{T_L}$ **do**
  if $\tau_{i,j}$ in $\mathbf{T_L}$ **then**
    if partition by preference failed **then**
      return Partition Failure
    end if
  else
    Failure $\leftarrow$ True
    while ℓ_i is no more than $\max\left\{\left\lceil\frac{C_i}{L_i}\right\rceil, \left\lceil\frac{C_i-L_i}{L_i(\gamma-1)}\right\rceil, b_i\right\}$ **do**
      if partition by preference failed **then**
        revoke all already partitioned $\tau_{i,1}, \tau_{i,2}, .., \tau_{i,j}$ belonging to DAG task τ_i
        $\ell_i \leftarrow \ell_i + 1$ {increment the number of reservation servers and re-try}
        for each j in $\{1, 2, .., \ell_i\}$ **do**
          $E_{i,j} \leftarrow \frac{C_i}{\ell_i} + (1-\frac{1}{\ell_i})\cdot L_i$ {compute evolved reservation budgets}
        end for
        continue
      else
        Partition $\tau_{i,j}$ to processor as chosen by preference
        Failure $\leftarrow$ False
        break
      end if
    end while
    if Failure is True **then**
      return Partition Failure
    end if
  end if
end for
return Partition and Reservations

In this section, we consider a class of algorithms where reservation server assignment is adapted based on whether the initially assigned servers are schedulable by an applied scheduling algorithm. In principle, reservation-based federated scheduling under any suitable multiprocessor scheduling algorithm for servers can be improved using this strategy. Here, we take DMP as an example, highlight one such adaptive approach and analyze its speedup bound.

We present the *Split-On-Fail* (SOF) algorithm in Alg. 2. The *SOF* algorithm attempts to partition all generated reservation servers according to a schedulability test for arbitrary-deadline tasks by preference, e.g., worst-fit, first-fit, and best-fit. In order to reduce the algorithmic complexity, the partitioning of each group of servers that belong to the same DAG task is done on the premise that all previously partitioned groups are already feasibly partitioned and their settings do not change. Additionally, light tasks are excluded from the adaptation process. Whenever a reservation server cannot be partitioned, an additional reservation server is added to the group and the individual reservation-budgets are decreased appropriately.

By the improved algorithm, the set of all possible reservation budgets is reduced from a theoretical feasible interval $L_i < E_i \leq D_i$. The feasible budget sizes can be calculated given the number of servers ℓ_i using $E_i = \frac{C_i}{\ell_i} + (1 - \frac{1}{\ell_i}) \cdot L_i$.

The intention is to improve schedulability by increasing the number of reservation servers, whilst decreasing their individual budgets. Note that any priority policy that assigns equal priorities to all reservations belonging to the same group results in non-equal reservations. This is due to the fact that if multiple reservations with the same priority are partitioned onto the same processor, this behaves as if there was only one reservation with the individual reservation budgets accumulated on that processor. Furthermore, note that since the reservation-budgets are determined upon partitioning, different partitioning strategies, i.e., *best-fit*, *worst-fit*, and *first-fit*, result in different reservation-budgets effectively.

By using *R-EQUAL* with $\gamma = \min_{\tau_i} \left\{ \frac{D_i}{L_i} \right\}$ to generate the initial reservations and for any $b_i > 1 \in \mathbb{N}$, we prove the following theorem assuming that $\gamma > 1$.

Theorem 11: The speedup factor of *SOF* under Deadline-Monotonic Partitioning (DMP), in which each processor uses EDF or DM for the reservation servers, is at most $\min\{\frac{\gamma^2+\gamma}{1-\gamma}, 4 - \min\left\{\frac{L_k}{C_k}, \frac{\gamma L_k - L_k}{C_k - L_k}, \frac{1}{b_k - 1}\right\} + 2\max_{i<k}\left\{\max\left\{\frac{C_i}{L_i}, \frac{C_i - L_i}{\gamma L_i - L_i}, b_k - 1\right\}\right\}\}$.

Proof: If *SOF* cannot find a feasible partition and reservations, then the initial reservation-budgets cannot be partitioned feasibly either. Due to Corollary 7 and Theorem 8, this yields that the speedup factor is at most $\frac{\gamma^2+\gamma}{1-\gamma}$. We use the same arguments as in Theorem 6, including the definition of τ_k and the index of the tasks according to DMP, i.e., $D_i \leq D_k$ for $i = 1, 2, \ldots, k-1$. For this case, we have

$$(2 - \frac{1}{c_k}) \cdot \frac{L_k}{\min\{T_k, D_k\}} + \sum_{i=1}^{k} \frac{C_i}{MT_i} + \frac{C_i}{MD_k} + \sum_{i=1}^{k} (c_i - 1) \cdot \left(\frac{L_i}{MT_i} + \frac{L_i}{MD_k} \right) > 1$$

Where due to the definition of the boundaries $c_i = \max\left\{ \left\lceil \frac{C_i}{L_i} \right\rceil, \left\lceil \frac{C_i - L_i}{L_i(\gamma-1)} \right\rceil, b_i \right\}$, it must be that $c_i \geq \left\lceil \frac{C_i}{L_i} \right\rceil \geq \frac{C_i}{L_i}$. Let $\alpha = \max\left\{ \frac{L_k}{\min\{T_k, D_k\}}, \sum_{i=1}^{k} \frac{C_i}{MT_i}, \sum_{i=1}^{k} \frac{C_i}{MD_k}, \right\}$ denote a necessary condition for schedulability. Then, the following inequality holds:

$$4 - \frac{1}{c_k} + 2 \cdot \max_{i<k} \{c_i - 1\} > \frac{1}{\alpha} \tag{21}$$

Since,

$$c_i - 1 = \max\left\{ \left\lceil \frac{C_i}{L_i} \right\rceil, \left\lceil \frac{C_i - L_i}{L_i(\gamma - 1)} \right\rceil, b_i \right\} - 1 \leq \max\left\{ \frac{C_i}{L_i}, \frac{C_i - L_i}{L_i(\gamma - 1)}, b_i - 1 \right\}$$

holds, the equation can be reformulated to

$$4 - \min\left\{ \frac{L_k}{C_k}, \frac{\gamma L_k - L_k}{C_k - L_k}, \frac{1}{b_k - 1} \right\} + 2 \cdot \max_{i<k}\left\{ \max\left\{ \frac{C_i}{L_i}, \frac{C_i - L_i}{\gamma L_i - L_i}, b_k - 1 \right\} \right\}$$

This proves the theorem. ∎

IX. Experimental Evaluation

This section presents the evaluation results. We first numerically evaluate the reservation-based federated scheduling under partitioned schedulers (as our optimization for reservation server allocation is mainly for partitioned schedulers) using synthetic task sets. We then empirically evaluate the reservation-based federated scheduling under the global DM scheduler (as there is no budget-based partitioned scheduler in the RTCG framework [32]) implemented on a real multicore platform.

A. Evaluations Based on Synthetic Task Sets

Task Set Generation: We used the *Randfixedsum* algorithm [21] to generate utilizations for task sets of fixed size with individual task utilizations $0 < U_i \leq M$, where M denotes the number of processors. Each task was characterized by $\tau_i = (C_i, L_i, D_i, T_i)$, where the periods were drawn uniformly from $(0, 100]$ to cover a wide range of characteristics. The range of the parameters $\alpha = \frac{D_i}{T_i}$ and $\beta = \frac{L_i}{D_i}$ for generating deadline and critical-path length is changed for different settings.

We evaluate task sets with implicit, constrained, and arbitrary deadlines. For each setting, task sets for 8, 16, and 32 processor systems were generated. For each normalized utilization (in five percent steps), 100 task sets were generated, where each task set consists of 20 DAG tasks. This task set size was selected to be large enough to allow smaller individual utilization and thus more options for the partitioning, whilst being small enough to still be difficult to partition.

In the following, the proposed reservation-based federated scheduling algorithms, namely *SOF* in the *R-MIN* and *R-EQUAL* variants as wells as the different partitioning heuristics first-fit, best-fit, and worst-fit were compared against the semi-federated scheduling approach by Jiang et al. [26]. Since the evaluations of Jiang et al. in [26] did not suggest a notable difference between the performance of their two proposed algorithms, only the algorithm $SF[X+1]$ was adopted and is referred to as *S-FED*. In semi-federated scheduling, the resource waste of federated scheduling is addressed by computing the required processors more tightly. That is, if federated scheduling requires at least $m_i \geq x$ many processors with $m_i \in \mathbb{N}$, then federated scheduling will allocate $\lceil x \rceil = (x+\epsilon)$-many processors for this task. This may lead to at most 200% of the required resources since $2 \geq \frac{x+1}{x+\epsilon}, x \geq 1$ for $\lim \epsilon \to 0$. Conversely, the proposed improvements decrease

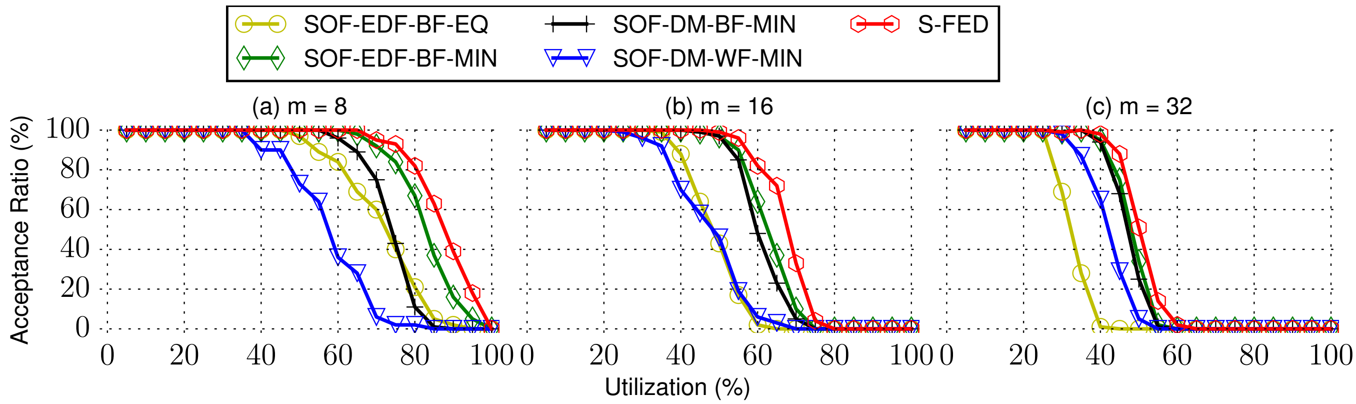


Fig. 2: Acceptance ratio for implicit-deadline DAG task sets with normalized utilizations in five percent steps on 8, 16, and 32 processors, respectively. The periods are drawn uniformly from $(0, 100]$ and the critical-path length is drawn uniformly from $(0.6T_i, 0.9T_i]$.

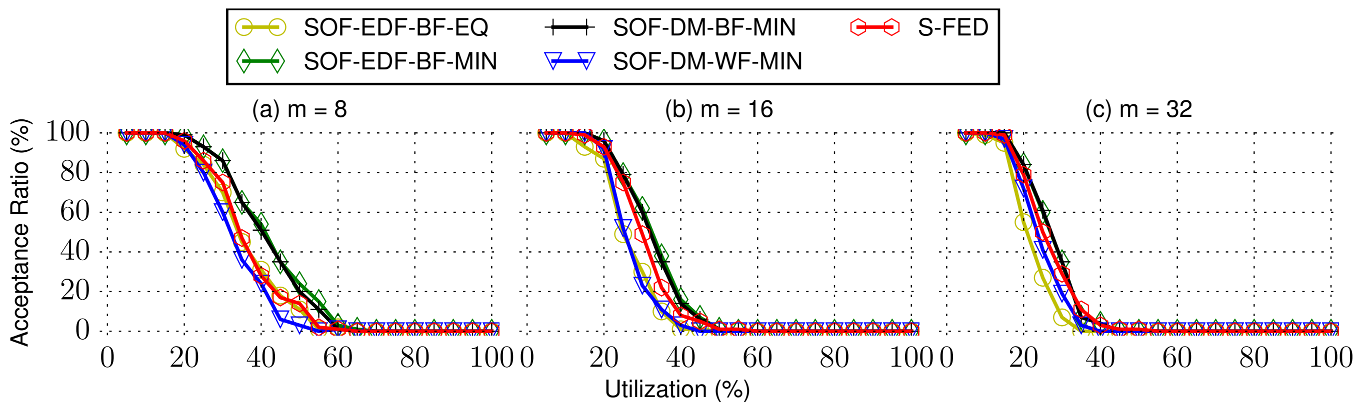


Fig. 3: Acceptance ratio for constrained-deadline DAG task sets on 8, 16, and 32 processors, respectively. The periods are drawn uniformly from $(0, 100]$, the deadline is drawn uniformly in $(0.1T_i, T_i]$, and the critical-path length is drawn uniformly from $(0.4D_i, 0.7D_i]$.

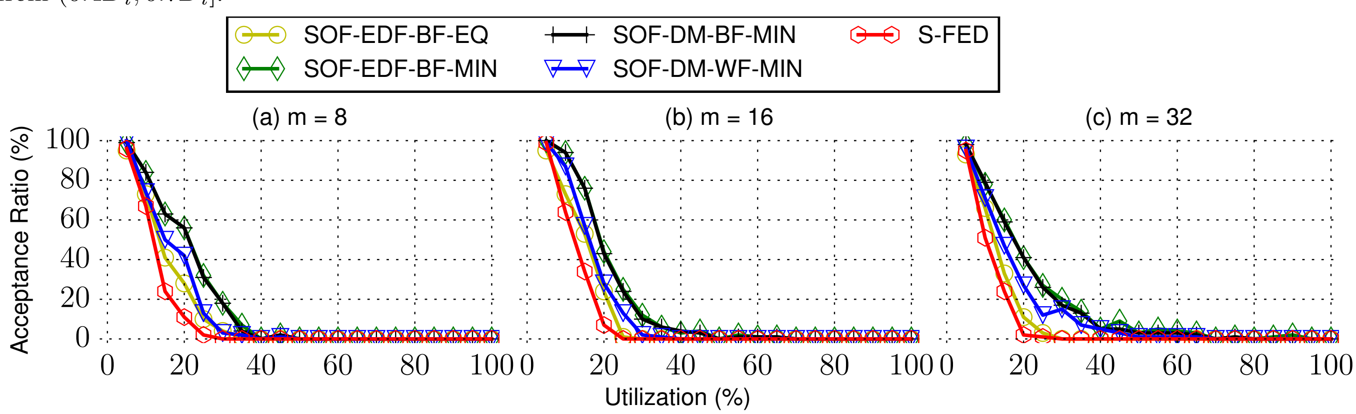


Fig. 4: Acceptance ratio for constrained-deadline DAG task sets on 8, 16, and 32 processors respectively. The periods are drawn uniformly from $(0, 100]$, the deadline is drawn uniformly in $(0, 0.5T_i]$, and the critical-path length is drawn uniformly from $(0, 0.5D_i]$.

with the number of required processors. To allow a fair evaluation and to make use of the advantage of the sporadic task model for the reservations, a *SOF* variant with partitioned EDF for arbitrary-deadlines using the linear demand-bound function approximation as proposed by Baruah [3] was adopted. Consequently, a scheduling test consisting of the reservation initialization method *R-MIN* or *R-EQUAL*, the uniprocessor scheduling algorithm EDF or DM, and a packing heuristic, results in 12 variants. Due to similar performance in the evaluations, only a subset of the variants that are representative of the group's performance is illustrated.

Experimental Results on Implicit-Deadline Task Sets: The first set of experiments is shown in Fig. 2. It can be seen that for tasks with implicit deadlines *S-FED* and the best performing variant *SOF-EDF-BF-MIN* behave similarly up to a cutoff utilization of 60%, 45%, and 35% for 8, 16, and

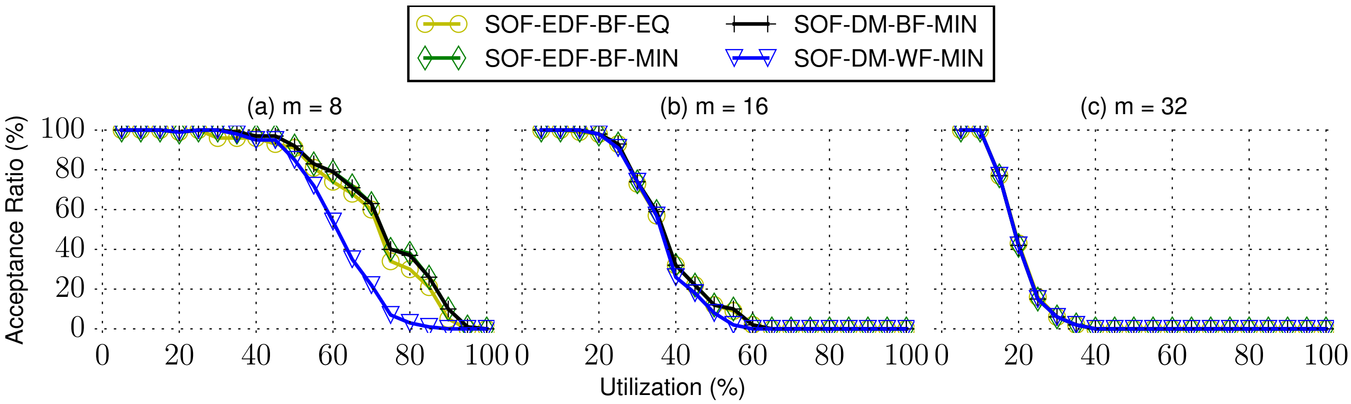


Fig. 5: Acceptance ratio for arbitrary-deadline DAG task sets on 8, 16, and 32 processors respectively. The periods are drawn uniformly from $(0, 100]$, the deadline is drawn uniformly in $(0.1T_i, 10T_i]$, and the critical-path length is drawn uniformly from $(0.4D_i, 0.7D_i]$.

32 processors. In general, *S-FED* declines slightly later than the best SOF variants, but the performance gap decreases with the number of processors. It can also be observed that the *R-EQUAL* variant compared to the best performing *SOF-EDF-BF-MIN* algorithm exhibits a substantially lowered cutoff utilization. This is due to the fact that *R-EQUAL* generates more reservations initially in comparison to *R-MIN* variants and thus decreases schedulability.

Experimental Results on Constrained-Deadline Task Sets: In Fig. 3, the deadlines of constrained-deadline tasks were chosen uniformly from $(0.1T_i, T_i]$. Here the variants *SOF-DM-BF-MIN* and *SOF-EDF-BF-MIN* slightly outperform *S-FED* for 8, 16, and 32 processors, whereas the gap decreases with the number of processors. The *S-FED* is especially sensitive to small L_i/D_i and D_i/T_i ratios, and performs much worse, which is demonstrated in an extreme-case shown in Fig. 4.

Experimental Results on Arbitrary-Deadline Task Sets: Finally, Fig. 5 presents arbitrary-deadline task sets where the deadline was chosen uniformly from $(0.1T_i, 10T_i]$. Since semi-federated scheduling does not support arbitrary-deadline DAG task sets, only *SOF* variants are evaluated. All *SOF* variants accept all task sets until 50%, 20%, and 10% cutoff utilizations in 8, 16, and 32 processor platforms, respectively. The variants *SOF-DM-BF-MIN* and *SOF-EDF-BF-EQ* exhibit the best performance and behave identically. Additionally, we observe that *SOF-DM-WF-MIN* demonstrates the worst performance.

B. Empirical Evaluation

We now describe our empirical results of reservation-based federated scheduling using it with DM scheduling for servers.

Platform Implementation: We implement a prototype platform that supports reservation-based federated scheduling under global DM by modifying an existing federated scheduling platform for OpenMP programs, namely RTCG in [32]. The main differences between the reservation-based federated scheduling under the global DM and the original federated scheduling include: (1) deadline monotonic priorities to the parallel threads of a task, (2) different numbers of threads generated for a task, where each thread corresponds to one reservation server in reservation-based federated scheduling in contrast to one dedicated core in the original federated scheduling, and (3) global execution of the threads of a task, instead of dedicated core assignment using federated scheduling. We modify RTCG correspondingly to address these differences.

Experiments were conducted on a 48-core machine composed of 4 AMD Opteron 6168 processors. We reserved one processor, i.e., 12 cores, for system tasks, leaving 36 experimental processing cores. Based on this hardware specification, we ran the single socket 12-core experiments and multi-socket 36-core experiments. Linux with the PREEMPT_RT kernel patch was used as the underlying RTOS.

Task Set Generation: We use the similar approach as in Section IX-A to randomly generate synthetic task sets. In these experiments, all units are expressed in millisecond (ms). In addition, we randomly selected task period from $\{4ms, 8ms, 16ms, 32ms, 64ms, 128ms\}$ to form task sets with harmonic periods. For all the experiments, each task set was run for 100 hyper-periods.

We compare reservation-based federated scheduling under global DM in the *R-MIN* and *R-EQUAL* variants (namely ***GDM-MIN*** and ***GDM-EQ***, respectively) against our implementation of global DM without any reservation (***GDM-ALL***). In addition, for experiments with implicit deadlines, we also compare against the original federated scheduling, i.e., RTCG platform (***FS***). To have a fair comparison, for experiments with constrained and arbitrary deadlines we modify the federated scheduling to use the $\min\{D_i, T_i\}$ instead of D_i to calculate the core allocation for each task.

Experimental Results: The experiments on 12 cores are shown in Fig. 6. In all settings, *GDM-MIN* outperforms *FS* and *FS* is comparable or better than *GDM-EQ*. As discussed in Section VII, reservation-based federated scheduling under global DM should choose reservation servers with the minimum reservation demands, i.e., *GDM-MIN*, instead of other alternatives such as *GDM-EQ*. Additionally, *GDM-MIN* outperforms *FS* since without the dedicated core allocation a core will not idle as long as there are some unfinished jobs. Therefore,

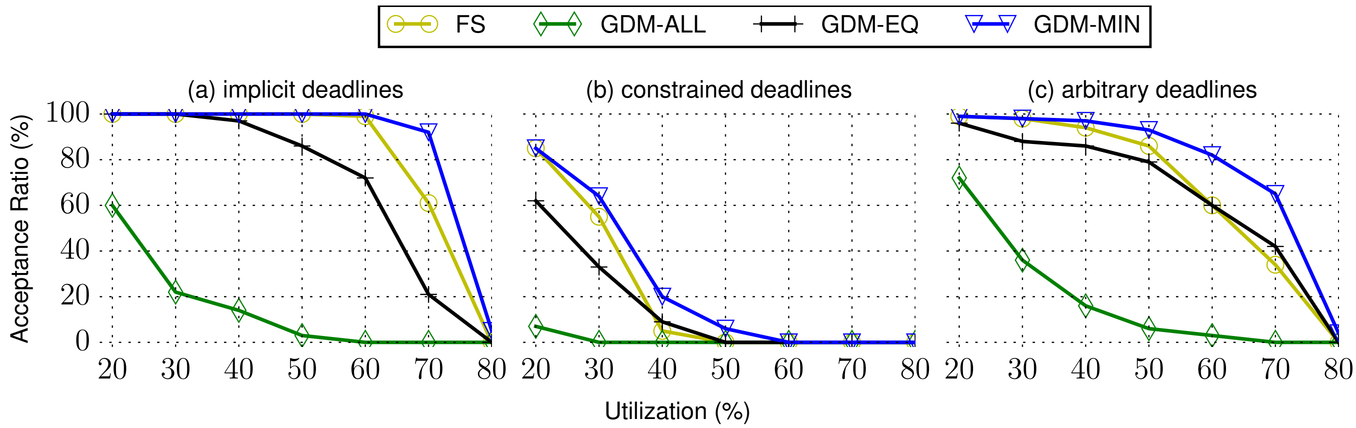


Fig. 6: Acceptance ratio under normalized utilization for task sets on 12 cores with implicit, constrained, and arbitrary deadlines. For each normalized utilization, 100 task sets each with 10 tasks are generated. The maximum critical-path length for each task is drawn uniformly from $(0.6T_i, 0.9T_i]$ for implicit and arbitrary deadlines, and is drawn from $(0.4D_i, 0.7D_i]$ for constrained deadlines. The deadline is drawn uniformly in $(0.1T_i, 10T_i]$ for arbitrary deadlines and in $(0.1T_i, 1T_i]$ for constrained deadlines.

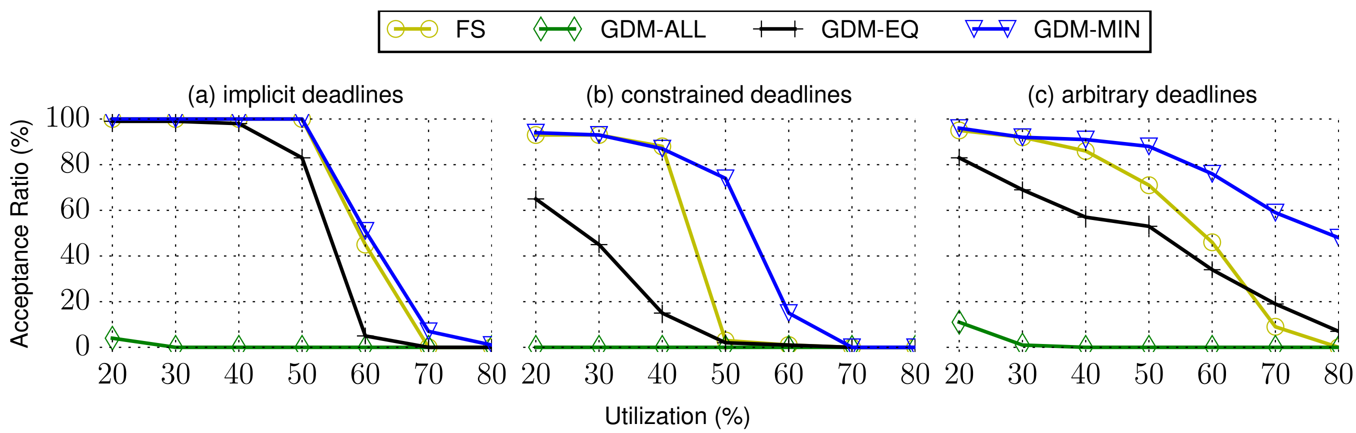


Fig. 7: Acceptance ratio displaying normalized utilizations for task sets on 36 cores with implicit, constrained, and arbitrary deadlines, respectively. For each normalized utilization, 100 task sets each with 20 tasks are generated. The maximum critical-path length for each task is drawn uniformly from $(0.6T_i, 0.9T_i]$ for implicit and arbitrary deadlines, and is drawn from $(0.4D_i, 0.7D_i]$ for constrained deadlines. The deadline is drawn uniformly in $(0.1T_i, 10T_i]$ for arbitrary deadlines and in $(0.5T_i, 1T_i]$ for constrained deadlines.

GDM-MIN can more efficiently utilize the multicore platform. Finally, *GDM-ALL* performs significantly worse, which shows the necessity of calculating the proper reservation demands.

In addition to the experiments on 12 cores, the results for the same experimental settings on 36 cores are shown in Fig. 7. The performance trends of different scheduling algorithms on 36 cores are similar to those on 12 cores. It can be seen that *GDM-MIN* outperforms all other scheduling algorithms in terms of acceptance ratios.

X. Conclusion and Future Research

In this work, we show that reservation-based federated scheduling is competitive with the state-of-the-art of sporadic DAG task scheduling for the parametric model. Especially in the constrained-deadline and arbitrary-deadline cases, the reservation-based federated scheduling demonstrates good performance as well as robustness with respect to varying parameters. Since reservation-based federated scheduling can be used in conjunction with any pre-existing scheduling algorithm that supports sporadic sequential tasks, we believe that it has a great potential to be applied in practical systems even though it has a slight performance loss for certain scenarios.

For future work, we will consider the memory and bus contention, as this has been either ignored or pessimistically overestimated in the literature. For constrained-deadline sporadic task systems (without DAG) under fixed-priority scheduling, there have been a few recent research results, e.g., [1], [25]. We plan to explore the impact of such resource contention when designing reservation-based federated scheduling.

Acknowledgment

This paper is supported by DFG, as part of the Collaborative Research Center SFB876, project B2 (http://sfb876.tu-dortmund.de/), by NSF Grants CCF-1337218 and AITF-1733873, and by NJIT Seed Grant. This research was initiated in the Dagstuhl Seminar 17131 — Mixed Criticality on Multicore/Manycore Platforms.

REFERENCES

[1] S. Altmeyer, R. I. Davis, L. S. Indrusiak, C. Maiza, V. Nélis, and J. Reineke. A generic and compositional framework for multicore response time analysis. In *Proceedings of the 23rd International Conference on Real Time Networks and Systems, RTNS*, pages 129–138, 2015.

[2] B. Andersson and D. de Niz. Analyzing global-edf for multiprocessor scheduling of parallel tasks. In *Principles of Distributed Systems, 16th International Conference, OPODIS*, pages 16–30, 2012.

[3] S. Baruah. The federated scheduling of constrained-deadline sporadic DAG task systems. In *Proceedings of the Design, Automation & Test in Europe Conference & Exhibition, DATE*, pages 1323–1328, 2015.

[4] S. Baruah. Federated scheduling of sporadic DAG task systems. In *IEEE International Parallel and Distributed Processing Symposium, IPDPS*, pages 179–186, 2015.

[5] S. Baruah. The federated scheduling of systems of conditional sporadic DAG tasks. In *Proceedings of the 15th International Conference on Embedded Software (EMSOFT)*, 2015.

[6] S. Baruah, V. Bonifaci, and A. Marchetti-Spaccamela. The global EDF scheduling of systems of conditional sporadic DAG tasks. In *27th Euromicro Conference on Real-Time Systems, ECRTS*, pages 222–231, 2015.

[7] S. Baruah and N. Fisher. Global deadline-monotonic scheduling of arbitrary-deadline sporadic task systems. In *Proceedings of the 11th International Conference on Principles of Distributed Systems*, OPODIS'07, pages 204–216, Berlin, Heidelberg, 2007. Springer-Verlag.

[8] S. K. Baruah, V. Bonifaci, A. Marchetti-Spaccamela, and S. Stiller. Improved multiprocessor global schedulability analysis. *Real-Time Systems*, 46(1):3–24, 2010.

[9] S. K. Baruah and N. Fisher. The partitioned multiprocessor scheduling of deadline-constrained sporadic task systems. *IEEE Trans. Computers*, 55(7):918–923, 2006.

[10] S. K. Baruah and N. Fisher. Global fixed-priority scheduling of arbitrary-deadline sporadic task systems. In *Distributed Computing and Networking, 9th International Conference, ICDCN*, pages 215–226, 2008.

[11] M. Bertogna, M. Cirinei, and G. Lipari. New schedulability tests for real-time task sets scheduled by deadline monotonic on multiprocessors. In *Principles of Distributed Systems, 9th International Conference, OPODIS*, pages 306–321, 2005.

[12] E. Bini, M. Bertogna, and S. Baruah. Virtual multiprocessor platforms: Specification and use. In *2009 30th IEEE Real-Time Systems Symposium*, pages 437–446. IEEE, 2009.

[13] E. Bini, T. H. C. Nguyen, P. Richard, and S. K. Baruah. A response-time bound in fixed-priority scheduling with arbitrary deadlines. *IEEE Trans. Computers*, 58(2):279–286, 2009.

[14] V. Bonifaci, A. Marchetti-Spaccamela, S. Stiller, and A. Wiese. Feasibility analysis in the sporadic dag task model. In *ECRTS*, pages 225–233, 2013.

[15] A. Burmyakov, E. Bini, and E. Tovar. Compositional multiprocessor scheduling: the gmpr interface. *Real-Time Systems*, 50(3):342–376, 2014.

[16] J.-J. Chen. Federated scheduling admits no constant speedup factors for constrained-deadline dag task systems. *Real-Time Systems*, 52(6):833–838, November 2016.

[17] J.-J. Chen. Partitioned multiprocessor fixed-priority scheduling of sporadic real-time tasks. In *Euromicro Conference on Real-Time Systems (ECRTS)*, pages 251–261, 2016.

[18] J.-J. Chen and S. Chakraborty. Resource augmentation for uniprocessor and multiprocessor partitioned scheduling of sporadic real-time tasks. *Real-Time Systems*, 49(4):475–516, 2013.

[19] J.-J. Chen, G. von der Brüggen, W.-H. Huang, and R. I. Davis. On the pitfalls of resource augmentation factors and utilization bounds in real-time scheduling. In *Euromicro Conference on Real-Time Systems, ECRTS*, pages 9:1–9:25, 2017.

[20] J.-J. Chen, G. von der Brüggen, and N. Ueter. Push forward: Global fixed-priority scheduling of arbitrary-deadline sporadic task systems. In *30th Euromicro Conference on Real-Time Systems, ECRTS*, pages 8:1–8:24, 2018. preprint in http://arxiv.org/abs/1802.10376.

[21] P. Emberson, R. Stafford, and R. I. Davis. Techniques for the synthesis of multiprocessor tasksets. In *International Workshop on Analysis Tools and Methodologies for Embedded and Real-time Systems (WATERS 2010)*, pages 6–11, 2010.

[22] N. Fisher, S. K. Baruah, and T. P. Baker. The partitioned scheduling of sporadic tasks according to static-priorities. In *ECRTS*, pages 118–127, 2006.

[23] N. Guan, M. Stigge, W. Yi, and G. Yu. New response time bounds for fixed priority multiprocessor scheduling. In *IEEE Real-Time Systems Symposium (RTSS)*, pages 387–397, 2009.

[24] W.-H. Huang and J.-J. Chen. Response time bounds for sporadic arbitrary-deadline tasks under global fixed-priority scheduling on multiprocessors. In *RTNS*, 2015.

[25] W.-H. Huang, J.-J. Chen, and J. Reineke. MIRROR: symmetric timing analysis for real-time tasks on multicore platforms with shared resources. In *Proceedings of the 53rd Annual Design Automation Conference, DAC*, pages 158:1–158:6, 2016.

[26] X. Jiang, N. Guan, X. Long, and W. Yi. Semi-federated scheduling of parallel real-time tasks on multiprocessors. In *IEEE Real-Time Systems Symposium RTSS*, pages 80–91, 2017.

[27] X. Jiang, X. Long, N. Guan, and H. Wan. On the decomposition-based global EDF scheduling of parallel real-time tasks. In *Real-Time Systems Symposium (RTSS)*, pages 237–246, 2016.

[28] J. Kim, H. Kim, K. Lakshmanan, and R. Rajkumar. Parallel scheduling for cyber-physical systems: analysis and case study on a self-driving car. In *ACM/IEEE 4th International Conference on Cyber-Physical Systems (with CPS Week 2013), ICCPS*, pages 31–40, 2013.

[29] K. Lakshmanan, S. Kato, and R. R. Rajkumar. Scheduling parallel real-time tasks on multi-core processors. In *Proceedings of the 2010 31st IEEE Real-Time Systems Symposium*, RTSS '10, pages 259–268, 2010.

[30] J. Li, K. Agrawal, C. Lu, and C. D. Gill. Analysis of global EDF for parallel tasks. In *Euromicro Conference on Real-Time Systems (ECRTS)*, pages 3–13, 2013.

[31] J. Li, J.-J. Chen, K. Agrawal, C. Lu, C. D. Gill, and A. Saifullah. Analysis of federated and global scheduling for parallel real-time tasks. In *26th Euromicro Conference on Real-Time Systems, ECRTS*, pages 85–96, 2014.

[32] J. Li, S. Dinh, K. Kieselbach, K. Agrawal, C. Gill, and C. Lu. Randomized work stealing for large scale soft real-time systems. In *Real-Time Systems Symposium (RTSS), 2016 IEEE*, pages 203–214. IEEE, 2016.

[33] G. Lipari and E. Bini. A framework for hierarchical scheduling on multiprocessors: from application requirements to run-time allocation. In *Real-Time Systems Symposium (RTSS), 2010 IEEE 31st*, pages 249–258. IEEE, 2010.

[34] A. Melani, M. Bertogna, V. Bonifaci, A. Marchetti-Spaccamela, and G. C. Buttazzo. Response-time analysis of conditional DAG tasks in multiprocessor systems. In *27th Euromicro Conference on Real-Time Systems, ECRTS*, pages 211–221, 2015.

[35] A. Melani, M. Bertogna, V. Bonifaci, A. Marchetti-Spaccamela, and G. C. Buttazzo. Schedulability analysis of conditional parallel task graphs in multicore systems. *IEEE Trans. Computers*, 66(2):339–353, 2017.

[36] G. Nelissen, V. Berten, J. Goossens, and D. Milojevic. Techniques optimizing the number of processors to schedule multi-threaded tasks. In *24th Euromicro Conference on Real-Time Systems, ECRTS*, pages 321–330, 2012.

[37] A. Saifullah, D. Ferry, J. Li, K. Agrawal, C. Lu, and C. D. Gill. Parallel real-time scheduling of dags. *IEEE Transactions on Parallel and Distributed Systems*, 25(12):3242–3252, 2014.

[38] M. A. Serrano, A. Melani, R. Vargas, A. Marongiu, M. Bertogna, and E. Quiones. Timing characterization of openmp4 tasking model. In *2015 International Conference on Compilers, Architecture and Synthesis for Embedded Systems (CASES)*, pages 157–166, Oct 2015.

[39] I. Shin, A. Easwaran, and I. Lee. Hierarchical scheduling framework for virtual clustering of multiprocessors. In *Real-Time Systems, 2008. ECRTS'08. Euromicro Conference on*, pages 181–190. IEEE, 2008.

[40] Y. Sun, G. Lipari, N. AGuan, W. Yi, et al. Improving the response time analysis of global fixed-priority multiprocessor scheduling. In *IEEE International Conference on Embedded and Real-Time Computing Systems and Applications (RTCSA)*, pages 1–9, 2014.

[41] K. Yang and J. H. Anderson. On the dominance of minimum-parallelism multiprocessor supply. Technical report, University of North Carolina at Chapel Hill Chapel Hill United States, 2016.