D/A-Converters

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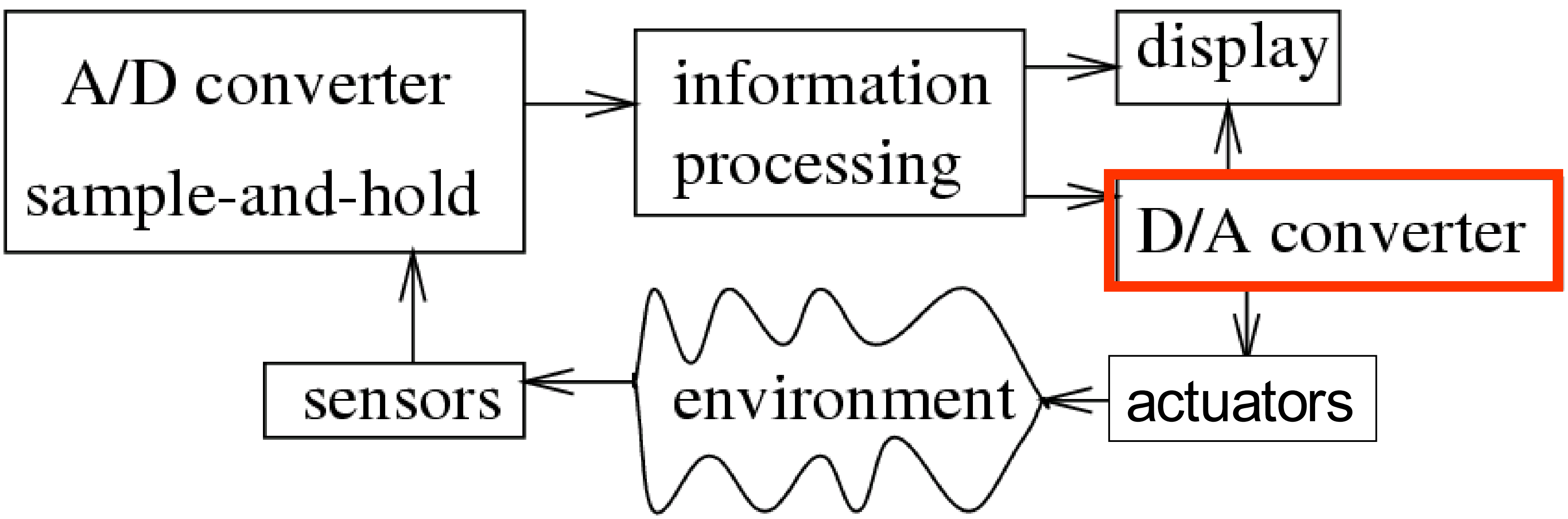
Embedded System Hardware

Embedded system hardware is frequently used in a loop („***hardware in a loop***“):



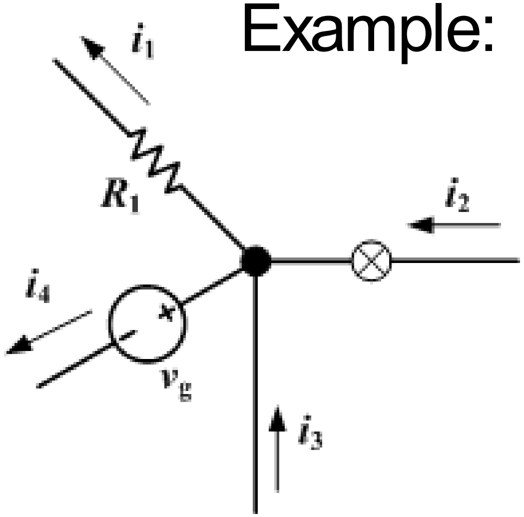
Kirchhoff's junction rule

Kirchhoff's Current Law, Kirchhoff's first rule

The principle of conservation of electric charge implies that:

At any point in an electrical circuit where charge density is not changing in time, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.

Example:



$$i_1 + i_4 = i_2 + i_3$$

www.wikipedia.org

Formally, for any node in a circuit:

$$\sum_k i_k = 0$$

$$-i_1+i_2+i_3-i_4=0$$

Count current flowing away from node as negative.

Kirchhoff's loop rule

Kirchhoff‘s Voltage Law, Kirchhoff's second rule

The principle of conservation of energy implies that:

The directed sum of the electrical potential differences around a closed circuit must be zero.

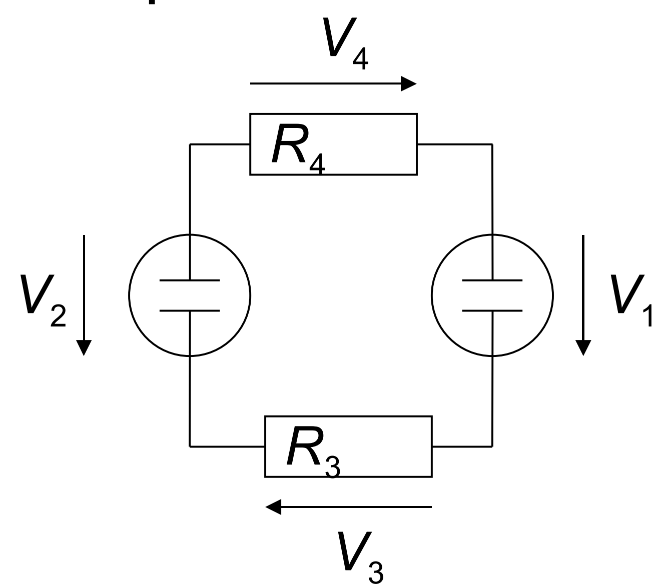
Otherwise, it would be possible to build a perpetual motion machine that passed a current in a circle around the circuit. [www.wikipedia.org]

Formally, for any loop in a circuit:

$$\sum_k V_k = 0$$

Count voltages traversed against arrow direction as negative

Example:



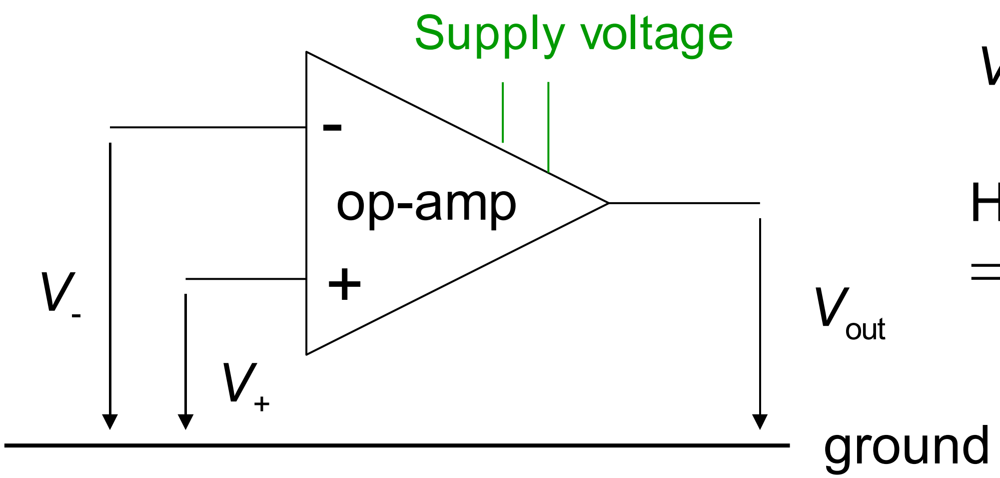
$V_1+V_3-V_2+V_4=0$

$V_3=R_3 \times I$ if current counted in the same direction as V_3

$V_3=-R_3 \times I$ if current counted in the opposite direction as V_3

Operational Amplifiers (Op-Amps)

Operational amplifiers (op-amps) are devices amplifying the voltage difference between two input terminals by a large gain factor *g*



$V_{out}=(V_+ - V_-) \cdot g$

High impedance input terminals
$\Rightarrow$ Currents into inputs ≈ 0

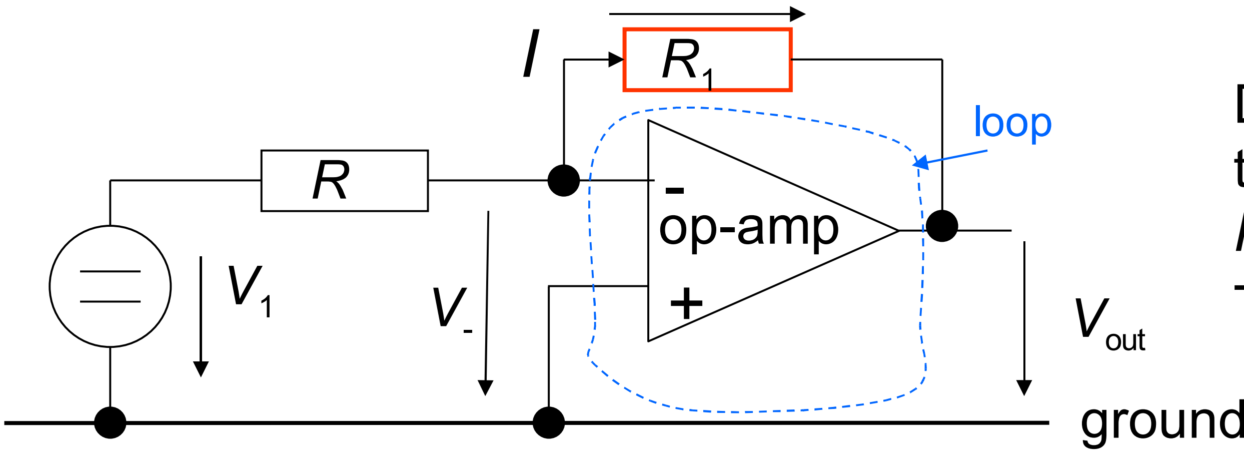
Op-amp in a separate package (TO-5) [wikipedia]

For an **ideal** op-amp: $g \rightarrow \infty$

(In practice: g may be around $10^4..10^6$)

Op-Amps with feedback

In circuits, negative feedback is used to define the actual gain



Due to the feedback to the *inverted* input, R_1 reduces voltage V_-. To which level?

$V_{out} = -g \cdot V_-$ (op-amp feature)

$I \cdot R_1 + V_{out} - V_- = 0$ (loop rule)

$\Rightarrow I \cdot R_1 + -g \cdot V_- - V_- = 0$

$\Rightarrow (1+g) \cdot V_- = I \cdot R_1$

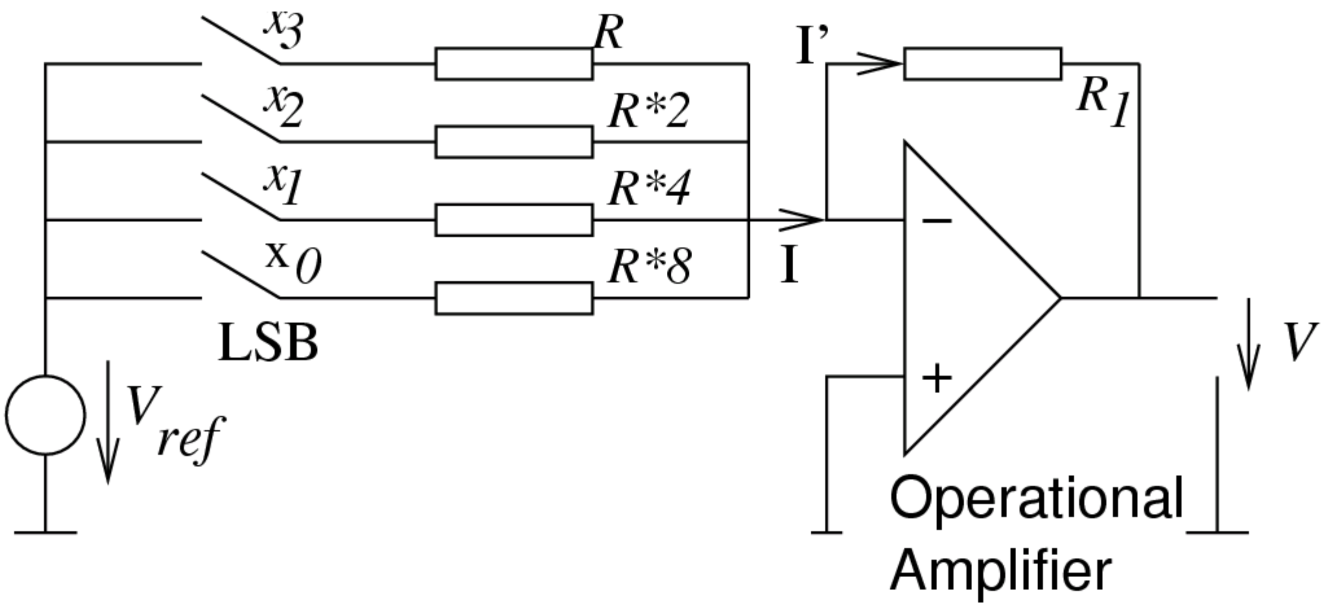
$$\Rightarrow V_- = \frac{I \cdot R_1}{1+g}$$

$$V_{-,ideal} = \lim_{g \to \infty} \frac{I \cdot R_1}{1+g} = 0$$

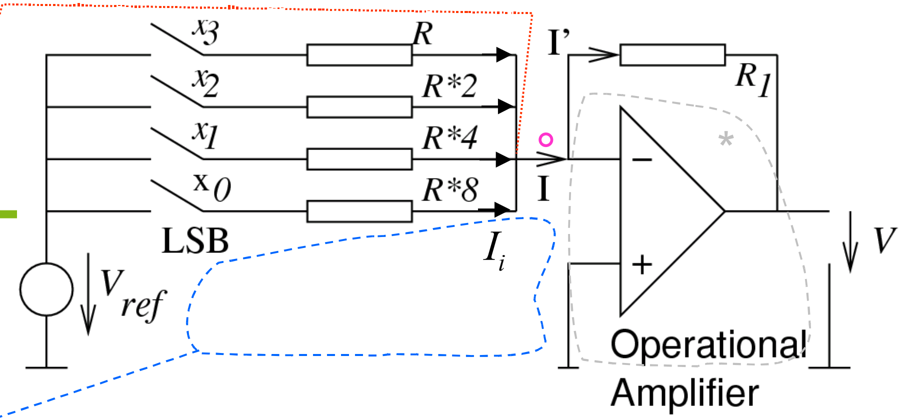
V_- is called **virtual ground**: the voltage is 0, but the terminal may not be connected to ground

Digital-to-Analog (D/A) Converters

Various types, can be quite simple, e.g.:



Output voltage ~ no. represented by x



Junction rule: $I = \sum_i I_i$

Loop rule: $I_i = x_i \times \frac{V_{ref}}{2^{3-i} \times R}$

☞ $$I = x_3 \times \frac{V_{ref}}{R} + x_2 \times \frac{V_{ref}}{2 \times R} + x_1 \times \frac{V_{ref}}{4 \times R} + x_0 \times \frac{V_{ref}}{8 \times R} = \frac{V_{ref}}{8 \times R} \times \sum_{i=0}^{3} x_i \times 2^i$$

Loop rule*: $V + R_1 \times I' = 0$

$I \sim nat(x)$, where $nat(x)$: natural number represented by x;

Junction rule°: $I = I'$

Hence: $V + R_1 \times I = 0$

Finally: $$-V = V_{ref} \times \frac{R_1}{8 \times R} \sum_{i=0}^{3} x_i \times 2^i = V_{ref} \times \frac{R_1}{8 \times R} \times nat(x)$$

Op-amp turns current $I \sim nat(x)$ into a voltage $\sim nat(x)$

Sampling Theorem

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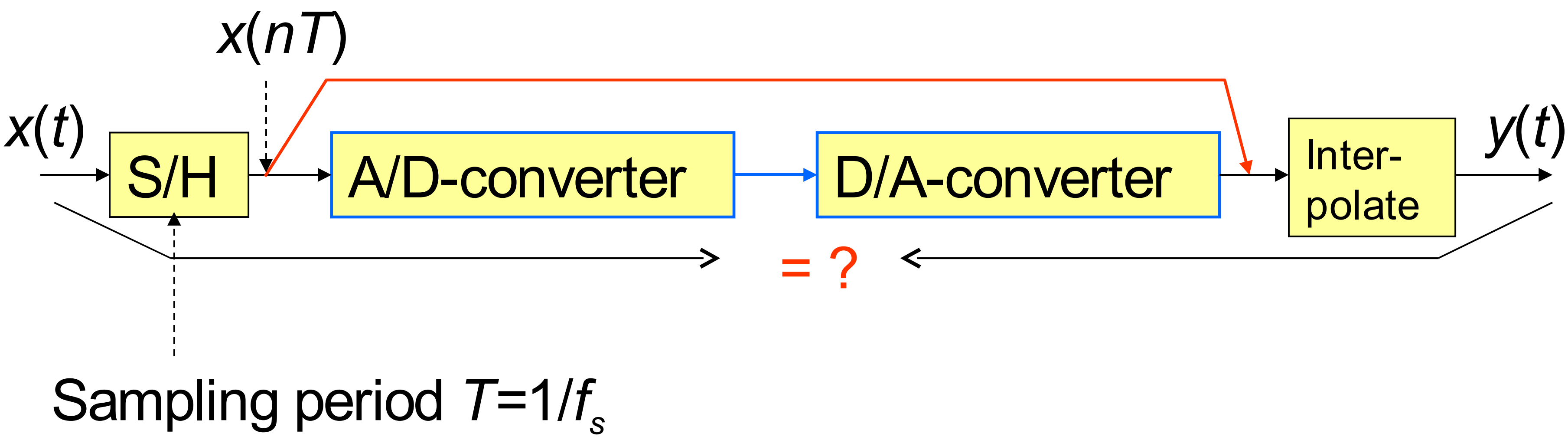
Possible to reconstruct original signal from discrete time series?

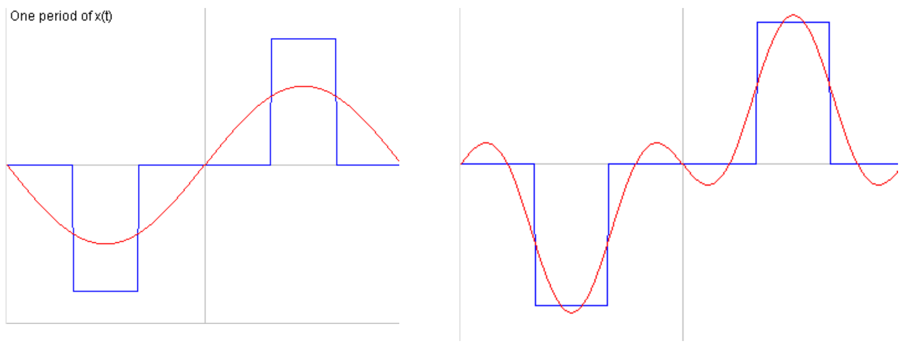


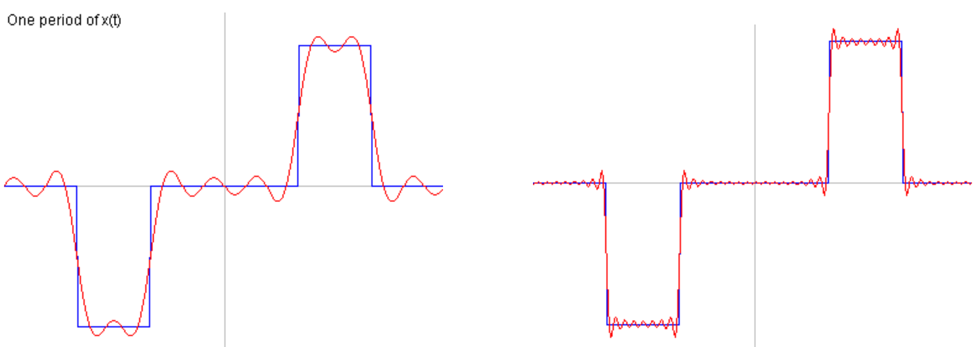
Approximation by sinoids

Fourier theorem: Any periodic signal with period $\omega_0 = 2\pi\nu_0$ can be approximated by sinoids of the form

$$x(t) = a_0 + a_1 \cos(\omega_o t + \theta_1) + a_2 \cos(2\omega_o t + \theta_2) + \ldots + a_N \cos(N\omega_o t + \theta_N)$$

Example:





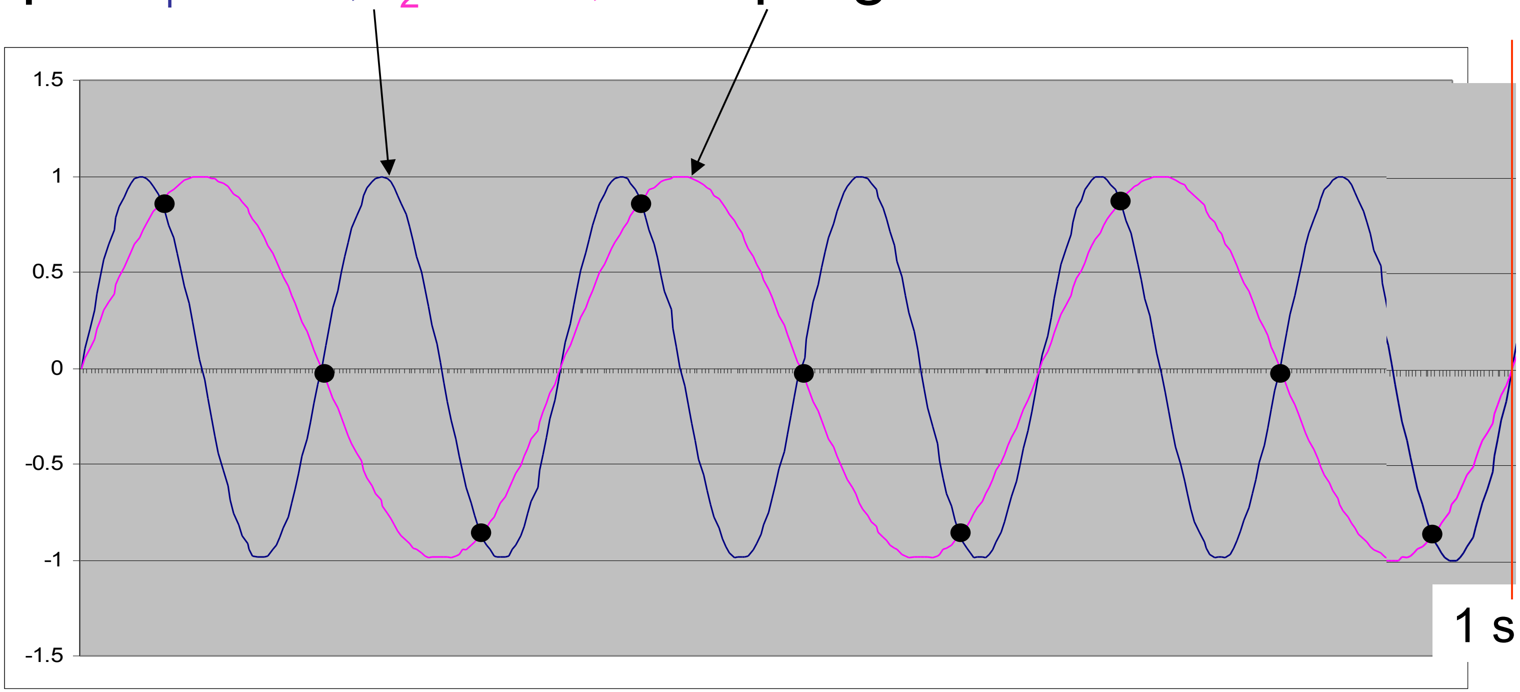
☞ We consider transformations of sinoids

© http://www.jhu.edu/~signals/fourier2/index.html

Limitations: example

Frequency components > $f_s/2$ cannot be distinguished from frequency components < $f_s/2$.

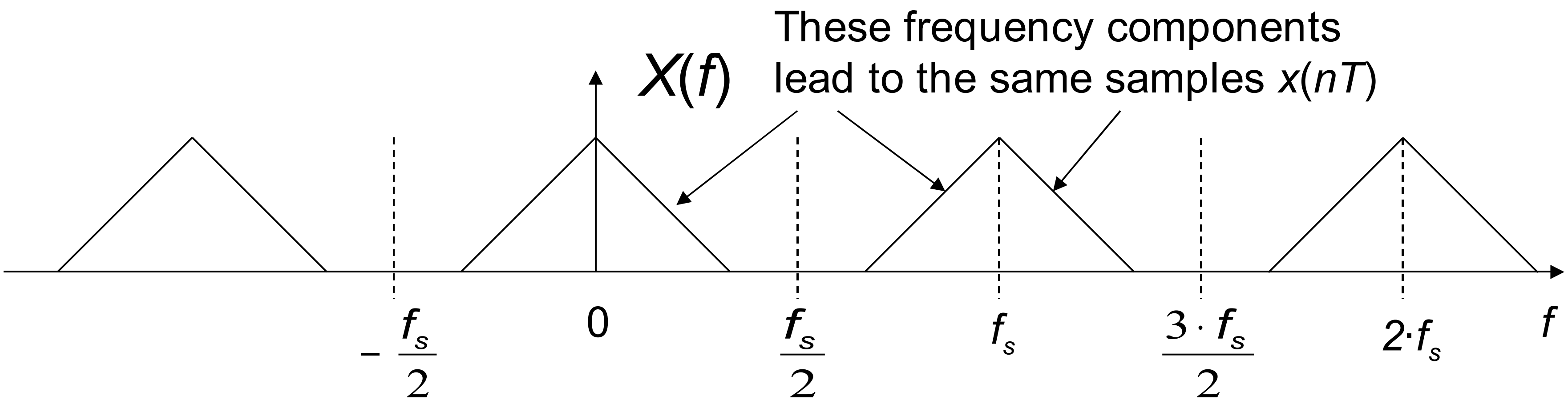
Example: f_1: 6 Hz; f_2: 3 Hz; Sampling: 9 Hz



$f_s/2=4.5$ Hz; $f_1-f_s/2=f_s/2-f_2=1.5$ Hz; samples for frequencies $f_s/2 \pm c$ identical

General view on frequencies leading to the same samples

Samples for frequencies $f_s/2 \pm c$ identical;
(Can be shown in general by considering symmetries of sine functions)



☞Reconstructing a time-continuous signal $x(t)$ requires that we **know** that only one of these frequency bands is used.

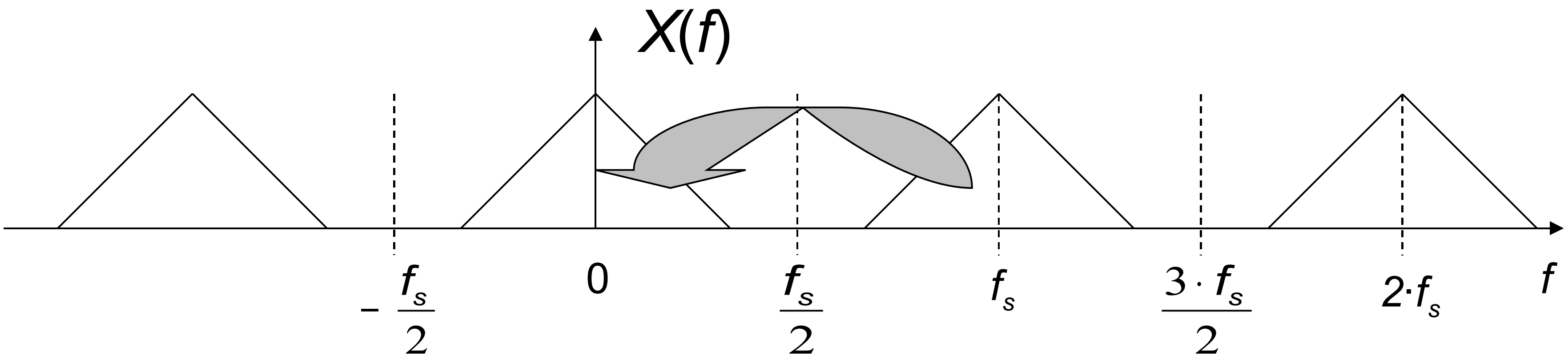
Assumption: we consider signals with $f \in [0..f_u]$ ("*base-band*") only.

☞Reconstructing a time-continuous signal after sampling with a sample frequency of f_s requires the signal to be bandwidth-limited to $f<f_s/2$.

$f_s/2$ is the **Nyquist frequency**.

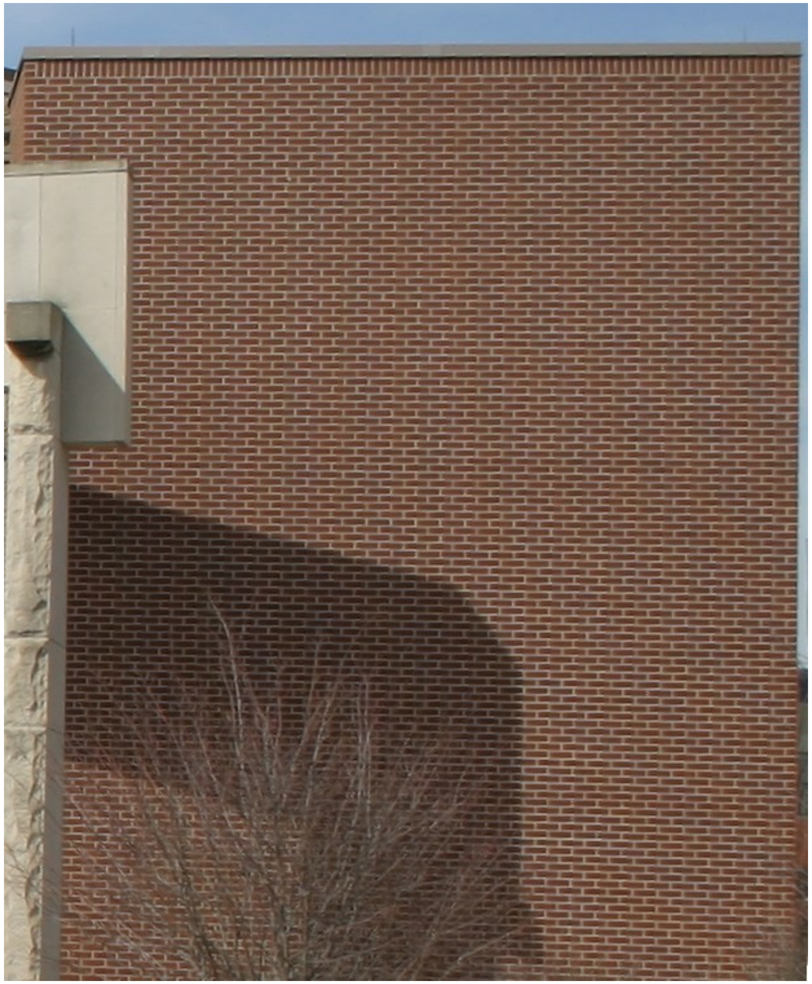
Aliasing

If the above condition is not met, **aliasing** occurs. Considering base-band signals only, frequency components with a frequency of $f_s/2+c$ are mapped to components at a frequency of $f_s/2-c$.



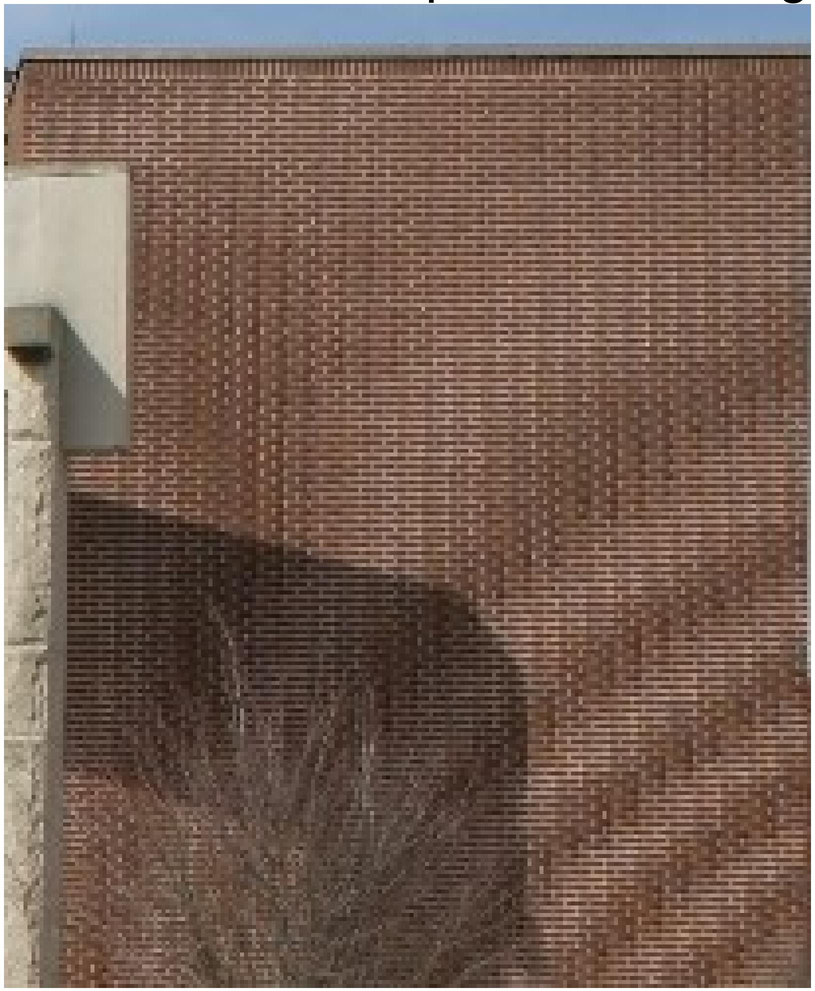
Examples of Aliasing in computer graphics*

Original



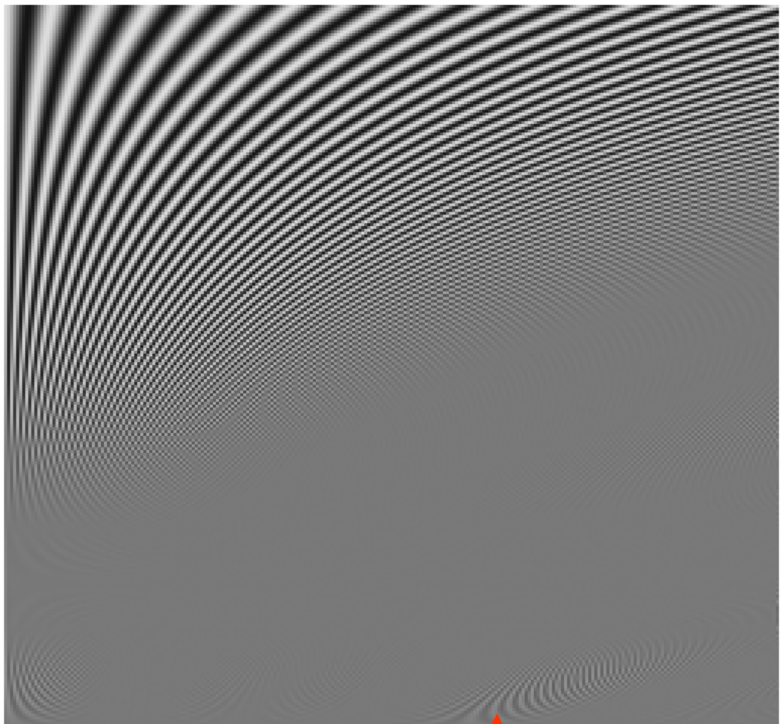
* sampling and filtering somewhat different from what we considered so far.

Sub-sampled, no filtering



Examples of Aliasing (2)

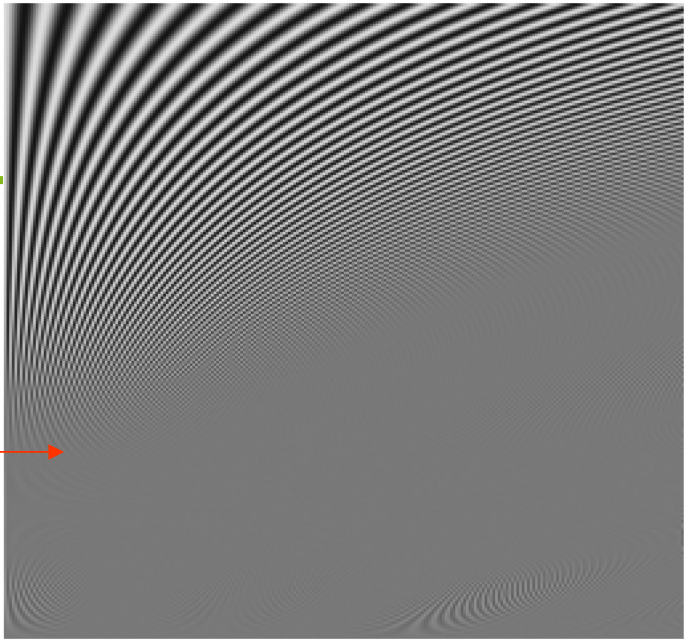
Original (pdf screen copy)



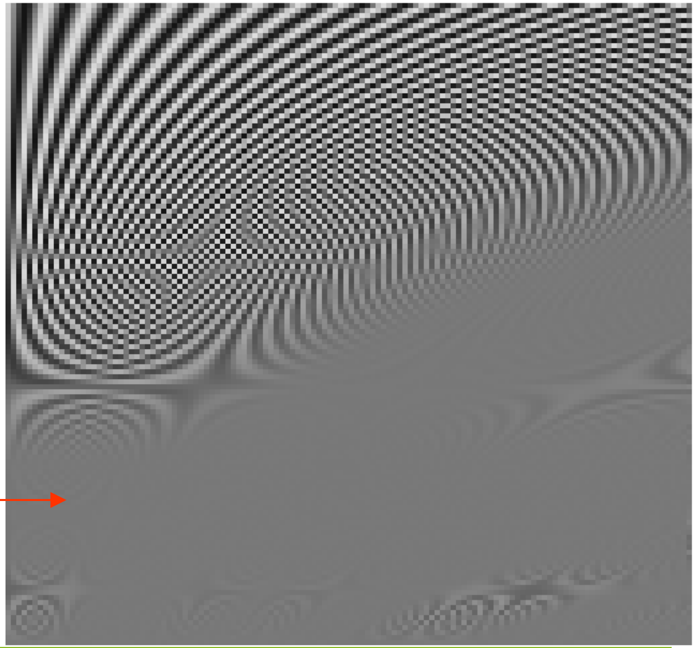
Impact of rasterization

http://www.niirs10.com/Resources/Reference Documents/Accuracy in Digital Image Processing.pdf

Filtered & sub-sampled

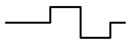


Sub-sampled, no filtering



How to generate good values for times $t \neq$ sampling times nT? (1)

A/D-converters do not interpolate between samples.



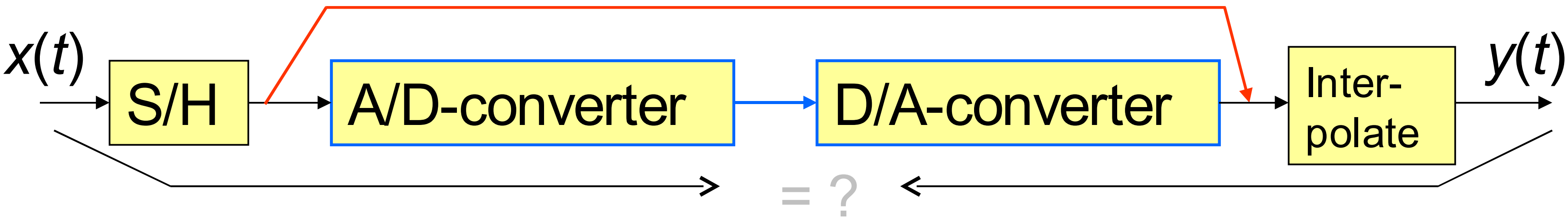
They generate step functions ☞ certainly not the original functions.

Theorem (Shannon and others):

Exact reconstruction of a continuous-time base-band signal from its samples is possible if the signal is band-limited and the sampling frequency is greater than twice the signal bandwidth.

$f=f_s/2$ has to be excluded as well.

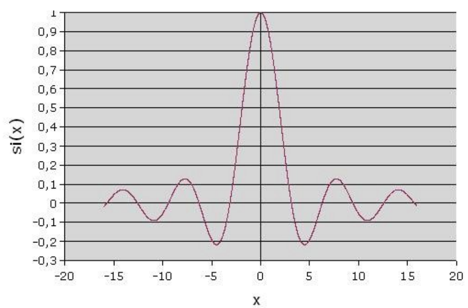
How to generate good values for times $t \neq$ sampling times nT? (2)



The necessary interpolation is based on the *sinc* function:

$$x(t) = y(t) := \sum_{k=-\infty}^{\infty} x_k \prod_{j\in\mathbb{Z},\, j\neq k} \frac{t-jT}{kT-jT} = \sum_{k=-\infty}^{\infty} x(kT)\,\mathrm{sinc}(t/T-k) \quad (1)$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



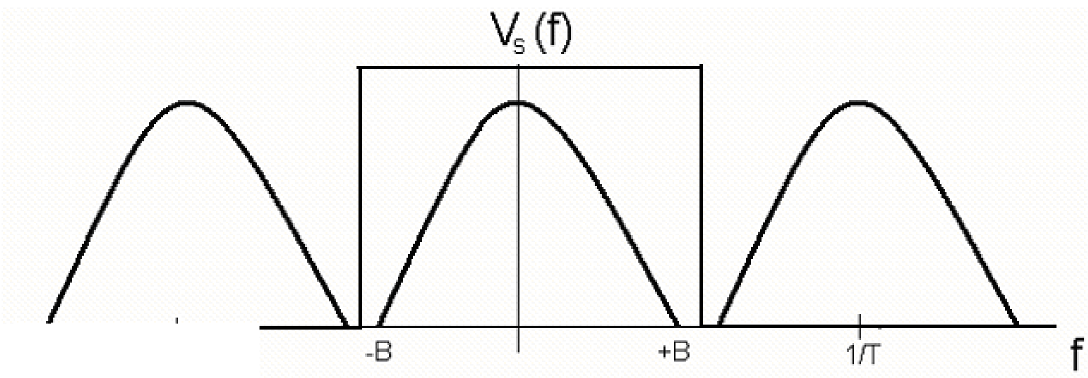
(1) is due to Whittacker (1915)

Each sample $x(nT)$ influences its neighborhood $t \neq nT$ with a weighting factor *sinc*($t/T-k$), which is zero at other sampling times.

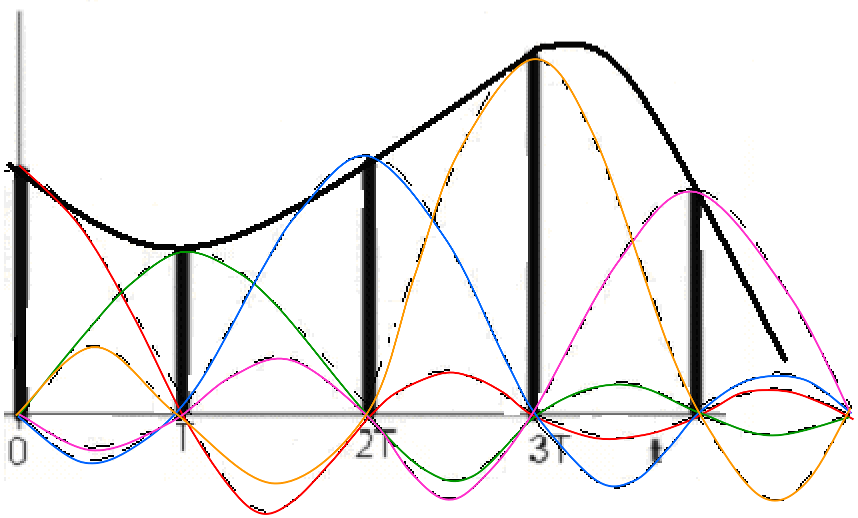
Equations & graphics: de.wikipedia.org

Motivating (1)

To recover the original signal, multiply by a pulse in the frequency domain.



sinc is Fourier transform of *rect* function



This is equivalent to convolving with a *sinc* in the time domain

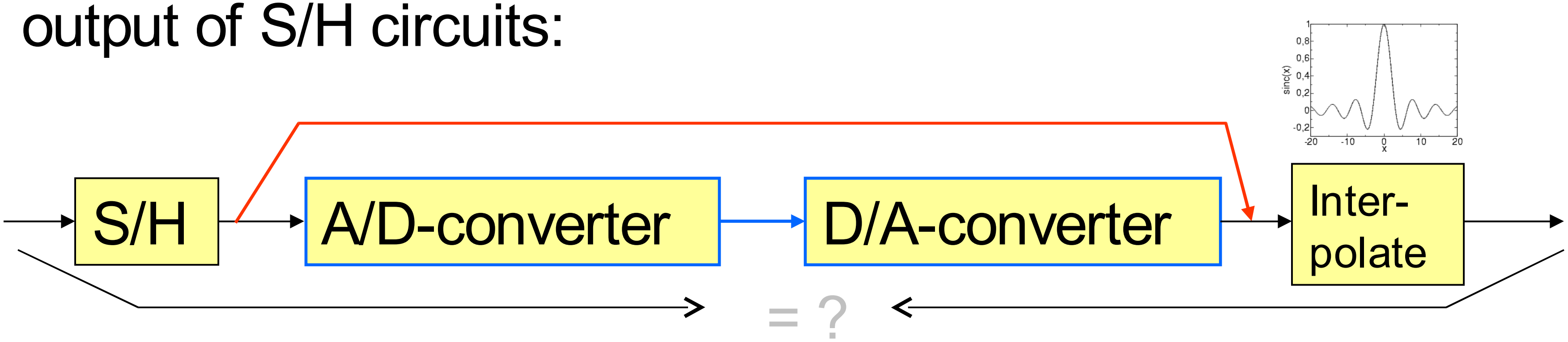
Limitations

$$x(t) = y(t) := \sum_{k=-\infty}^{\infty} x_k \prod_{j\in\mathbb{Z},\, j\neq k} \frac{t - jT}{kT - jT} = \sum_{k=-\infty}^{\infty} x(kT)\,\mathrm{sinc}(t/T - k)$$

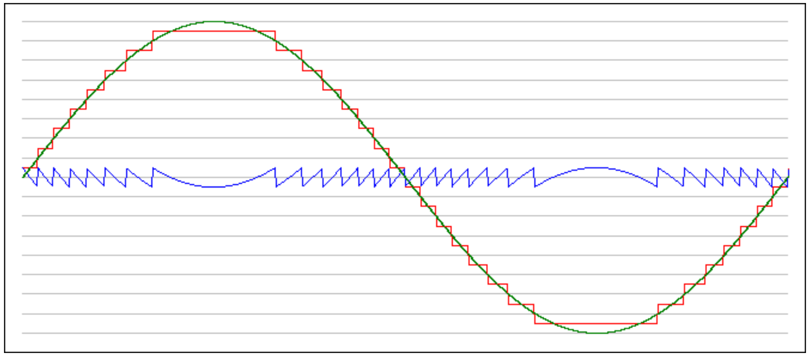
- Actual A/D converters do not compute the *sinc* function. Actually, other functions allow reconstruction as well. In practice, filters are used as an approximation. Computing good filters is an art itself!
- All samples must be known to completely reconstruct $x(t)$; this means, we have to wait indefinitely before we can generate output. In practice, we have to generate output from a finite set of samples.
- Actual signals are never perfectly bandwidth limited.

Impact of quantization noise

We can only hope to reconstruct originals signals from the output of S/H circuits:



Signals from the output of the A/D-converter contain quantization noise;



* [http://www.beis.de/Elektronik/DeltaSigma/DeltaSigma.html]

This noise cannot be removed.

Actuators

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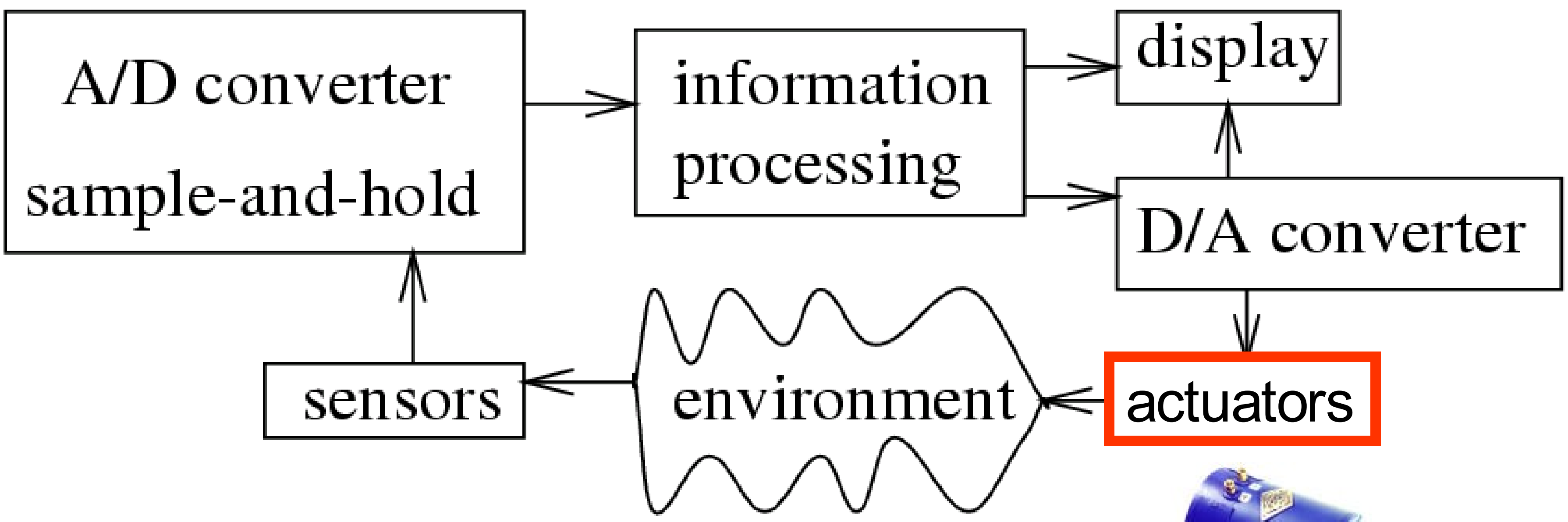
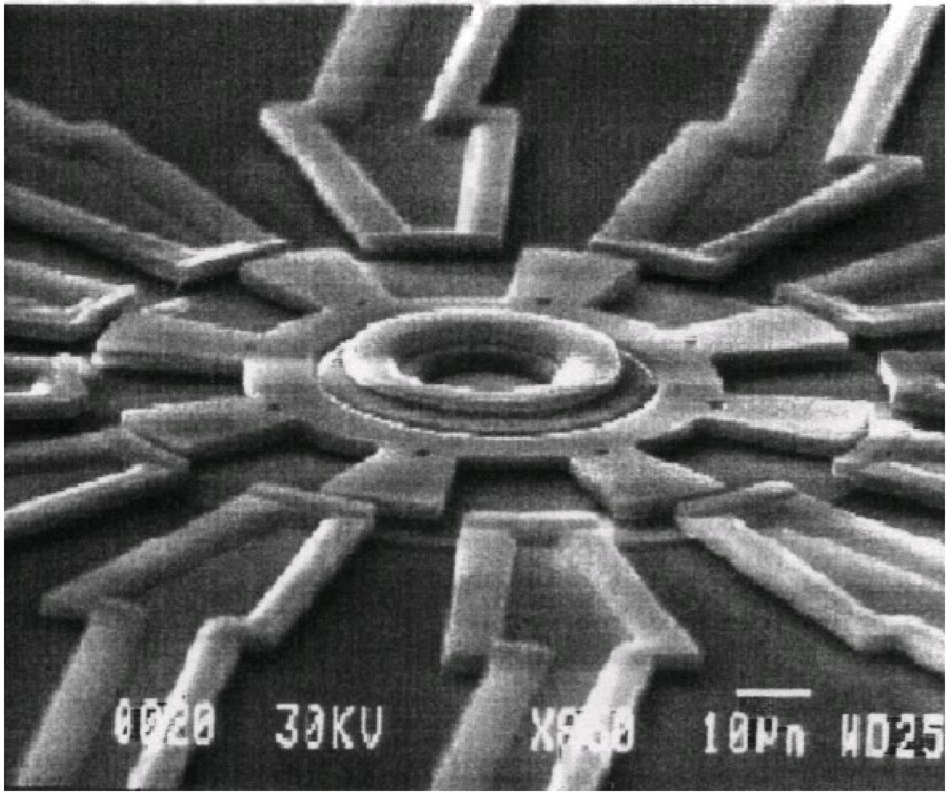
Embedded System Hardware

Embedded system hardware is frequently used in a loop („***hardware in a loop***“):



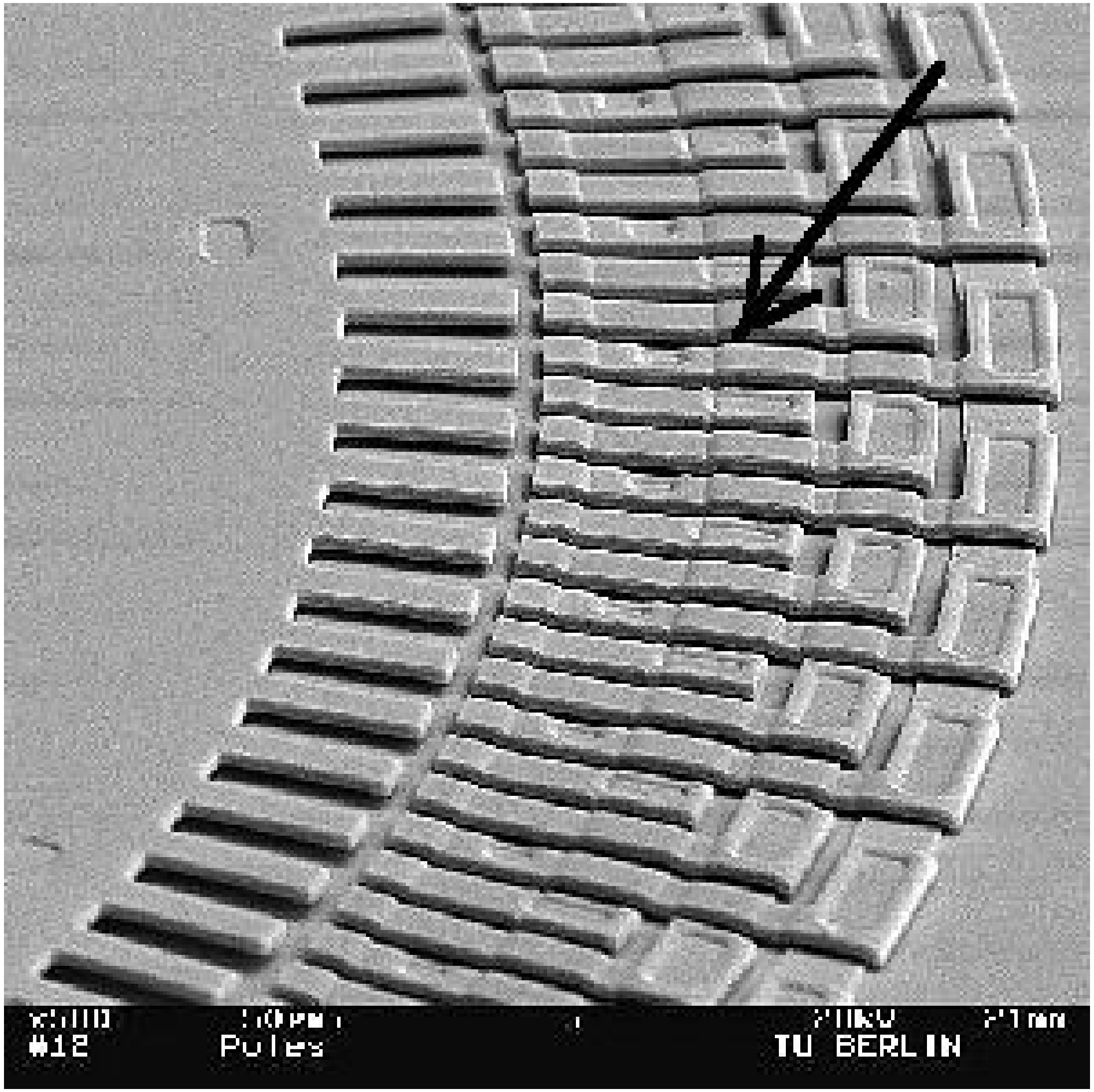
Actuators

Huge variety of actuators and output devices, impossible to present all of them.
Microsystems motors as examples (© MCNC):



(© MCNC)

Actuators (2)



Courtesy and ©:
E. Obermeier, MAT,
TU Berlin

Stepper Motor

- Stepper motor: rotates fixed number of degrees when given a “step” signal.
- In contrast, DC motor just rotates when power applied.
- Rotation achieved by applying specific voltage sequence to coils
- Controller greatly simplifies this

http://www.cise.ufl.edu/~prabhat/Teaching/cis6930-f04/comp4.pdf

Summary

Hardware in a loop

- Sensors
- Discretization
- Information processing
  - Importance of energy efficiency
  - Special purpose HW very expensive
  - Energy efficiency of processors
  - Code size efficiency
  - Run-time efficiency
  - Reconfigurable Hardware
- D/A converters
- Actuators