Mapping:
Applications → Processors

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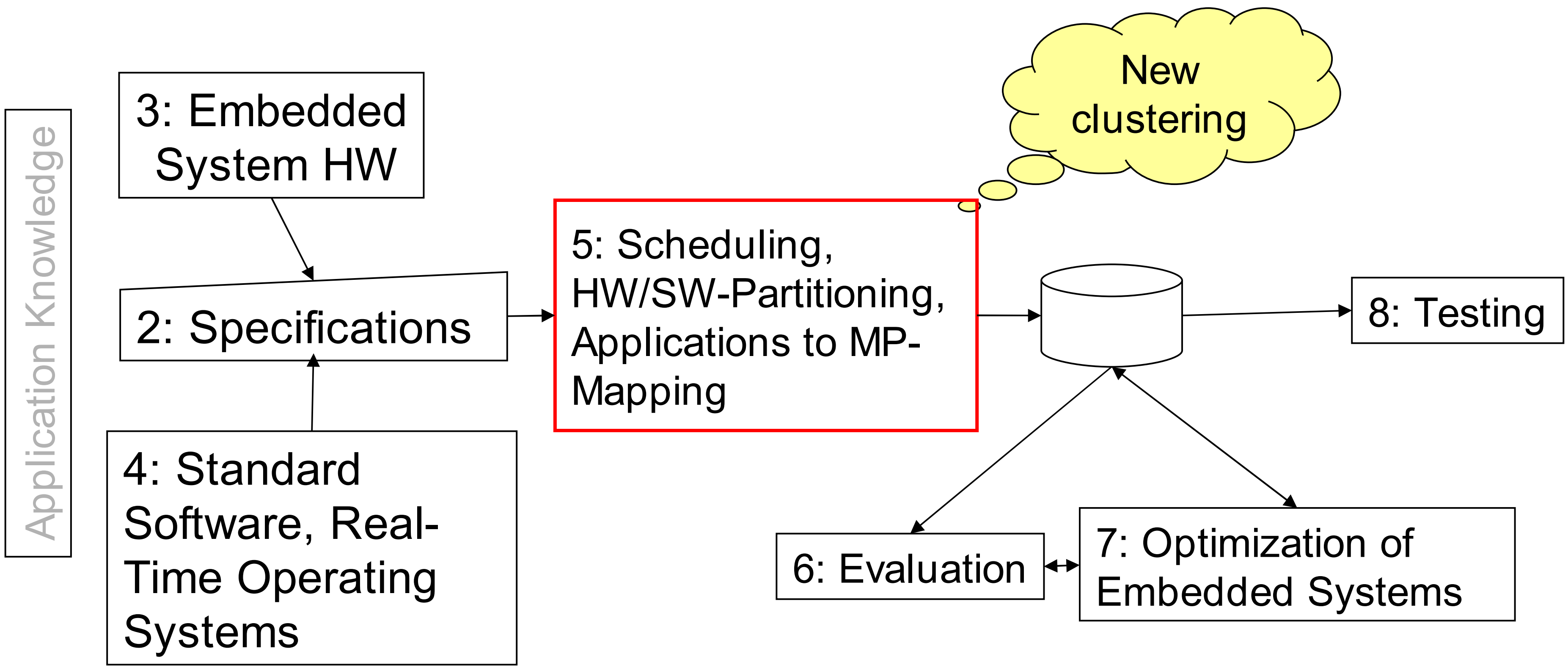
Structure of this course



Book chapter 4.2

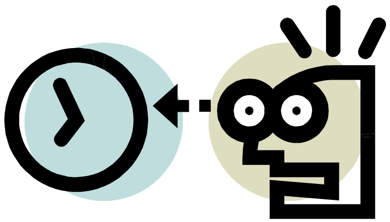
Scope of mapping algorithms

Useful terms from hardware synthesis:

- **Resource Allocation**
  Decision concerning type and number of available resources
- **Resource Assignment**
  Mapping: Task → (Hardware) Resource
- **xx to yy binding:**
  Describes a mapping from behavioral to structural domain, e.g. task to processor binding, variable to memory binding
- **Scheduling**
  Mapping: Tasks → Task start times
  Sometimes, resource assignment is considered being included in scheduling.

Real-time scheduling

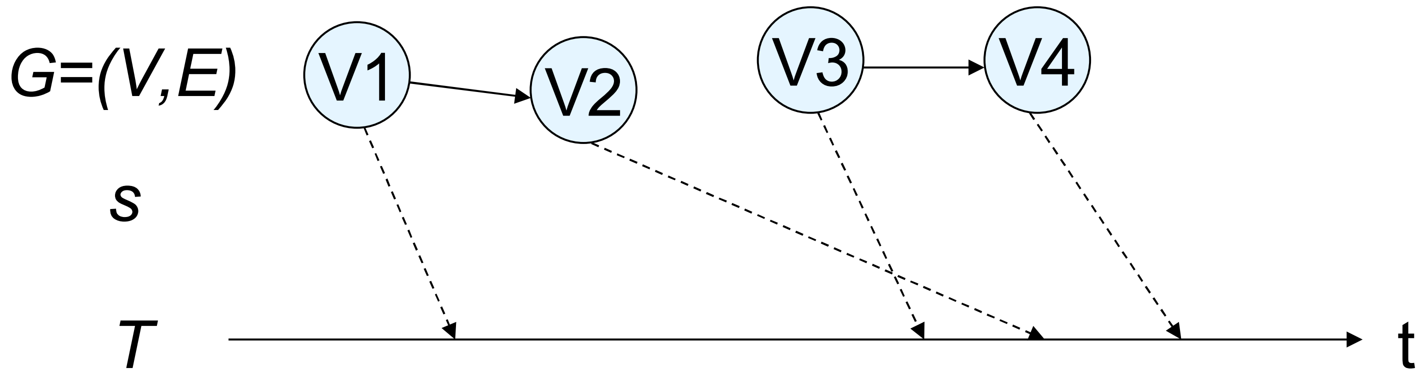
Assume that we are given a task graph *G=(V,E)*.



Def.: A **schedule** *s* of *G* is a mapping

$$V \rightarrow T$$

of a set of tasks *V* to start times from domain *T*.

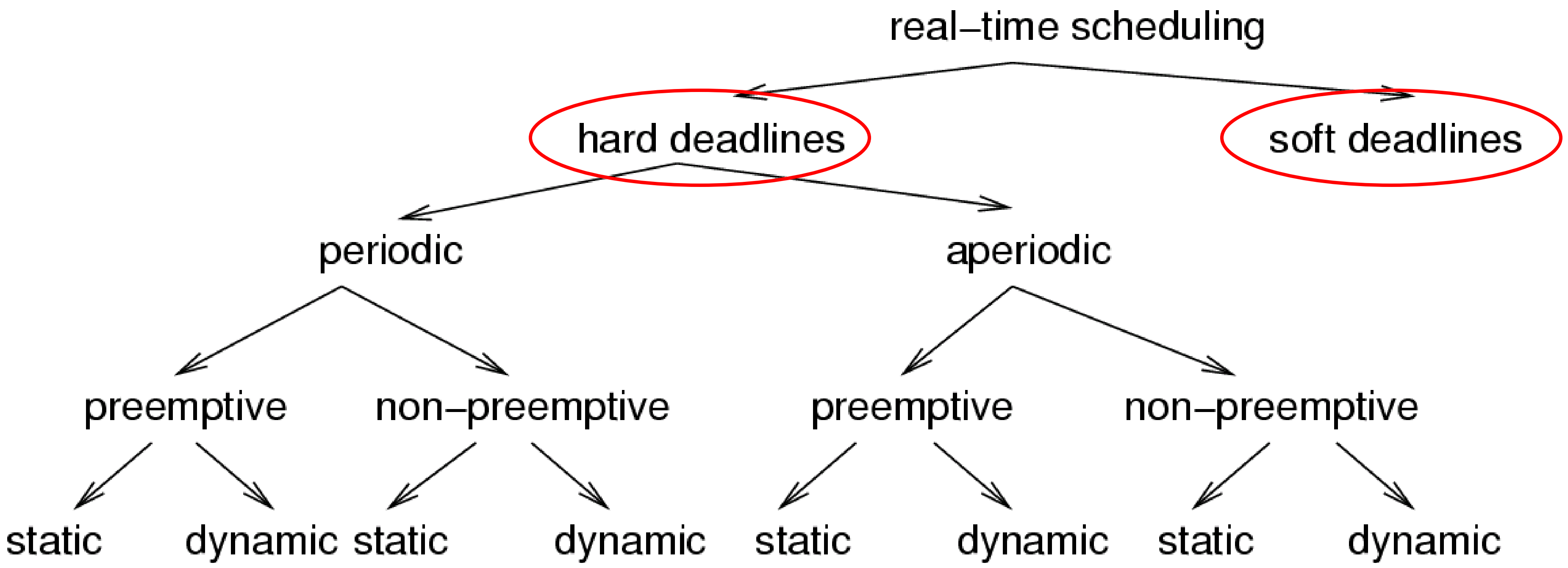


Typically, schedules have to respect a number of constraints, incl. resource constraints, dependency constraints, deadlines.
Scheduling = finding such a mapping.

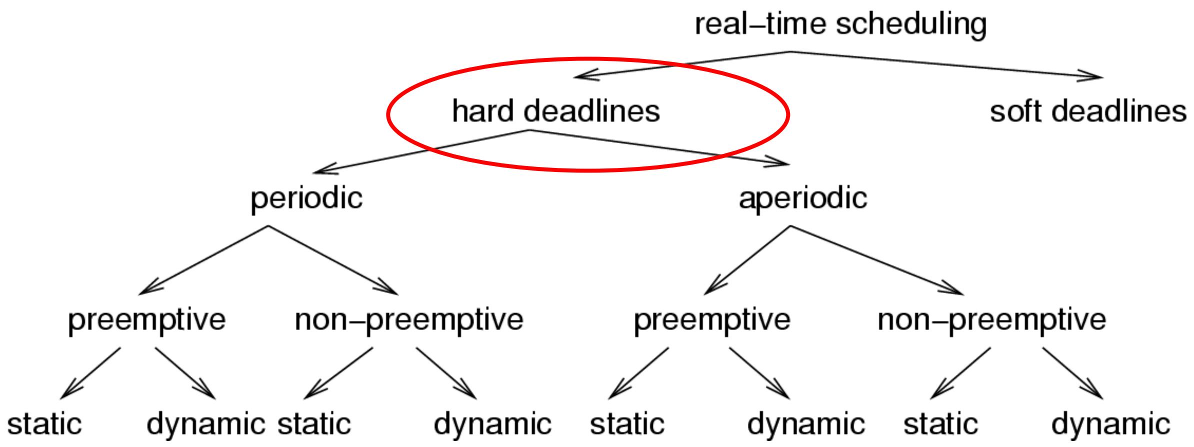
Classes of mapping algorithms considered in this course

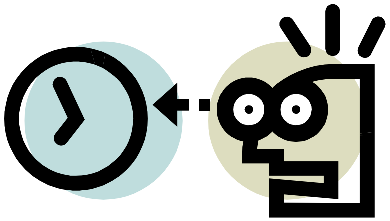
- **Classical scheduling algorithms**
  Mostly for independent tasks & ignoring communication, mostly for mono- and homogeneous multiprocessors
- **Hardware/software partitioning**
  Dependent tasks, heterogeneous systems, focus on resource assignment
- **Dependent tasks as considered in architectural synthesis**
  Initially designed in different context, but applicable
- **Design space exploration using genetic algorithms**
  Heterogeneous systems, incl. communication modeling

Classification of scheduling algorithms



Hard and soft deadlines



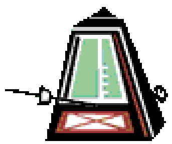


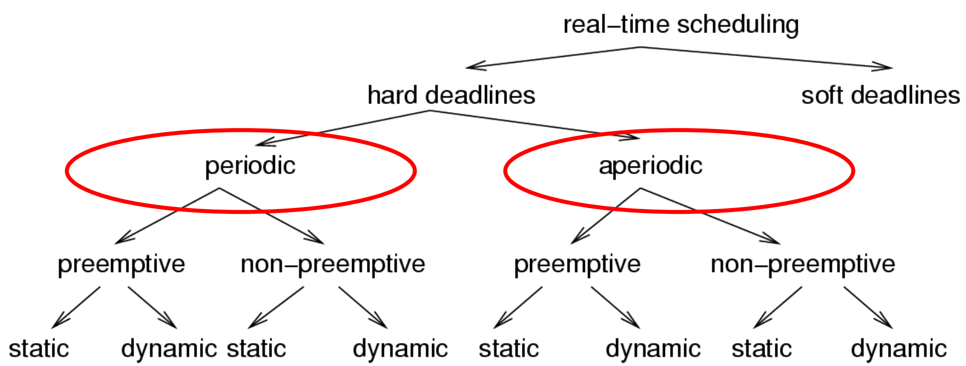
Def.: A time-constraint (deadline) is called **hard** if not meeting that constraint could result in a catastrophe [Kopetz, 1997].

All other time constraints are called **soft**.

We will focus on hard deadlines.

Periodic and aperiodic tasks



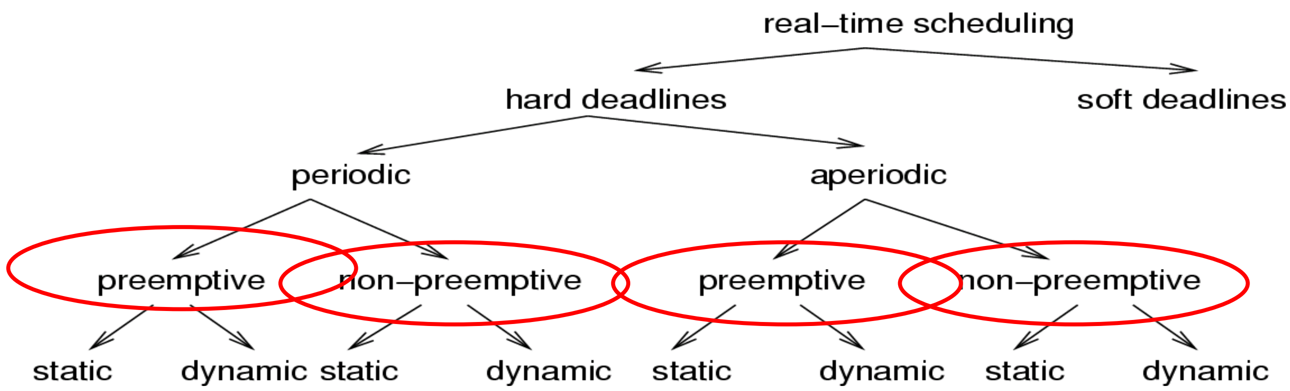


Def.: Tasks which must be executed once every *p* units of time are called **periodic** tasks. *p* is called their period. Each execution of a periodic task is called a **job**.

All other tasks are called **aperiodic**.

Def.: Tasks requesting the processor at unpredictable times are called **sporadic**, if there is a minimum separation between the times at which they request the processor.

Preemptive and non-preemptive scheduling



- **Non-preemptive schedulers:**

  Tasks are executed until they are done.

  Response time for external events may be quite long.
- **Preemptive schedulers:** To be used if
  - some tasks have long execution times or
  - if the response time for external events to be short.

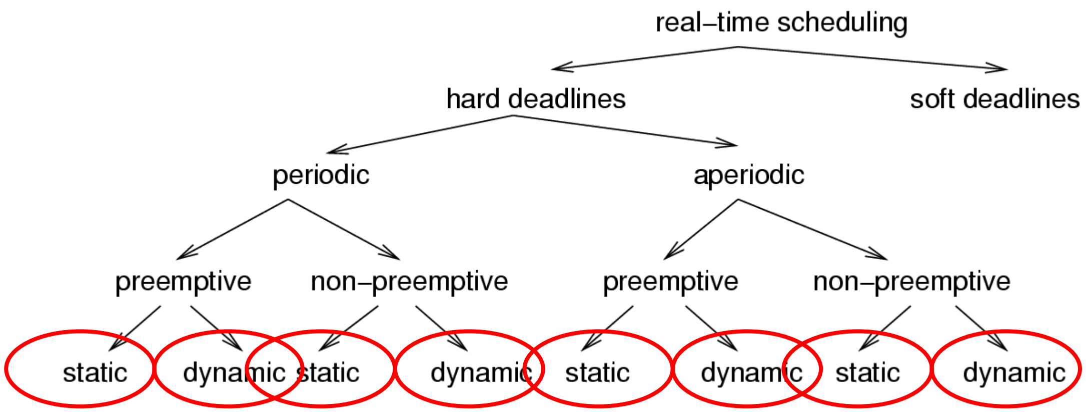
Centralized and distributed scheduling

- **Centralized and distributed scheduling:** Multiprocessor scheduling either locally on 1 or on several processors.

- **Mono- and multi-processor scheduling:**
  - Simple scheduling algorithms handle single processors,
  - more complex algorithms handle multiple processors.
    - algorithms for homogeneous multi-processor systems
    - algorithms for heterogeneous multi-processor systems (includes HW accelerators as special case).

Dynamic/online scheduling

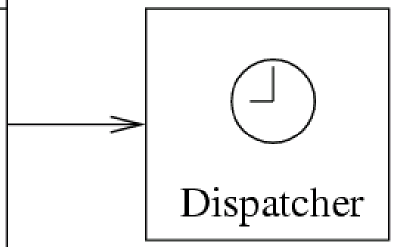
- **Dynamic/online scheduling:** Processor allocation decisions (scheduling) at run-time; based on the information about the tasks arrived so far.



Static/offline scheduling

- **Static/offline scheduling:**
  Scheduling taking a priori knowledge about arrival times, execution times, and deadlines into account. Dispatcher allocates processor when interrupted by timer. Timer controlled by a table generated at design time.

Time	Action	WCET
10	start T1	12
17	send M5	
22	stop T1	
38	start T2	20
47	send M3	



Time-triggered systems (1)

In an entirely time-triggered system, the temporal control structure of all tasks is established ***a priori*** *by off-line support-tools. This temporal control structure is encoded in a* ***Task-Descriptor List (TDL)*** *that contains the cyclic schedule for all activities of the node. This schedule considers the required precedence and mutual exclusion relationships among the tasks such that an explicit coordination of the tasks by the operating system at run time is not necessary. ..*

The dispatcher is activated by the synchronized clock tick. It looks at the TDL, and then performs the action that has been planned for this instant [Kopetz].

Time	Action	WCET
10	start T1	12
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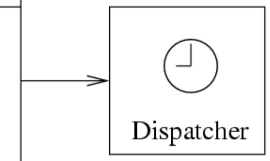


Time-triggered systems (2)

… pre-run-time scheduling is often the only practical means of providing predictability in a complex system. [Xu, Parnas].

It can be easily checked if timing constraints are met.
The disadvantage is that the response to sporadic events may be poor.

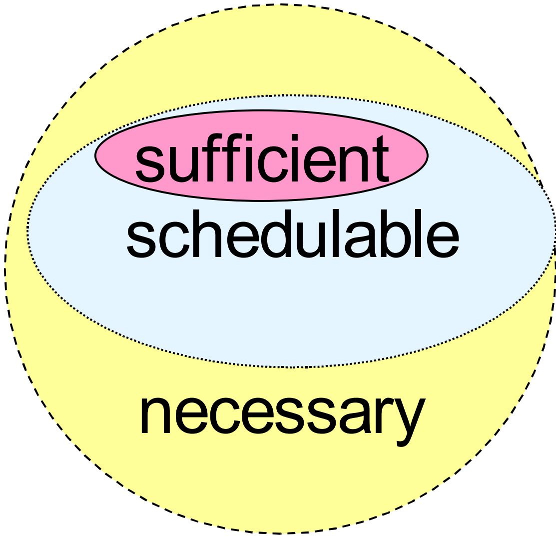
Schedulability

Set of tasks is **schedulable** under a set of constraints, if a schedule exists for that set of tasks & constraints.

Exact tests are NP-hard in many situations.

Sufficient tests: sufficient conditions for schedule checked. (Hopefully) small probability of not guaranteeing a schedule even though one exists.

Necessary tests: checking necessary conditions. Used to show no schedule exists. There may be cases in which no schedule exists & we cannot prove it.



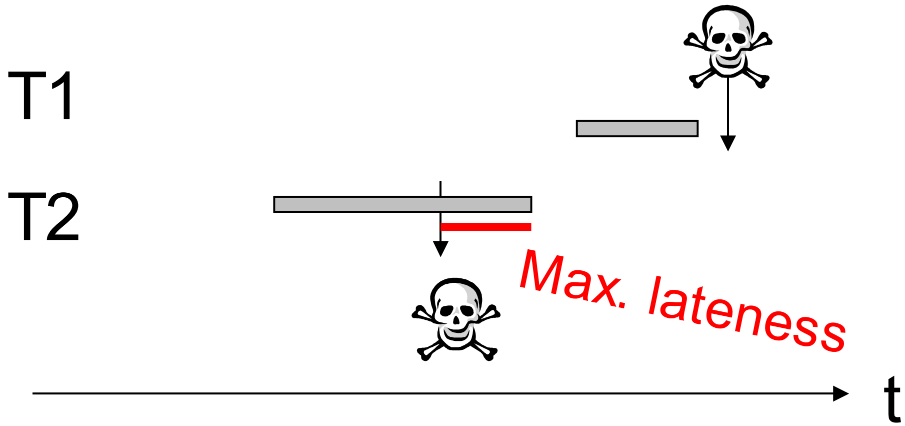
Cost functions

Cost function: Different algorithms aim at minimizing different functions.

Def.: Maximum lateness =

$\mathbf{max}_{\text{all tasks}}$ (completion time – deadline)

Is <0 if all tasks complete before deadline.



Classical scheduling algorithms for aperiodic systems

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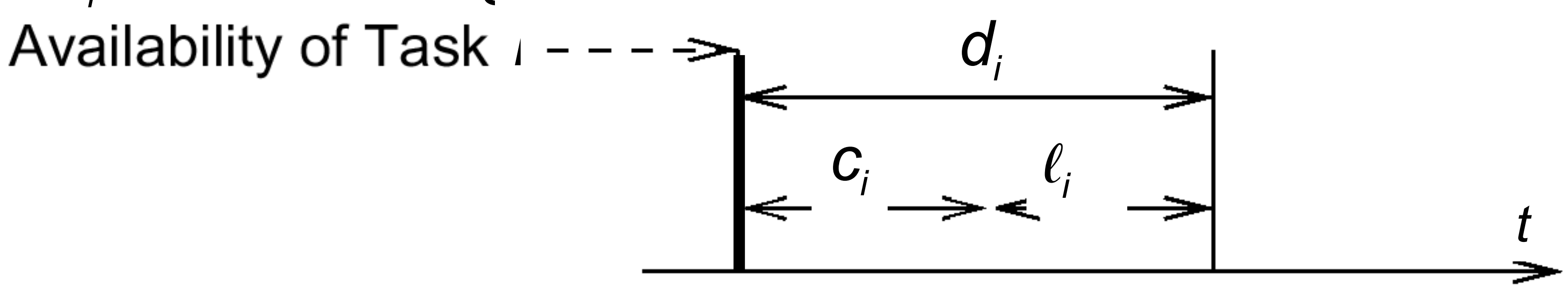
Aperiodic scheduling
- Scheduling with no precedence constraints -

Let $\{T_i\}$ be a set of tasks. Let:

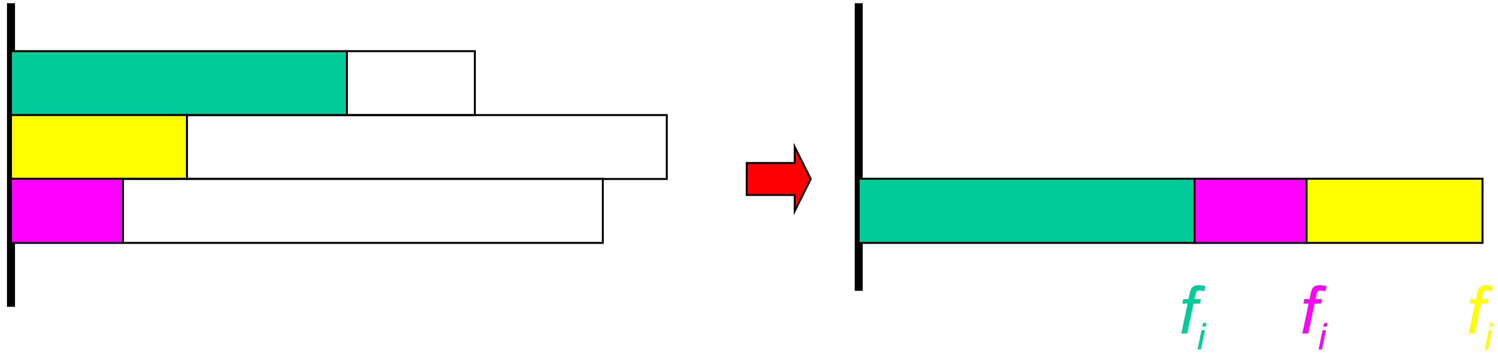
- c_i be the execution time of T_i,
- d_i be the **deadline interval**, that is, the time between T_i becoming available and the time until which T_i has to finish execution.
- ℓ_i be the **laxity** or **slack**, defined as $\ell_i = d_i - c_i$
- f_i be the finishing time.



Uniprocessor with equal arrival times

Preemption is useless.

Earliest Due Date (EDD): Execute task with earliest due date (deadline) first.



EDD requires all tasks to be sorted by their (absolute) deadlines. Hence, its complexity is $O(n \log(n))$.

Optimality of EDD

EDD is optimal, since it follows Jackson's rule:
Given a set of *n* independent tasks, any algorithm that executes the tasks in order of non-decreasing (absolute) deadlines is optimal with respect to minimizing the maximum lateness.

Proof (See Buttazzo, 2002):

- Let σ be a schedule produced by any algorithm *A*
- If $A \neq$ EDD $\rightarrow \exists\, T_a, T_b, d_a \leq d_b, T_b$ immediately precedes T_a in σ.
- Let σ' be the schedule obtained by exchanging T_a *and* T_b.

Exchanging T_a and T_b cannot increase lateness

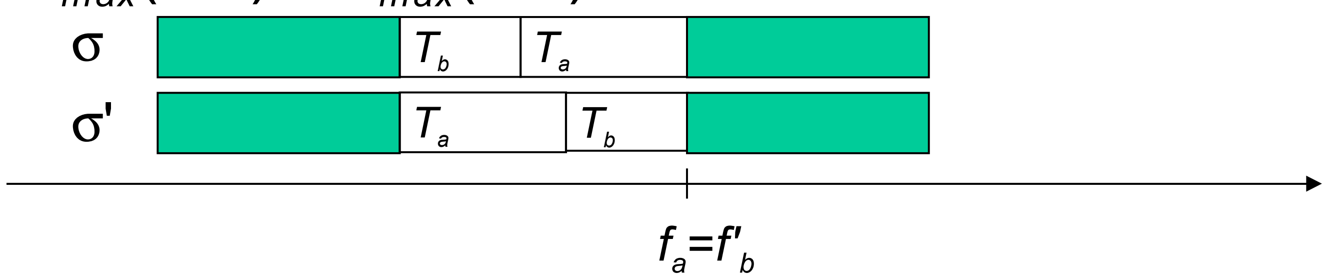
Max. lateness for T_a and T_b in σ is $L_{max}(a,b)=f_a\text{-}d_a$

Max. lateness for T_a and T_b in σ' is $L'_{max}(a,b)=\max(L'_a,L'_b)$

Two possible cases

1. $L'_a \geq L'_b: \rightarrow L'_{max}(a,b) = f'_a - d_a < f_a - d_a = L_{max}(a,b)$
   since T_a starts earlier in schedule σ'.
2. $L'_a \leq L'_b: \rightarrow L'_{max}(a,b) = f'_b - d_b = f_a - d_b \leq f_a - d_a = L_{max}(a,b)$ since $f_a = f'_b$ and $d_a \leq d_b$

☞ $L'_{max}(a,b) \leq L_{max}(a,b)$



EDD is optimal

- ☞ Any schedule σ with lateness L can be transformed into an EDD schedule σ^n with lateness $L^n \leq L$, which is the minimum lateness.
- ☞ EDD is optimal (q.e.d.)

Earliest Deadline First (EDF) - Horn's Theorem -

Different arrival times: Preemption potentially reduces lateness.

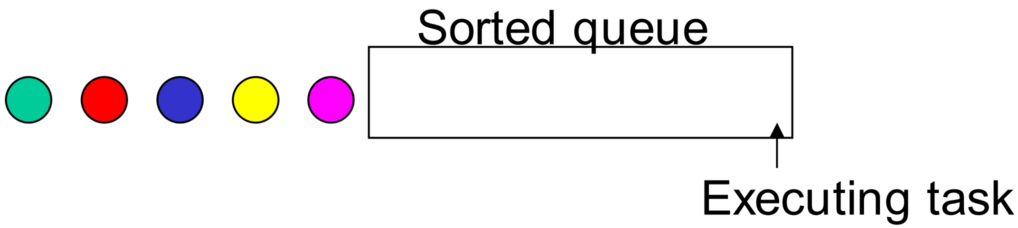
Theorem [Horn74]: Given a set of *n* independent tasks with arbitrary arrival times, any algorithm that at any instant executes the task with the earliest absolute deadline among all the ready tasks is optimal with respect to minimizing the maximum lateness.

Earliest Deadline First (EDF) - Algorithm -

Earliest deadline first (EDF) algorithm:

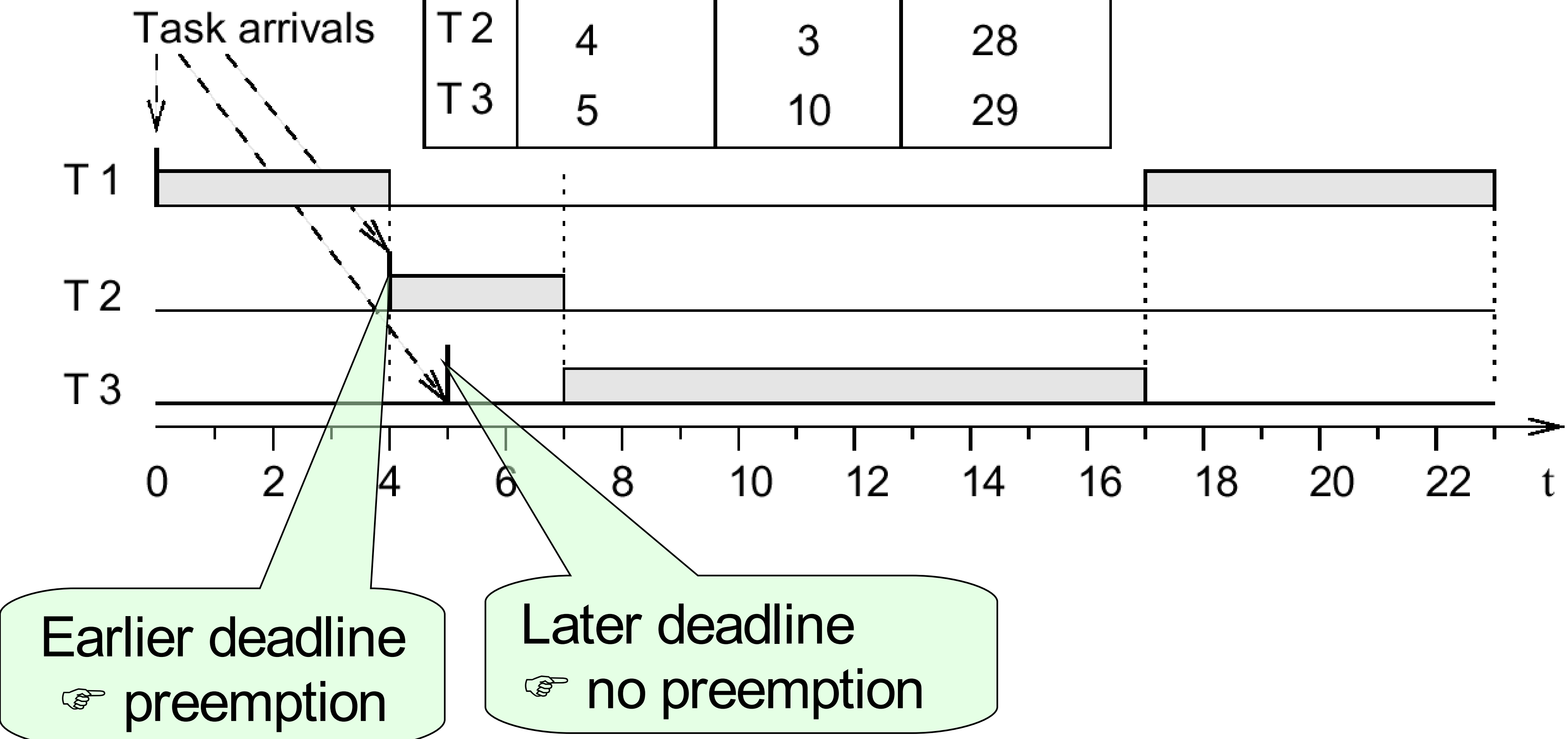
- Each time a new ready task arrives:
- It is inserted into a queue of ready tasks, sorted by their **absolute** deadlines. Task at head of queue is executed.
- If a newly arrived task is inserted at the head of the queue, the currently executing task is preempted.

Straightforward approach with sorted lists (full comparison with existing tasks for each arriving task) requires run-time $O(n^2)$; (less with binary search or bucket arrays).



Earliest Deadline First (EDF)
- Example -

	arrival	duration	deadline
T 1	0	10	33
T 2	4	3	28
T 3	5	10	29



Optimality of EDF

To be shown: EDF minimizes maximum lateness.

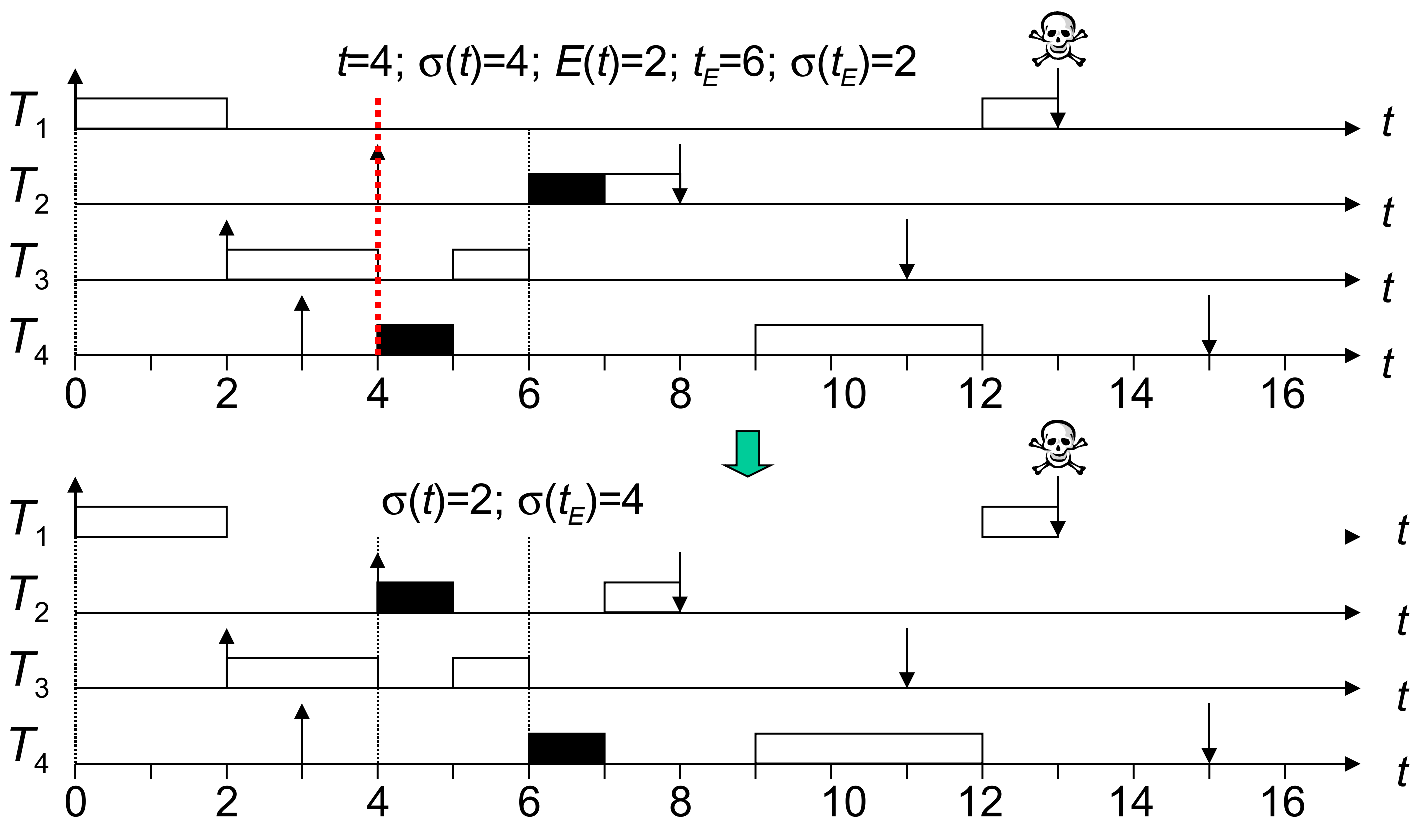
Proof (Buttazzo, 2002):

- Let σ be a schedule produced by generic schedule *A*
- Let σ_{EDF}: schedule produced by EDF
- Preemption allowed: tasks executed in disjoint time intervals
- σ divided into time slices of 1 time unit each
- Time slices denoted by $[t, t+1)$
- Let $\sigma(t)$: task executing in $[t, t+1)$
- Let $E(t)$: task which, at time t, has the earliest deadline
- Let $t_E(t)$: time ($\geq t$) at which the next slice of task $E(t)$ begins its execution in the current schedule

Optimality of EDF (2)

If $\sigma \neq \sigma_{EDF}$, then there exists time t: $\sigma(t) \neq E(t)$

Idea: swapping $\sigma(t)$ and $E(t)$ cannot increase max. lateness.



If $\sigma(t)$ starts at t=0 and $D=\max_i\{d_i\}$ then σ_{EDF} can be obtained from σ by at most D transpositions.

[Buttazzo, 2002]

Optimality of EDF (3)

Algorithm `interchange`:
{ **for** (t=0 **to** D-1) {
  if ($\sigma(t) \neq E(t)$) {
   $\sigma(t_E) = \sigma(t)$;
   $\sigma(t) = E(t)$; }}}

Using the same argument as in the proof of Jackson's algorithm, it is easy to show that swapping cannot increase maximum lateness; hence EDF is optimal.

Does `interchange` preserve schedulability?

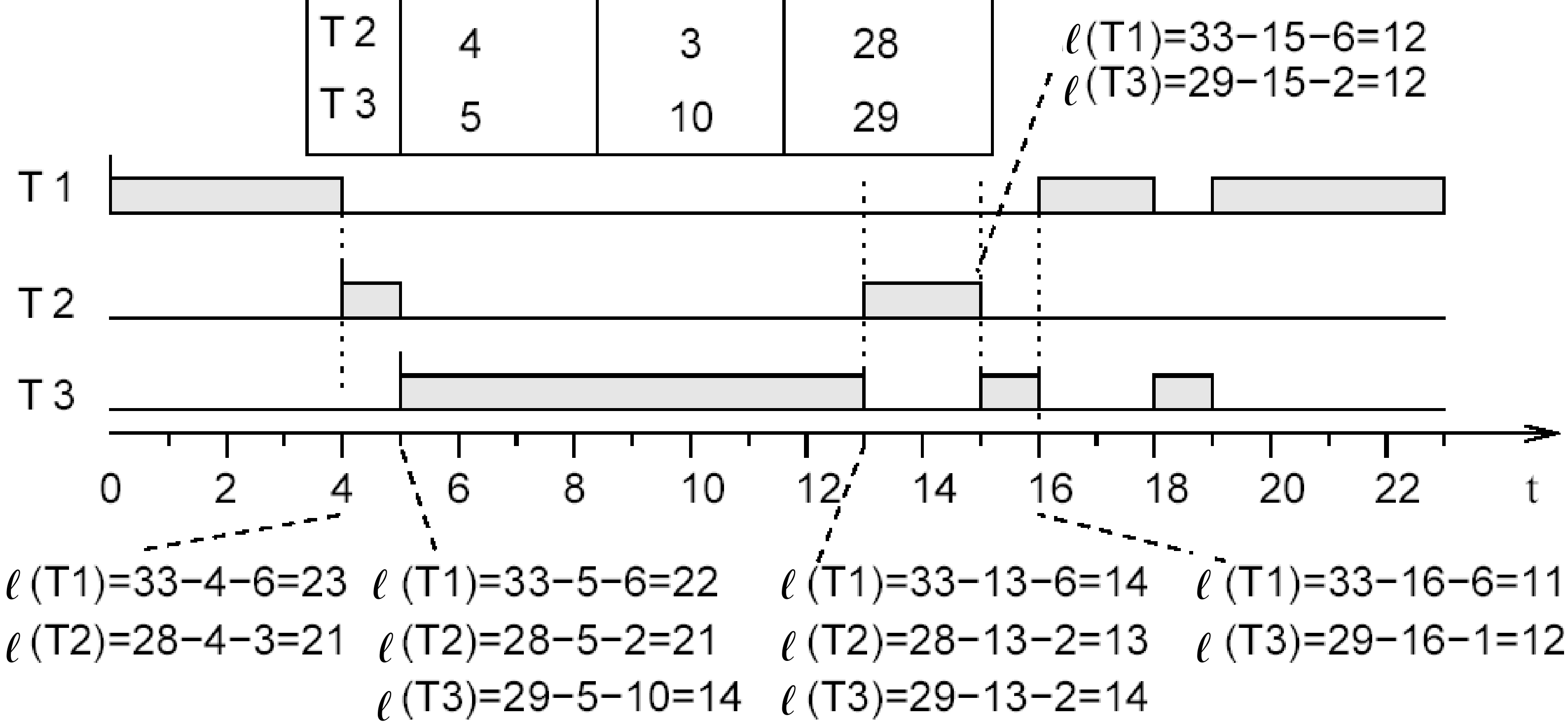
1. task $E(t)$ moved ahead: meeting deadline in new schedule if meeting deadline in σ
2. task $\sigma(t)$ delayed: if $\sigma(t)$ is feasible, then $(t_E+1) \leq d_E$, where d_E is the earliest deadline. Since $d_E \leq d_i$ for any i, we have $t_E+1 \leq d_i$, which guarantees schedulability of the delayed task.

q.e.d.

Least laxity (LL), Least Slack Time First (LST)

Priorities = decreasing function of the laxity (the less laxity, the higher the priority); dynamically changing priority; preemptive.

	arrival	duration	deadline
T 1	0	10	33
T 2	4	3	28
T 3	5	10	29



Properties

- Not sufficient to call scheduler & re-compute laxity just at task arrival times.
- Overhead for calls of the scheduler.
- Many context switches.
- **Detects missed deadlines early.**
- LL is also an optimal scheduling for mono-processor systems.
- Dynamic priorities ☞ cannot be used with a fixed prio OS.
- LL scheduling requires the knowledge of the execution time.

Scheduling without preemption (1)

Lemma: If preemption is not allowed, optimal schedules may have to leave the processor idle at certain times.

Proof: Suppose: optimal schedulers never leave processor idle.

Scheduling without preemption (2)

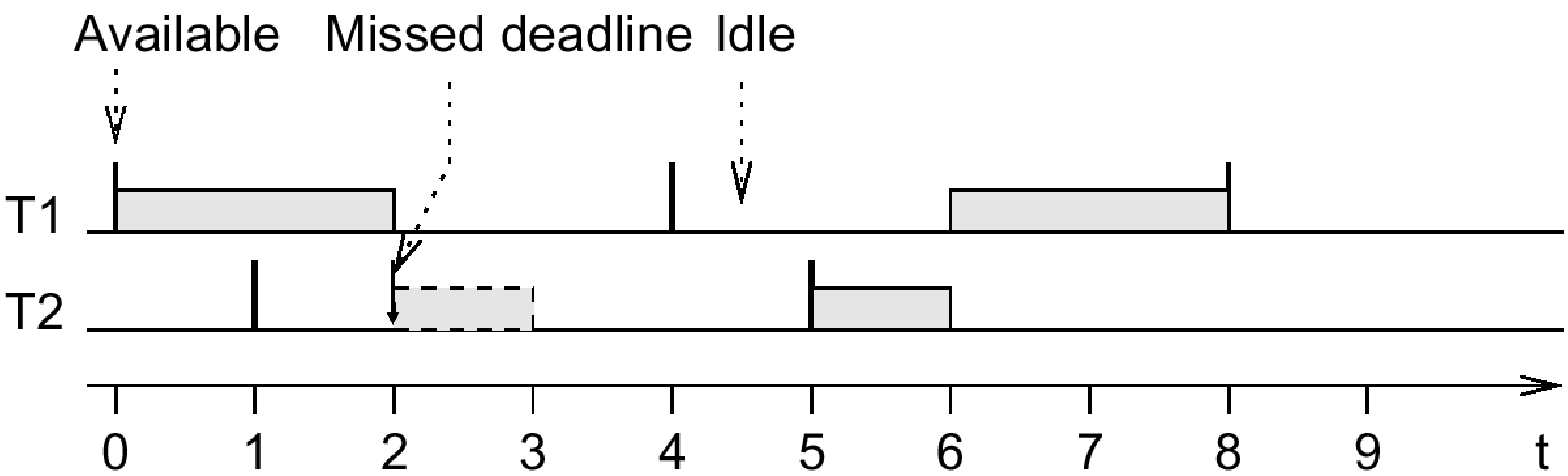
T1: periodic, $c_1 = 2$, $p_1 = 4$, $d_1 = 4$

T2: occasionally available at times $4*n+1$, $c_2 = 1$, $d_2 = 1$

T1 has to start at $t=0$

☞ deadline missed, but schedule is possible (start T2 first)

☞ scheduler is not optimal ☞ contradiction! q.e.d.



Scheduling without preemption

Preemption not allowed: ☞ optimal schedules may leave processor idle to finish tasks with early deadlines arriving late.

☞ Knowledge about the future is needed for optimal scheduling algorithms

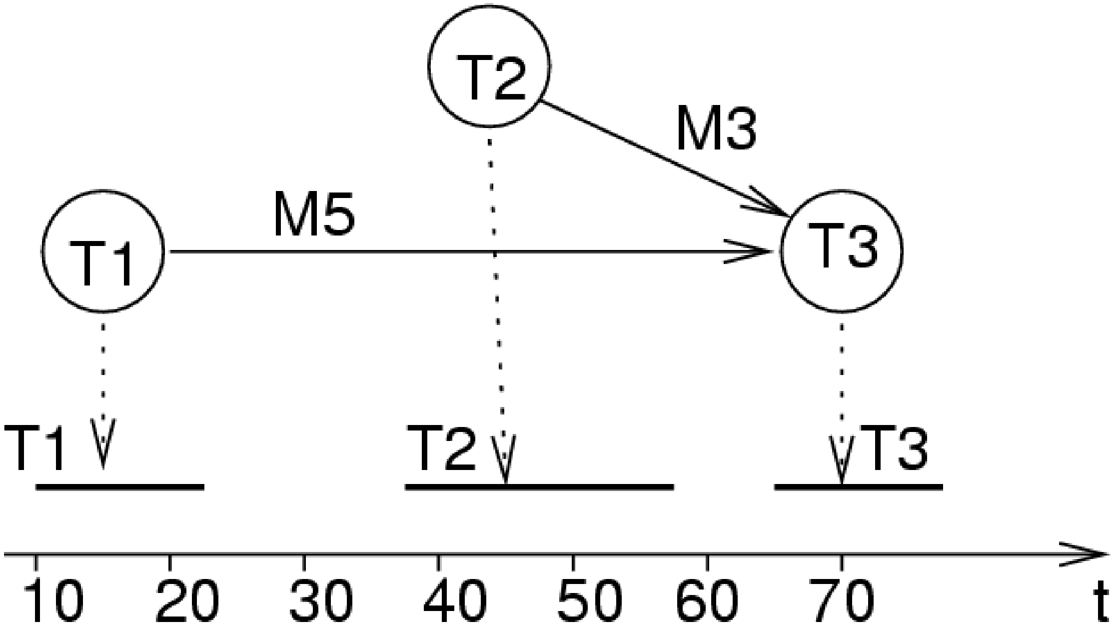
☞ No online algorithm can decide whether or not to keep idle.

EDF is optimal among all scheduling algorithms not keeping the processor idle at certain times.

If arrival times are known a priori, the scheduling problem becomes NP-hard in general. B&B typically used.

Scheduling with precedence constraints

Task graph and possible schedule:

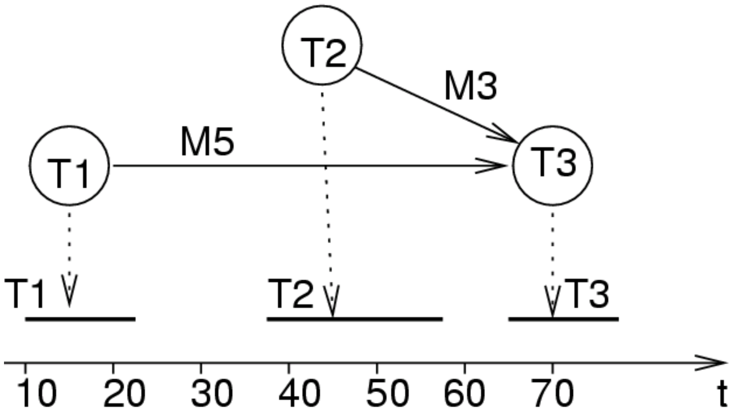


Simultaneous Arrival Times: The Latest Deadline First (LDF) Algorithm

LDF [Lawler, 1973]: reads the task graph and among the tasks with no successors inserts the one with the latest deadline into a queue. It then repeats this process, putting tasks whose successor have all been selected into the queue.

At run-time, the tasks are executed in the generated total order.

LDF is non-preemptive and is optimal for mono-processors.



If no local deadlines exist, LDF performs just a topological sort.

Asynchronous Arrival Times: Modified EDF Algorithm

This case can be handled with a modified EDF algorithm.
The key idea is to transform the problem from a given set of dependent tasks into a set of independent tasks with different timing parameters [Chetto90].
This algorithm is optimal for mono-processor systems.

If preemption is not allowed, the heuristic algorithm developed by Stankovic and Ramamritham can be used.

Overview

- Scheduling of ***aperiodic tasks*** with real-time constraints:
  - Table with some known algorithms:

	Equal arrival times non preemptive	Arbitrary arrival times preemptive
Independent tasks	EDD (Jackson)	EDF (Horn)
Dependent tasks	LDF (Lawler)	EDF* (Chetto)

Summary

Definition mapping terms

- Resource allocation, assignment, binding, scheduling
- Hard vs. soft deadlines
- Static vs. dynamic ☞TT-OS
- Schedulability

Classical scheduling

- Aperiodic tasks
  - No precedences
    - Simultaneous (☞EDD)
      & Asynchronous Arrival Times (☞EDF, LL)
  - Precedences
    - Simultaneous Arrival Times (☞ LDF)
    - Asynchronous Arrival Times (☞ mEDF)