Scheduling Aperiodic Jobs on Uniprocessor Systems

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Aperiodic Job (Task) Models (Revisited)

A job J is characterized as follows:

- Arrival time (a_j) or release time (r_j) is the time at which the job becomes ready for execution
- Computation (execution) time (C_j) is the time necessary to the processor for executing the job without interruption (= WCET).
- Absolute deadline (d_j) is the time at which the job should be completed.

Earliest Due Date Algorithm

Theorem

Given a set of n independent jobs that arrive synchronously (release time is 0), any algorithm that executes tasks in order of non-decreasing absolute deadlines is optimal with respect to minimizing the maximum lateness.
Denoted as Earliest Due Date (EDD) Algorithm [Jackson, 1955]

Proof

Let σ be the schedule for J produced by scheduling algorithm A. We can transform A to EDD schedule A' without increasing $L_{\max}$. Details are in the textbook by Buttazzo [Theorem 3.1].

A Sketched Proof

EDD is optimal for minimizing the maximum lateness: Given a set of n independent tasks, any algorithm that executes the tasks in order of non-decreasing (absolute) deadlines is optimal with respect to minimizing the maximum lateness.

Proof (Buttazzo, 2002):

- Let S be a schedule produced by any algorithm A
- If $S \neq$ the schedule of EDD
  $\rightarrow \exists\ J_a,\ J_b :\ d_a \leq d_b,\ J_b$ immediately precedes J_a in S.
- Let S' be the schedule obtained by exchanging J_a and J_b

A Sketched Proof (contd.)

Exchanging J_a and J_b cannot increase lateness

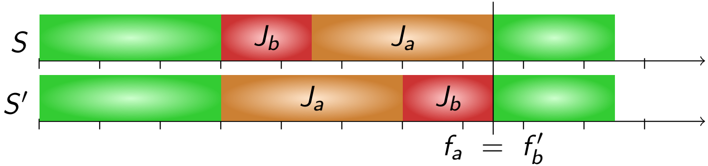
Max. lateness for J_a and J_b in S is $L_{max}(a, b) = f_a - d_a$

Max. lateness for J_a and J_b in S' is $L_{max}(a, b) = max(L'_a, L'_b)$

Two possible cases:

1. $L'_a \geq L'_b :\rightarrow L'_{max}(a, b) = f'_a - d_a < f_a - d_a = L_{max}(a, b)$ since J_a starts earlier in schedule S'.
2. $L'_a < L'_b :\rightarrow L'_{max}(a, b) = f'_b - d_b = f_a - d_b \leq f_a - d_a = L_{max}(a, b)$ since $f_a = f'_b$ and $d_a \leq D_b$

$\rightarrow\ L'_{max}(a, b) \leq L_{max}(a, b)$



Optimality of EDF

Theorem

Given a set of n independent aperiodic tasks (jobs) with arbitrary arrival times, if the aperiodic task set is feasible on a single processor then any algorithm that executes tasks with earliest deadline (among the set of active tasks) is guaranteed to meet all tasks' deadlines.

- Several proofs of optimality exist: Liu and Layland (1973), Horn (1974), and Dertouzos (1974).
- Similar to Jackson Algorithm proof of optimality, but need to account for preemption. (The steps are almost the same as the previous slide, by accounting for preemptions. I leave this proof out.)

Exact Schedulability Test for EDF

Theorem

A set of aperiodic tasks is schedulable (by EDF) if and only if

$$\forall a_i < d_k, \sum_{\tau_j: a_i \leq a_j \text{ and } d_j \leq d_k} C_j \leq d_k - a_i$$

- Proof for only if (necessary test): this simply comes from the fact that the demand must be no more than the available time
- Now: Proof for if (sufficient test): use contrapositive: $A \rightarrow B \Leftrightarrow \overline{B} \rightarrow \overline{A}$

Proof: Sufficient Schedulability Test for EDF

Proof

Proof by contrapositive:

- Suppose that *EDF* schedule does not meet the deadline
- Let τ_k be the first task which misses its absolute deadline d_k
- Let t_0 be the last instant before d_k, at which either the processor is idle or the processor executes a task with absolute deadline larger than d_k
- By *EDF*, t_0 must be an arrival time of a job, called τ_i
- Therefore, t_0 is equal to a_i
- The processor executes only the jobs arriving no earlier than a_i and with absolute deadline less than or equal to d_k
- Therefore, we conclude the proof by showing that

$$\exists a_i < d_k, \sum_{\tau_j : a_i \leq a_j \text{ and } d_j \leq d_k} C_j > d_k - a_i$$

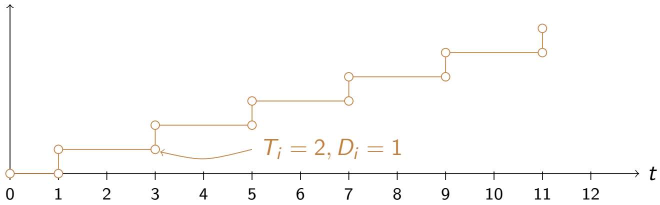
Link this to DBF

Theorem

Define demand bound function $dbf(\tau_i, t)$ as

$$dbf(\tau_i, t) = \max\left\{0, \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor\right\} C_i = \max\left\{0, \left\lfloor \frac{t - D_i}{T_i} \right\rfloor + 1\right\} C_i.$$

A task set $\mathcal{T}$ of independent, preemptable, periodic tasks can be feasibly scheduled (under EDF) on one processor if and only if $\forall\ L \geq 0,\ \sum_{i=1}^{n} dbf(\tau_i, L) \leq L$.



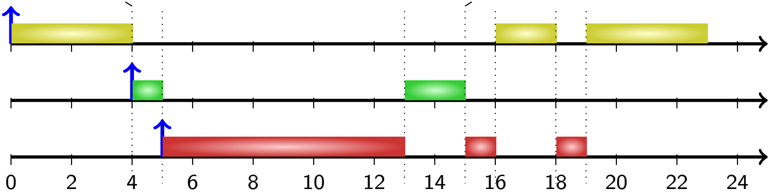
Least Laxity First

Priorities = decreasing function of the laxity
(lower laxity = higher priority); changing priority; preemptive.

	τ_1	τ_2	τ_3
a_j	0	4	5
C_j	10	3	10
d_j	33	28	29

$l(\tau_1) = 33 - 4 - 6 = 23$
$l(\tau_2) = 28 - 4 - 3 = 21$

$l(\tau_1) = 33 - 15 - 6 = 12$
$l(\tau_3) = 29 - 15 - 2 = 12$



$l(\tau_1) = 33 - 5 - 6 = 22$
$l(\tau_2) = 28 - 5 - 2 = 21$
$l(\tau_3) = 29 - 5 - 10 = 14$

$l(\tau_1) = 33 - 13 - 6 = 14$
$l(\tau_2) = 28 - 13 - 2 = 13$
$l(\tau_3) = 29 - 13 - 2 = 14$

$l(\tau_1) = 33 - 16 - 6 = 11$
$l(\tau_3) = 29 - 16 - 1 = 12$