Classical scheduling algorithms for periodic systems

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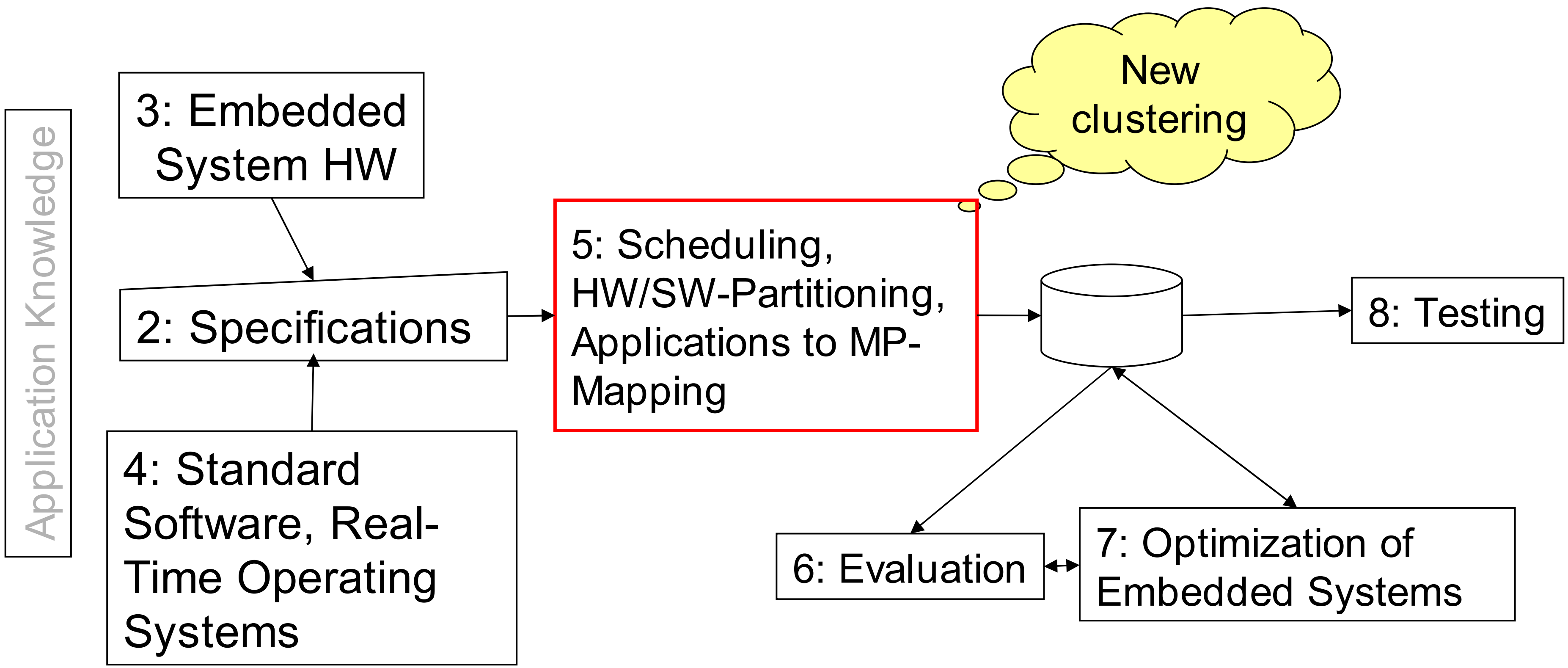
2008/06/21

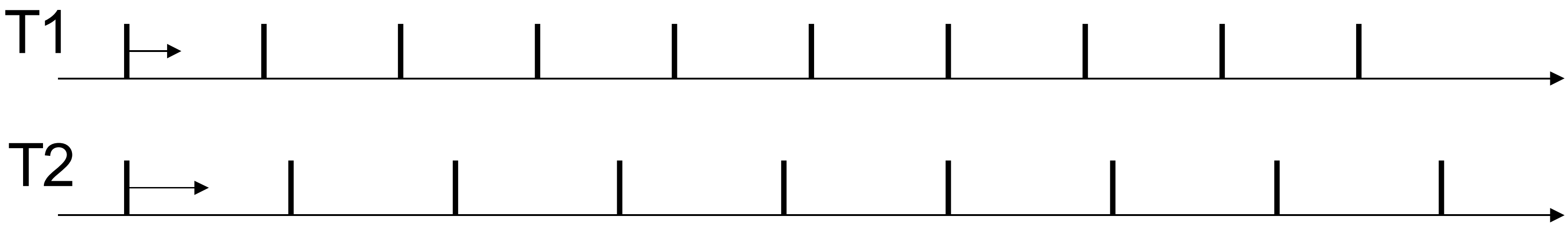
Structure of this course



Classes of mapping algorithms considered in this course

- **Classical scheduling algorithms**
  Mostly for independent tasks & ignoring communication, mostly for mono- and homogeneous multiprocessors
- **Dependent tasks as considered in architectural synthesis**
  Initially designed in different context, but applicable
- **Hardware/software partitioning**
  Dependent tasks, heterogeneous systems, focus on resource assignment
- **Design space exploration using genetic algorithms**
  Heterogeneous systems, incl. communication modeling

Periodic scheduling



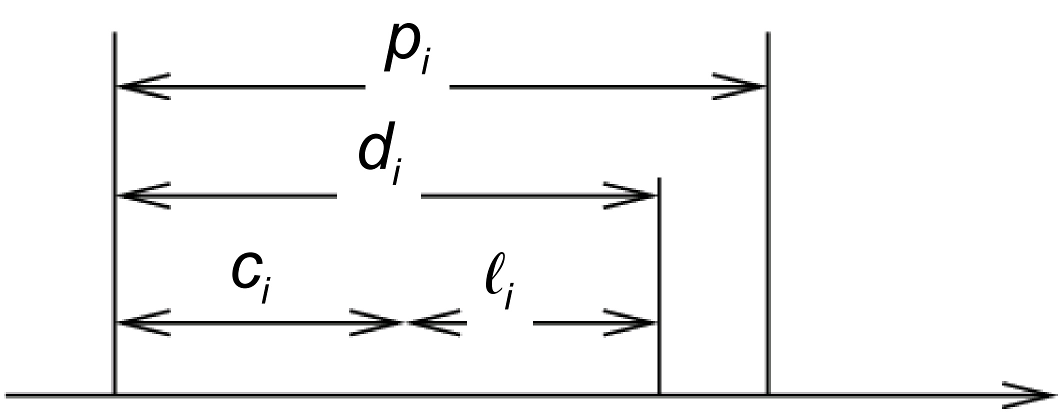
For periodic scheduling, the best that we can do is to design an algorithm which will always find a schedule if one exists.

☞ A scheduler is defined to be **optimal** iff it will find a schedule if one exists.

Periodic scheduling

Let

- p_i be the period of task T_i,
- c_i be the execution time of T_i,
- d_i be the deadline ***interval***, that is, the time between a job of T_i becoming available and the time until the same job T_i has to finish execution.
- ℓ_i be the **laxity** or **slack**, defined as $\ell_i = d_i - c_i$



Average utilization

Average utilization:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i}$$

Necessary condition for schedulability (with m=number of processors):

$$\mu \leq m$$

Independent tasks: Rate monotonic (RM) scheduling

Most well-known technique for scheduling independent periodic tasks [Liu, 1973].

Assumptions:

- All tasks that have hard deadlines are periodic.
- All tasks are independent.
- $d_i = p_i$, for all tasks.
- c_i is constant and is known for all tasks.
- The time required for context switching is negligible.
- For a single processor and for n tasks, the following equation holds for the average utilization μ:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq n(2^{1/n} - 1)$$

Rate monotonic (RM) scheduling - The policy -

RM policy: **The priority of a task is a monotonically decreasing function of its period.**

At any time, a highest priority task among all those that are ready for execution is allocated.

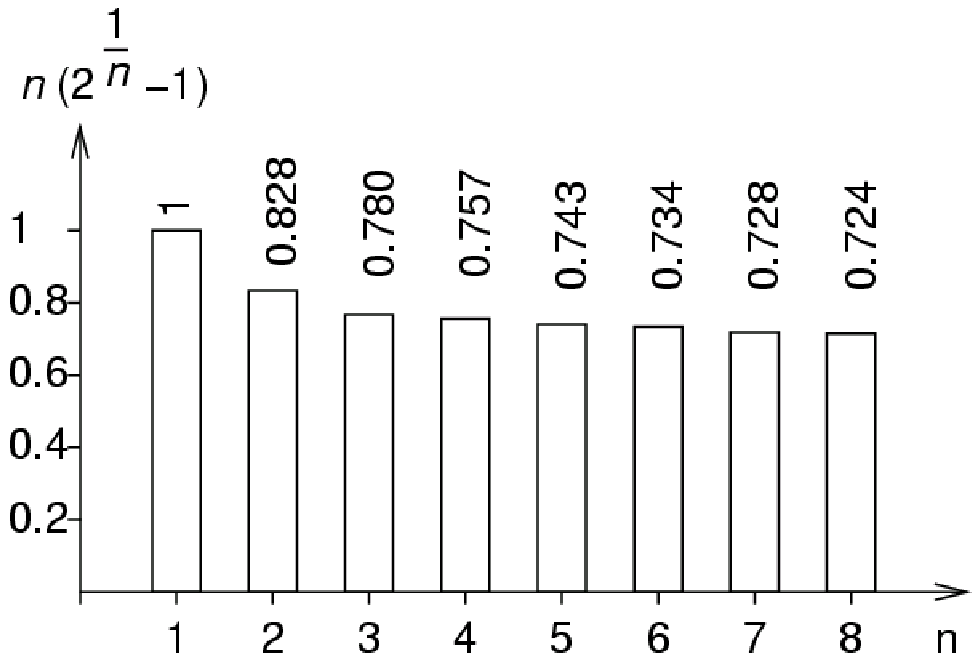
Theorem: If all RM assumptions are met, schedulability is guaranteed.

Maximum utilization for guaranteed schedulability

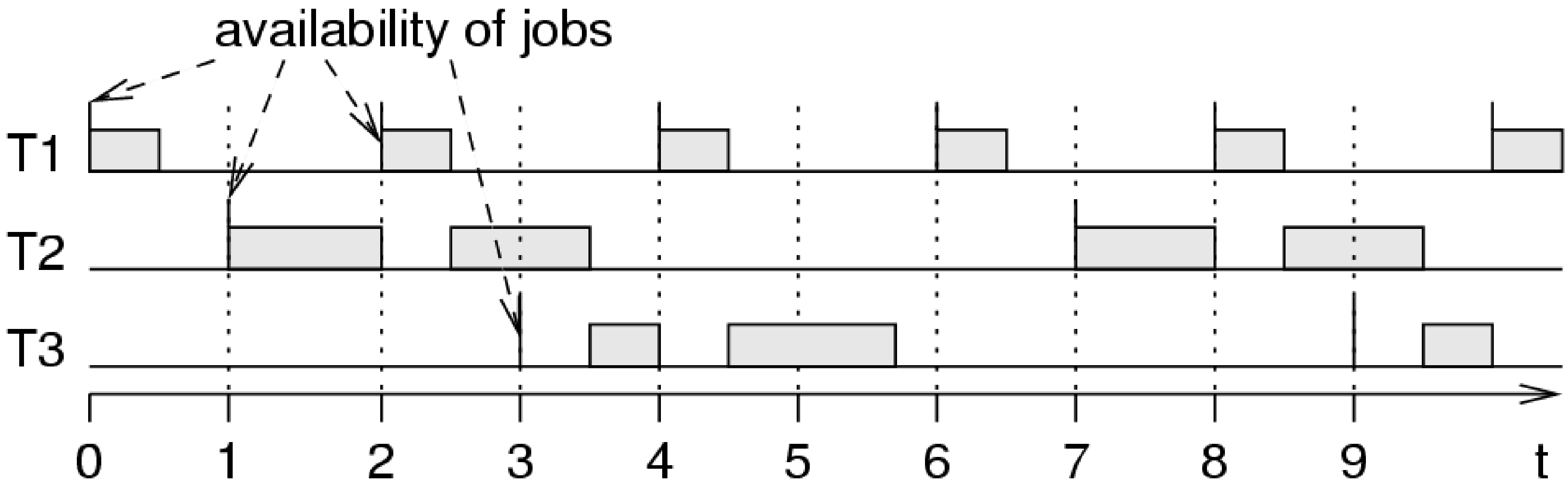
Maximum utilization as a function of the number of tasks:

$$\mu = \sum_{i=1}^{n} \frac{c_i}{p_i} \leq n(2^{1/n} - 1)$$

$$\lim_{n \to \infty} (n(2^{1/n} - 1)) = \ln(2)$$



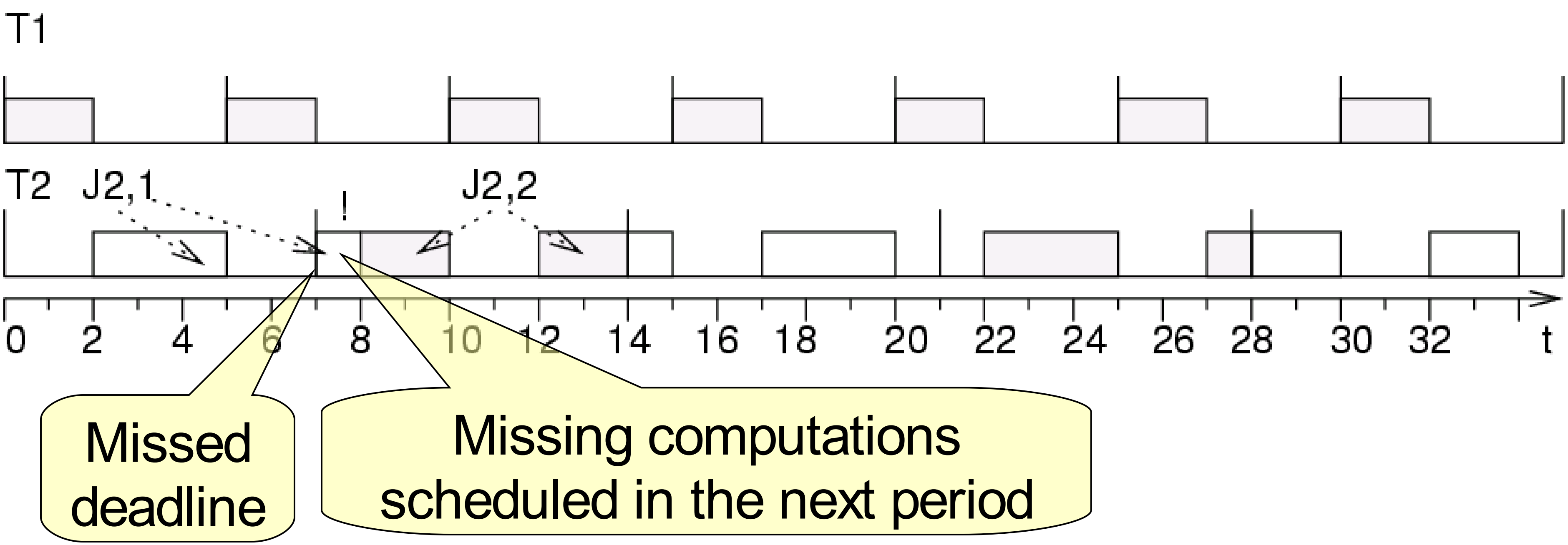
Example of RM-generated schedule



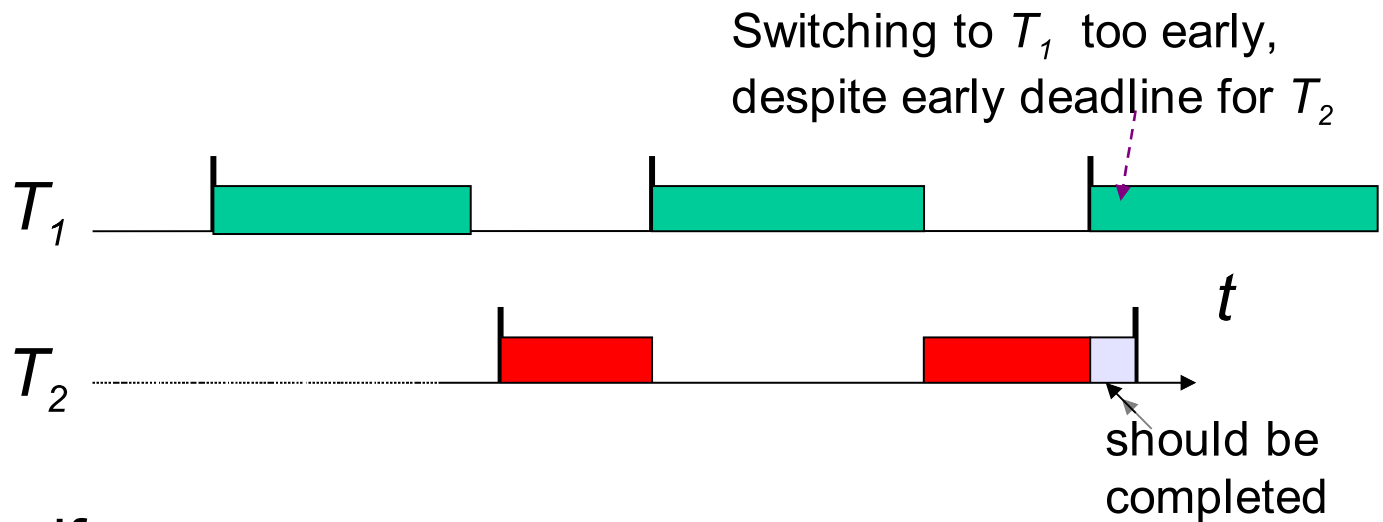
T1 preempts T2 and T3.
T2 and T3 do not preempt each other.

Case of failing RM scheduling

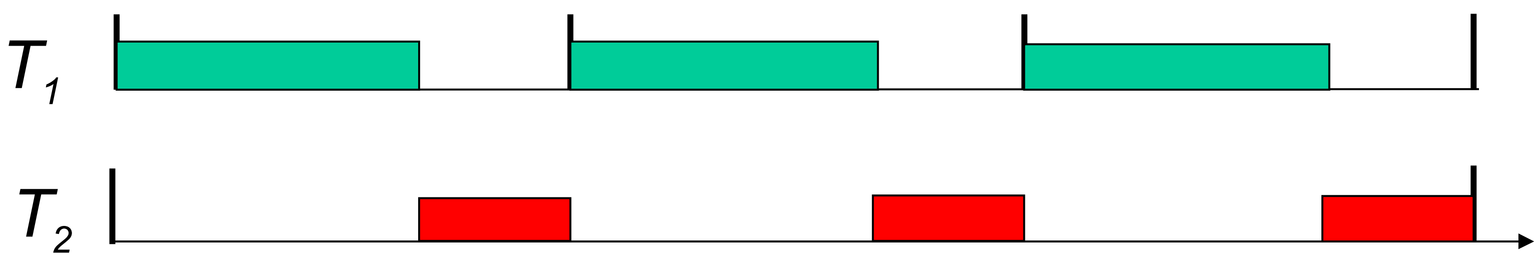
Task 1: period 5, execution time 2
Task 2: period 7, execution time 4
$\mu = 2/5 + 4/7 = 34/35 \approx 0.97$
$2(2^{1/2}-1) \approx 0.828$



Intuitively: Why does RM fail ?



No problem if $p_2 = m\, p_1,\ m \in \mathbb{N}$:



fits

Critical instants

Definition: A **critical instant** of a task is the time at which the release of a task will produce the largest response time.

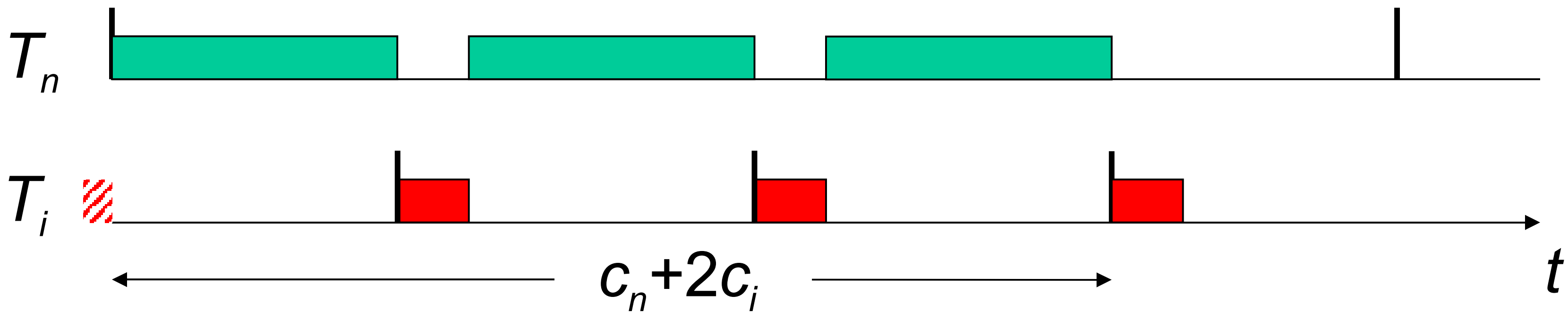
Lemma: For any task, the **critical instant** occurs if that task is simultaneously released with all higher priority tasks.

Proof: Let $T=\{T_1, \ldots, T_n\}$: periodic tasks with $\forall i$: $p_i \leqq p_{i+1}$.

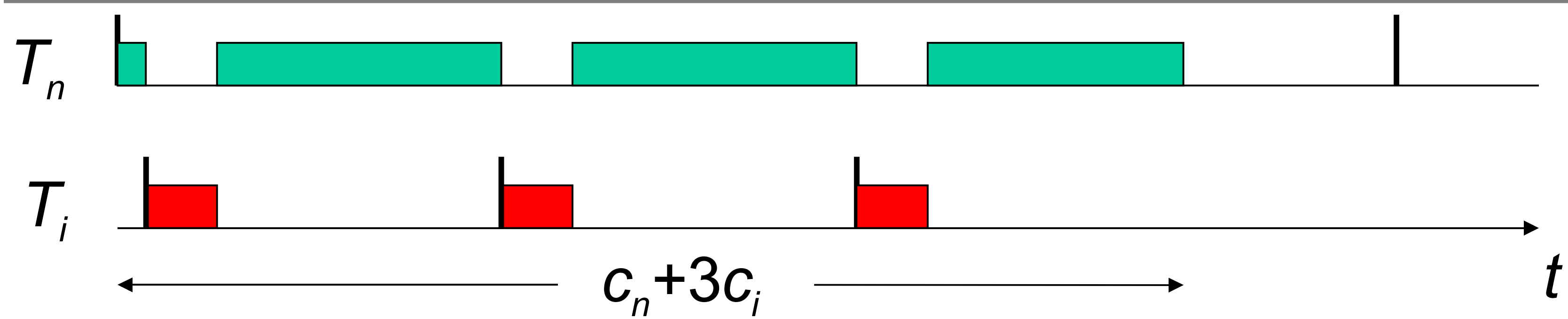
Source: G. Buttazzo, Hard Real-time Computing Systems, Kluwer, 2002

Critical instances (1)

Response time of T_n is delayed by tasks T_i of higher priority:



Delay may increase if T_i starts earlier



Maximum delay achieved if T_n and T_i start simultaneously.

Critical instants (2)

Repeating the argument for all *i* = 1, … *n-1:*

- ☞ The worst case response time of a task occurs when it is released simultaneously with all higher-priority tasks. q.e.d.

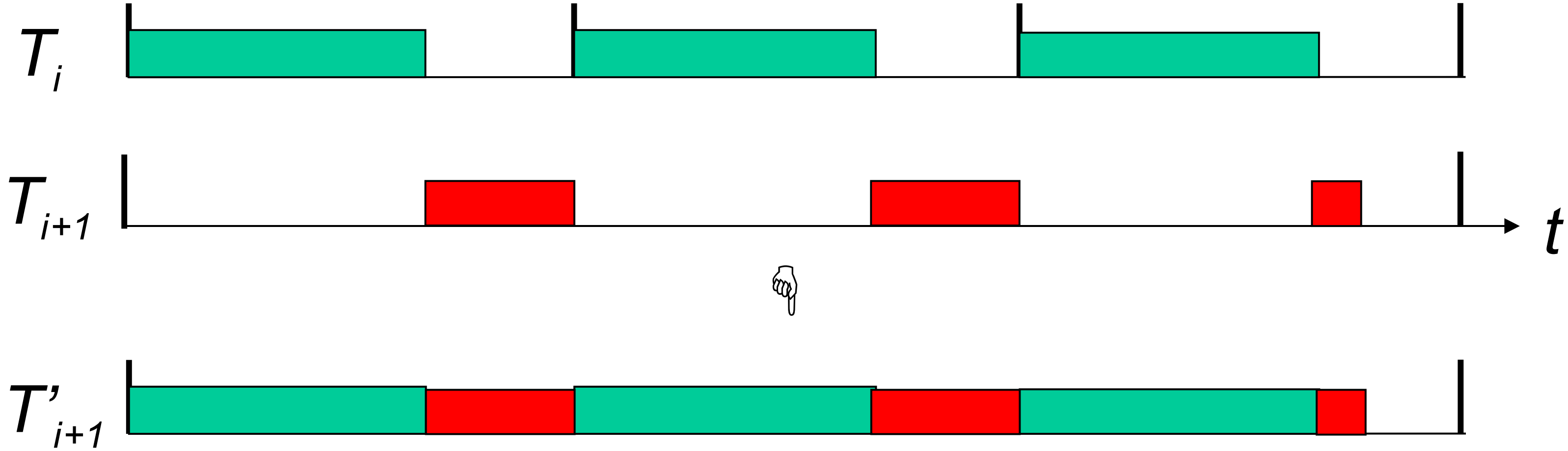
- ☞ Schedulability is checked at the critical instants.
- ☞ If all tasks of a task set are schedulable at their critical instants, they are schedulable at all release times.
- ☞ Observation helps designing examples

The case $\forall i$: $p_{i+1} = m_i \, p_i$

Lemma*: **If each task period is a multiple of the period of the next higher priority task**, then schedulability is also guaranteed if μ ≤ 1.

Proof: Assume schedule of T_i is given. Incorporate T_{i+1}:

T_{i+1} fills idle times of T_i; T_{i+1} completes in time, if μ ≤ 1.



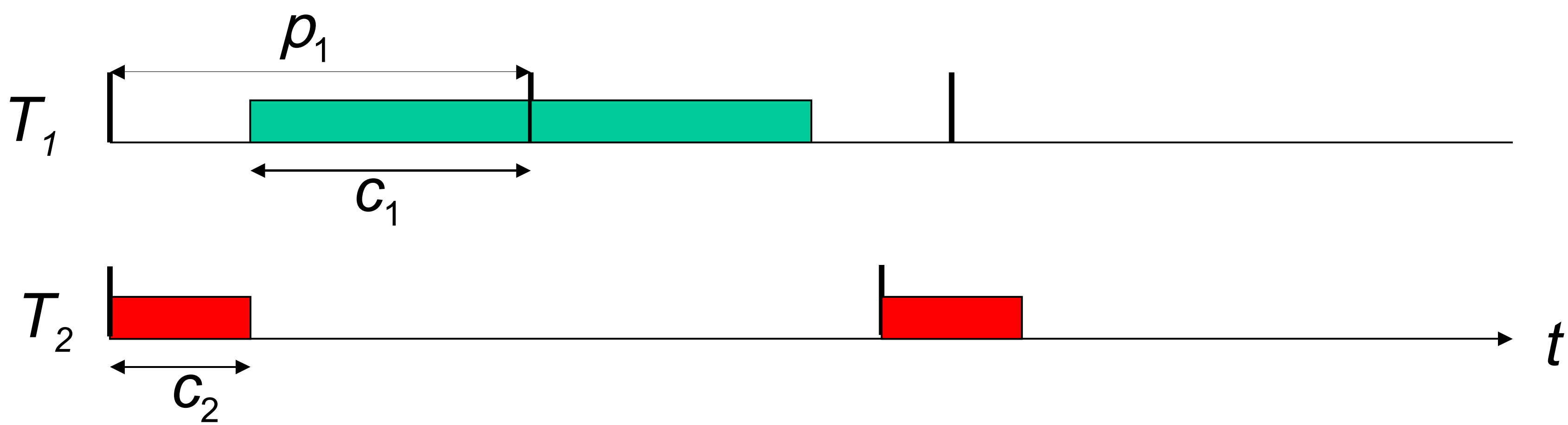
Used as the higher priority task at the next iteration.

More in-depth:

Proof of the RM theorem

Let $T=\{T_1, T_2\}$ with $p_1 < p_2$.

Assume RM is **not** used → prio(T_2) is highest:



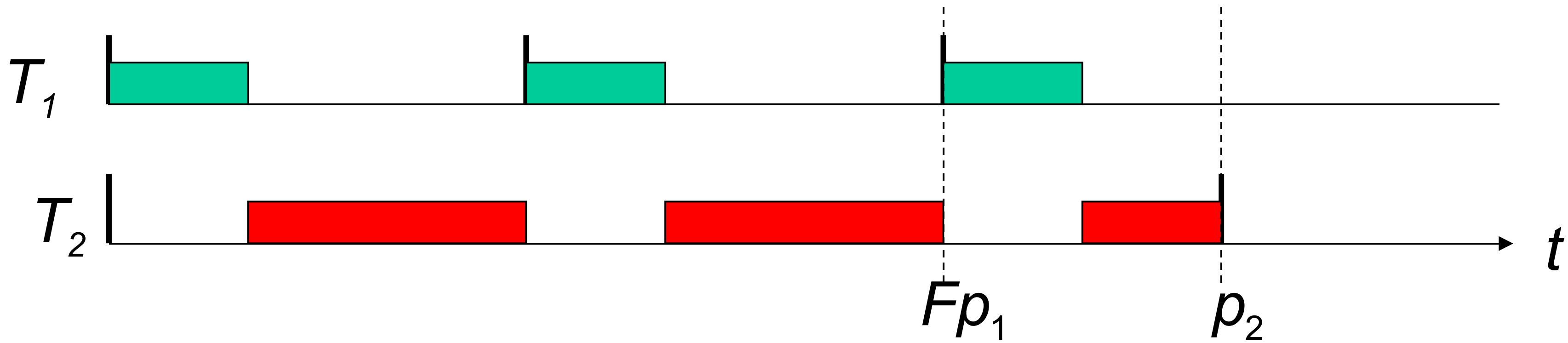
Schedule is feasible if $c_1+c_2 \leqq p_1$ (1)

Define $F=\lfloor p_2/p_1 \rfloor$: # of periods of T_1 fully contained in T_2

Case 1: $c_1 \leq p_2 - Fp_1$

Assume RM is used → prio(T_1) is highest:

Case 1*: $c_1 \leq p_2 - Fp_1$
(c_1 small enough to be finished before 2nd instance of T_2)



Schedulable if $(F+1)\, c_1 + c_2 \leq p_2$ (2)

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* Typos in [Buttazzo 2002]: < and ≤ mixed up]

Proof of the RM theorem (3)

Not RM: schedule is feasible if $c_1+c_2 \leq p_1$ (1)

RM: schedulable if $(F+1)\,c_1 + c_2 \leq p_2$ (2)

From (1): $Fc_1+Fc_2 \leq Fp_1$

Since F ≥ 1: $Fc_1+c_2 \leq Fc_1+Fc_2 \leq Fp_1$

Adding c_1: $(F+1)c_1+c_2 \leq Fp_1 +c_1$

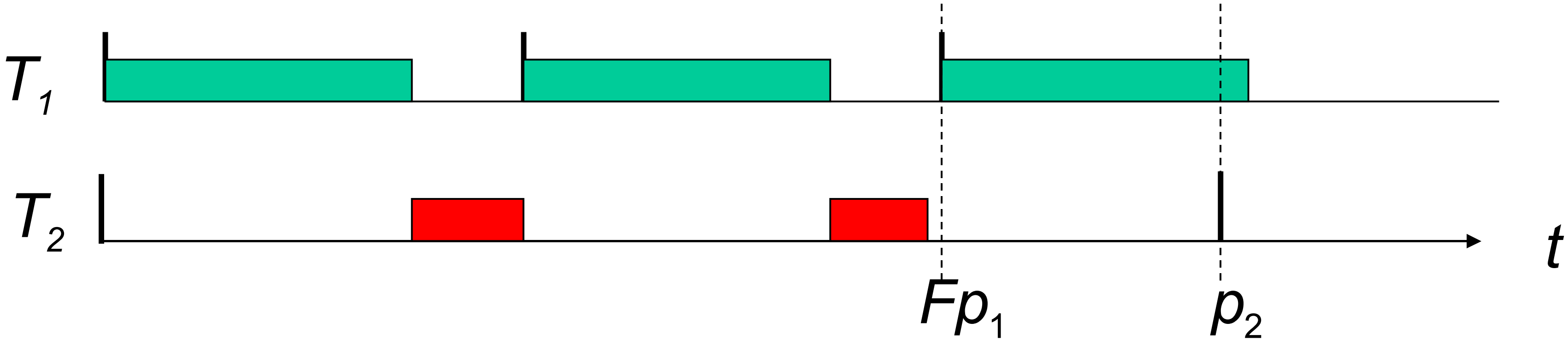
Since $c_1 \leq p_2 - Fp_1$: $(F+1)c_1+c_2 \leq Fp_1 +c_1 \leq p_2$

Hence: if (1) holds, (2) holds as well

☞For case 1: Given tasks T_1 and T_2 with $p_1 < p_2$, then if the schedule is feasible by an arbitrary (but fixed) priority assignment, it is also feasible by RM.

Case 2: $c_1 > p_2 - Fp_1$

Case 2: $c_1 > p_2 - Fp_1$
(c_1 large enough not to finish before 2nd instance of T_2)



Schedulable if $\quad F\,c_1 + c_2 \leq F\,p_1 \quad (3)$

$\quad c_1 + c_2 \leq p_1 \quad (1)$

Multiplying (1) by F yields $\quad F\,c_1 + F\,c_2 \leq F\,p_1$

Since F ≥ 1: $\quad F\,c_1 + c_2 \leq F\,c_1 + Fc_2 \leq F\,p_1$

☞ Same statement as for case 1.

Calculation of the least upper utilization bound

Let $T=\{T_1, T_2\}$ with $p_1 < p_2$.

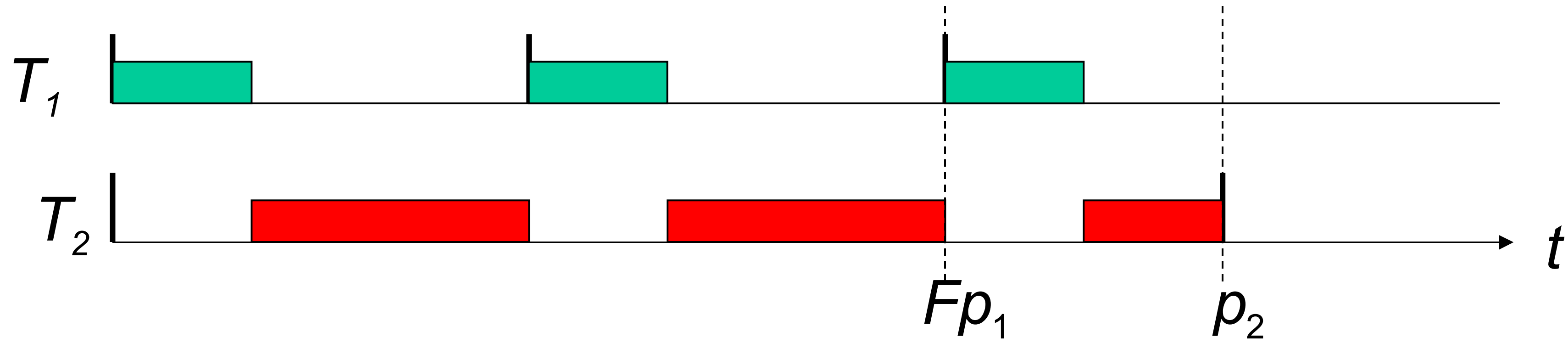
Proof procedure: compute least upper bound U_{lup} as follows

- Assign priorities according to RM
- Compute upper bound U_{up} by setting computation times to fully utilize processor
- Minimize upper bound with respect to other task parameters

As before: $F=\lfloor p_2/p_1 \rfloor$

c_2 adjusted to fully utilize processor.

Case 1: $c_1 \le p_2 - Fp_1$



Largest possible value of c_2 is $c_2 = p_2 - c_1\,(F+1)$

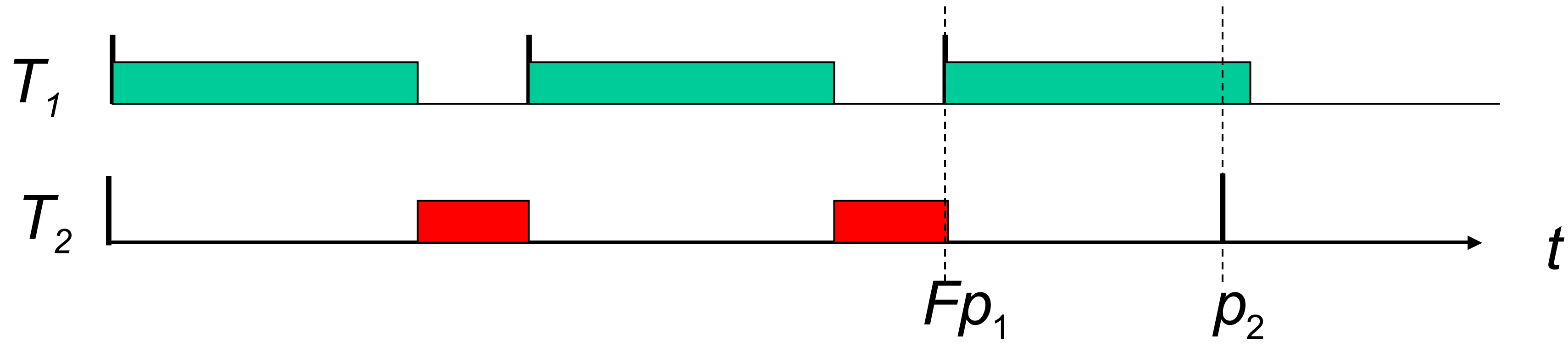
Corresponding upper bound is

$$U_{ub} = \frac{c_1}{p_1} + \frac{c_2}{p_2} = \frac{c_1}{p_1} + \frac{p_2 - c_1\,(F+1)}{p_2} = 1 + \frac{c_1}{p_1} - \frac{c_1\,(F+1)}{p_2} = 1 + \frac{c_1}{p_2}\left\{\frac{p_2}{p_1} - (F+1)\right\}$$

{ } is <0 → U_{ub} monotonically decreasing in c_1

Minimum occurs for $c_1 = p_2 - Fp_1$

Case 2: $c_1 \geq p_2 - Fp_1$



Largest possible value of c_2 is $c_2 = (p_1 - c_1)F$

Corresponding upper bound is:

$$U_{ub} = \frac{c_1}{p_1} + \frac{c_2}{p_2} = \frac{c_1}{p_1} + \frac{(p_1 - c_1)\,F}{p_2} = \frac{p_1}{p_2}F + \frac{c_1}{p_1} - \frac{c_1}{p_2}F = \frac{p_1}{p_2}F + \frac{c_1}{p_2}\left\{\frac{p_2}{p_1} - F\right\}$$

{ } is ≥ 0 → U_{ub} monotonically increasing in c_1 (independent of c_1 if {}=0)

Minimum occurs for $c_1 = p_2 - Fp_{1,}$ as before.

Utilization as a function of $G = p_2/p_1 - F$

For minimum value of c_1:

$$U_{ub} = \frac{p_1}{p_2}F + \frac{c_1}{p_2}\left(\frac{p_2}{p_1} - F\right) = \frac{p_1}{p_2}F + \frac{(p_2 - p_1F)}{p_2}\left(\frac{p_2}{p_1} - F\right) = \frac{p_1}{p_2}\left\{F + \left(\frac{p_2}{p_1} - F\right)\left(\frac{p_2}{p_1} - F\right)\right\}$$

Let $G = \frac{p_2}{p_1} - F; \qquad \Rightarrow$

$$U_{ub} = \frac{p_1}{p_2}\left(F + G^2\right) = \frac{\left(F + G^2\right)}{p_2 / p_1} = \frac{\left(F + G^2\right)}{\left(p_2 / p_1 - F\right) + F} = \frac{\left(F + G^2\right)}{F + G} = \frac{\left(F + G\right) - (G - G^2)}{F + G}$$

$$= 1 - \frac{G(1 - G)}{F + G}$$

Since $0 \leq G < 1$: $\quad G(1-G) \geq 0 \quad \rightarrow \quad U_{ub}$ increasing in F $\rightarrow$

Minimum of U_{ub} for min(F): $\quad F=1 \rightarrow$

$$U_{ub} = \frac{1 + G^2}{1 + G}$$

Proving the RM theorem for *n*=2

end

$$U_{ub} = \frac{1+G^2}{1+G}$$

Using derivative to find minimum of U_{ub} :

$$\frac{dU_{ub}}{dG} = \frac{2G(1+G)-(1+G^2)}{(1+G)^2} = \frac{G^2+2G-1}{(1+G)^2} = 0$$

$$G_1 = -1-\sqrt{2}; \qquad G_2 = -1+\sqrt{2};$$

Considering only G_2, since $0 \le G < 1$:

$$U_{lub} = \frac{1+(\sqrt{2}-1)^2}{1+(\sqrt{2}-1)} = \frac{4-2\sqrt{2}}{\sqrt{2}} = 2(\sqrt{2}-1) = 2(2^{\frac{1}{2}}-1) \cong 0.83$$

This proves the RM theorem for the special case of *n*=2

Properties of RM scheduling

- RM scheduling is based on **static** priorities. This allows RM scheduling to be used in standard OS, such as Windows NT.
- No idle capacity is needed if $\forall i$: $p_{i+1}=F\ p_i$:
  i.e. if the **period of each task is a multiple of the period of the next higher priority task**, schedulability is then also guaranteed if $\mu \leq 1$.
- A huge number of variations of RM scheduling exists.
- In the context of RM scheduling, many formal proofs exist.

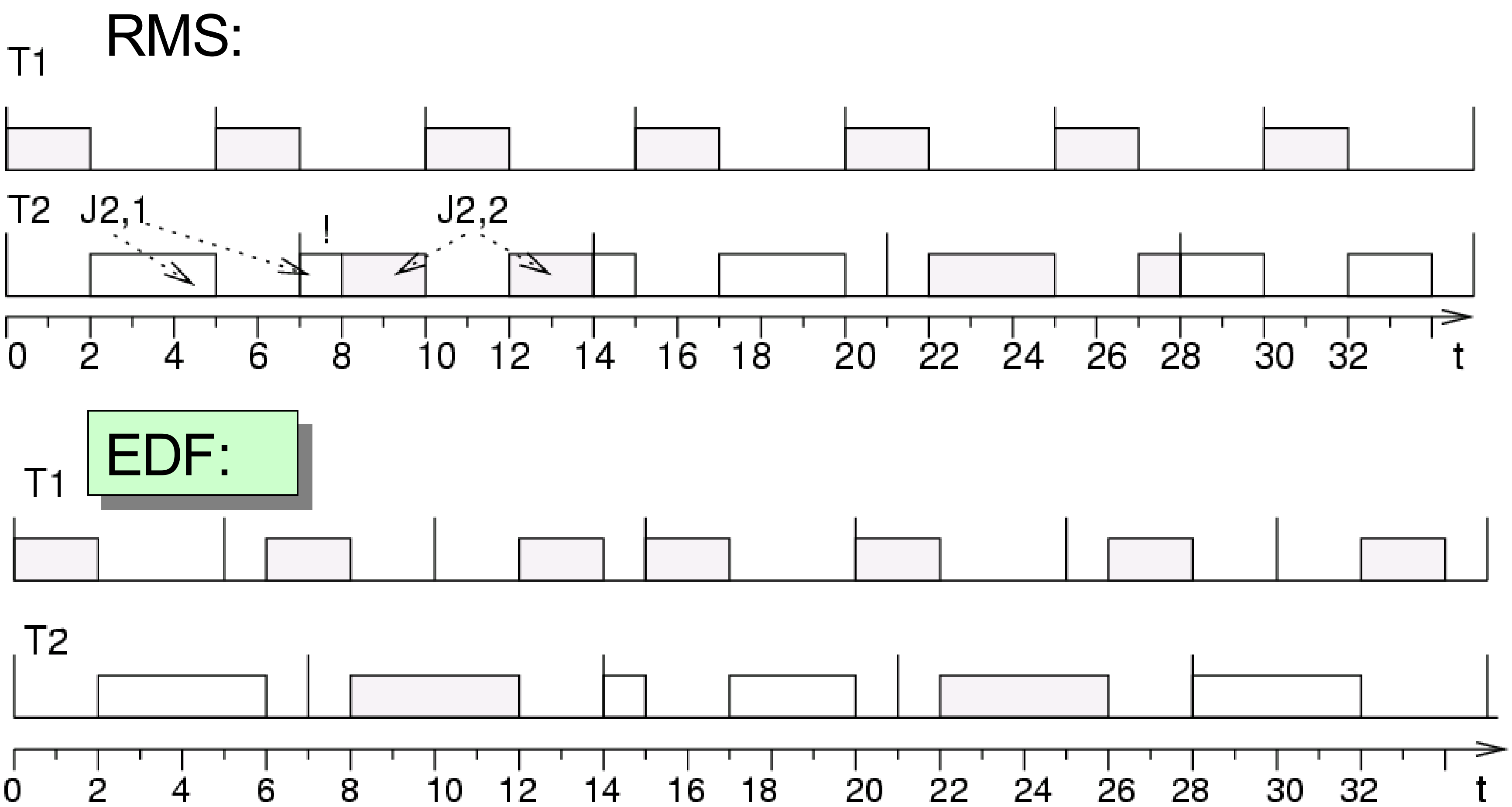
EDF

EDF can also be applied to periodic scheduling.

EDF optimal for every period

- ☞ Optimal for periodic scheduling
- ☞ EDF must be able to schedule the example in which RMS failed.

Comparison EDF/RMS



T2 not preempted, due to its earlier deadline.

EDF: Properties

EDF requires dynamic priorities

☞ EDF cannot be used with a standard operating system just providing static priorities.

Comparison RMS/EDF

	RMS	EDF
Priorities	Static	Dynamic
Works with std. OS with fixed priorities	**Yes**	**No**
Uses full computational power of processor	No, just up till $\mu=n(2^{1/n}-1)$	Yes
Possible to exploit full computational power of processor without provisioning for slack	**No**	**Yes**

Sporadic tasks

If sporadic tasks were connected to interrupts, the execution time of other tasks would become very unpredictable.

☞ Introduction of a sporadic task server, periodically checking for ready sporadic tasks;

☞ Sporadic tasks are essentially turned into periodic tasks.

Dependent tasks

The problem of deciding whether or not a schedule exists for a set of dependent tasks and a given deadline is NP-complete in general [Garey/Johnson].

Strategies:

2. Add resources, so that scheduling becomes easier
3. Split problem into static and dynamic part so that only a minimum of decisions need to be taken at run-time.
4. Use scheduling algorithms from high-level synthesis

Summary

Periodic scheduling

- Rate monotonic scheduling
  - Proof of the utilization bound for n=2.
- EDF
- Dependent and sporadic tasks (briefly)