Schedulability of Sporadic Tasks on Uniprocessor Systems

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07, May 2014
Schedulability Condition for Rate Monotonic

The time-demand function $W_i(t)$ of the task $\tau_i$ is defined as follows:

$$W_i(t) = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{t}{T_j} \right\rceil C_j.$$ 

Theorem

A system $T$ of periodic, independent, preemptable tasks is schedulable on one processor by rate monotonic scheduling if

$$\forall \tau_i \in T \exists t \text{ with } 0 < t \leq D_i \text{ and } W_i(t) \leq t.$$ 

Theorem

Eisenbrand and Rothvoss [RTSS 2008]: Fixed-Priority Real-Time Scheduling: Response Time Computation Is $\mathcal{NP}$-Hard
Schedulability Conditions for EDF Scheduling

**Theorem**

A task set $T$ of independent, preemptable, periodic tasks with relative deadlines equal to or less than their periods can be feasibly scheduled (under EDF) on one processor if and only if

$$\forall t \geq 0, \sum_{i=1}^{n} dbf(\tau_i, t) = \sum_{i=1}^{n} \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor C_i \leq t,$$

where $dbf(\tau_i, t) = \left\lfloor \frac{t + T_i - D_i}{T_i} \right\rfloor C_i$ is the definition of the demand bound function of task $\tau_i$ at time $t$.

**Theorem**

Eisenbrand and Rothvoss [SODA 2010]: testing EDF schedulability of such a task set is (weakly) coNP-hard. That is, deciding whether a task set is not schedulable by EDF is (weakly) NP-hard.
Schedulability

- The issue for uniprocessor scheduling is on how to analyze the schedulability.
  - EDF is optimal
  - DM is optimal for fixed-priority scheduling when $D_i \leq T_i$
  - Ausley’s iterative approach (1992) can also be applied for fixed-priority scheduling when $D_i > T_i$
- As verifying the schedulability is $NP$-hard or co$NP$-hard, there does not exist any polynomial-time algorithm for schedulability tests unless $P = NP$.
- Approximations are possible, but what do we approximate when only binary decisions (Yes or No) have to be made?
  - Answers like probabilistic guarantee are unlikely preferred, e.g., the task set is 99% schedulable.
  - Deadline relaxation: requires modifications of task specification
  - Period relaxation: requires modifications of task specification
- Resource augmentation by speeding up: a faster platform
- Resource augmentation by allocating more processors: a better platform
Resource Augmentation

For an algorithm $A$ with a $\rho$ resource augmentation factor, it guarantees that

$\Rightarrow$

if the task set (system) is schedulable (feasible), Algorithm $A$ will also returns a schedulable (feasible) answer by speeding up the system by a factor $\rho$, or

$\Leftarrow$

if Algorithm $A$ does not return a schedulable (feasible) answer, the system is also unschedulable (infeasible) by slowing down by a factor $\rho$. 
Schedulability by Least Utilization Bound

Algorithm: Given $n$ periodic tasks with relative deadline equal to the period

- If the total utilization is less than $n(2^{\frac{1}{n}} - 1)$, the task set is schedulable;
- otherwise, the task set is probably not schedulable.

The algorithm is with a $\frac{1}{0.693}$ (or $\frac{1}{\ln 2}$) resource augmentation factor for deciding whether a task set is schedulable by the rate monotonic scheduling algorithm.

- The resource augmentation factor analysis is tight
  - $n$ jobs with the same parameters $C = (2^{\frac{1}{n}} - 1) + \epsilon, D = P = 1$ where $\epsilon > 0$ and $\epsilon \to 0^+$.  
  - The task set is schedulable, but the above testing algorithm says that it is probably not schedulable.
Time Demand Function Revisit for RM/DM

Let $w_i(t)$ of the task $\tau_i$ be defined as follows

$$w_i(t) = \left\lceil \frac{t}{T_i} \right\rceil C_i.$$ 

We need approximation to enforce polynomial-time schedulability test.

$$w_i^*(t) = C_i + \frac{t}{T_i} C_i.$$
Resource Augmentation

The approximated time-demand function $W_i^*(t)$ of $\tau_i$ is defined as follows:

$$W_i^*(t) = C_i + \sum_{j=1}^{i-1} w_j^*(t).$$

- If $W_i^*(t) \leq t$, then $W_i(t) \leq t$.
- If $W_i^*(t) > t$, then $W_i(t) > 0.5t$.

Theorem

[Fisher and Baruah, 2005] A system $T$ of periodic, independent, preemttable tasks is schedulable on one processor by RM/DM if

$$\forall \tau_i \in T \exists t \text{ with } 0 < t \leq D_i \text{ and } W_i^*(t) \leq t.$$  

Otherwise, the system is not schedulable when slowing down by a factor 2 (i.e., running at 0.5 of the original speed).
Resource Augmentation

The approximated time-demand function $W_i^*(t)$ of $\tau_i$ is defined as follows:

$$W_i^*(t) = C_i + \sum_{j=1}^{i-1} w_j^*(t).$$

- If $W_i^*(t) \leq t$, then $W_i(t) \leq t$.
- If $W_i^*(t) > t$, then $W_i(t) > 0.5t$.

The analysis is tight by considering the following example:
- A task with period $P = D = 1$ and $C = 0.5 + \epsilon$.
- Since $(0.5 + \epsilon)(1 + x) > x$ for all $x \geq 0$ and $\epsilon > 0$, the above test does not succeed.
- The system is still schedulable if it is slowed by to run at $0.5 + \epsilon$ of the original speed.
Given a precision factor $\delta$, we can approximate $\left\lceil \frac{t}{T_j} \right\rceil$ by $w'_j(t)$

$$w'_j(t) = \begin{cases} 
\left\lceil \frac{t}{T_j} \right\rceil C_j & \text{if } t \leq (\left\lceil \frac{1}{\delta} \right\rceil - 2) T_j \\
(1 + \frac{t}{T_j}) C_j & \text{if } t > (\left\lceil \frac{1}{\delta} \right\rceil - 2) T_j
\end{cases}$$
Resource Augmentation

The approximated time-demand function $W'_i(t)$ of $\tau_i$ is defined as follows:

$$W'_i(t) = C_i + \sum_{j=1}^{i-1} w'_j(t).$$

- If $W'_i(t) \leq t$, then $W_i(t) \leq t$.
- If $W'_i(t) > t$, then $W_i(t) > (1 - \frac{1}{\lceil \frac{1}{\delta} \rceil})t$.

Theorem

[Fisher and Baruah, 2005] A system $T$ of periodic, independent, pre-emptable tasks is schedulable on one processor by RM/DM if

$$\forall \tau_i \in T \ \exists t \text{ with } 0 < t \leq D_i \text{ and } W'_i(t) \leq t.$$  

Otherwise, the system is not schedulable when slowing down to run at speed $(1 - \delta)$ of the original speed.
Exercise

Please provide pseudo code and analyze the complexity and resource augmentation factor so that

- the algorithm runs in polynomial time with respect to $\frac{1}{\delta}$ and the number of tasks, and
- the resource augmentation factor is $\frac{1}{1-\delta}$. 
Demand Bound Function Revisit for EDF

Define demand bound function $dbf(\tau_i, t)$ as

$$dbf(\tau_i, t) = \max \left\{ 0, \left\lceil \frac{t + T_i - D_i}{T_i} \right\rceil \right\}$$

$$C_i = \max \left\{ 0, \left\lceil \frac{t - D_i}{T_i} \right\rceil + 1 \right\}$$

We need approximation to enforce polynomial-time schedulability test.

$$dbf^*(\tau_i, t) = \begin{cases} 0 & \text{if } t < D_i \\ \left( \frac{t-D_i}{T_i} + 1 \right) C_i & \text{otherwise.} \end{cases}$$

$$dbf(\tau_i, t) \leq dbf^*(\tau_i, t) \leq 2dbf(\tau_i, t)$$
Resource Augmentation by EDF

- If $\sum_{i=1}^{n} dbf^*(\tau_i, t) \leq t$, then $\sum_{i=1}^{n} dbf(\tau_i, t) \leq t$.
- If $\sum_{i=1}^{n} dbf^*(\tau_i, t) > t$, then $\sum_{i=1}^{n} dbf(\tau_i, t) > 0.5t$.

With similar strategy, we can prove that such an approach has a resource augmentation factor 2.

- For all $t$, if $\sum_{i=1}^{n} dbf^*(\tau_i, t) \leq t$, then it is schedulable by EDF;
- otherwise, it is probably not schedulable.

Similarly, we can also extend to approximate with a given error tolerate parameter $\delta$. [Albers and Slomka, 2004]
Is the Approximation for EDF Tight?

\[ dbf^*(\tau_i, t) = \begin{cases} 
0 & \text{if } t < D_i \\
(t - D_i) \frac{T_i}{T_i} + 1 C_i & \text{otherwise.}
\end{cases} \]

- Not really, when \( t \) is very close to \( t + D_i \), we can find a sharp increase of the demand bound function.
- Even though a factor 2 in is tight to bound \( dbf \) and \( dbf^* \), it is not tight for resource augmentation even for a uniprocessor system.
Theorem

Chen and Chakraborty [RTSS 2011]

- There exists a set of input instances such that the resource augmentation factor for one-step approximation of DBF is 1.5.
- The resource augmentation factor for one-step approximation of DBF is at most \( \frac{2e-1}{e} \approx 1.6322 \).

Proofs and details are omitted.