Multiprocessor Scheduling III: Semi-Partitioned Scheduling

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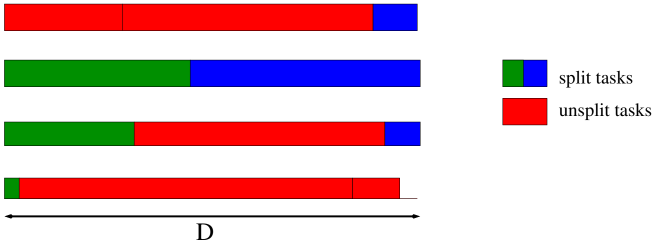
Weakness of Partitioned Scheduling

- Restricting a task on a processor reduces the schedulability
- Restricting a task on a processor makes the problem $\mathcal{NP}$-hard
- Example: Suppose that there are M processors and $M+1$ tasks with the same period T and the (worst-case) execution times of all these $M+1$ tasks are $\frac{T}{2}+\epsilon$ with $\epsilon > 0$
  - With partitioned scheduling, it is not schedulable
  - The least upper bound (LUB) for partitioned scheduling is no more than 50% of the EDF or RM.
- Within this part, we will focus only for *identical processors* and periodic tasks with *implicit deadlines*.

Benefits by Allowing Task Migration

- The schedulability can be improved.
- The $\mathcal{N}P$-completeness for EDF does no hold any more if the migration has *no overhead*.
- The $\mathcal{N}P$-completeness for schedulability test of RM still holds since 1 processor is already $\mathcal{N}P$-complete.
- For the above example, we will have 100% LUB if all the tasks have the same period with $D_i = T_i$, and $C_i \le T_i$ for task τ_i.



Limits of Task Migration

- Migration indeed requires overhead
  - Migration should be performed only when it is necessary.
  - The number of migrations should be reduced as much as possible.
  - This property is the killing argument for fine-grained task migration schedulers, e.g., p-fair scheduling.
- Migration has to be done carefully so that a job of a task should not be executed simultaneously on more than one processor.
  - The execution of a migrating task should be divided into blocks on different processors such that the blocks do not overlap.
  - This property forces the scheduler to make smarter decisions for task migrations

Articles for This Module

- Björn Andersson, Konstantinos Bletsas: Sporadic Multiprocessor Scheduling with Few Preemptions. ECRTS 2008: 243-252

Dynamic Priority

- Nan Guan, Martin Stigge, Wang Yi, Ge Yu: Fixed-Priority Multiprocessor Scheduling with Liu and Layland's Utilization Bound. IEEE Real-Time and Embedded Technology and Applications Symposium 2010: 165-174

Static Priority

Further Readings

- Nan Guan, Martin Stigge, Yu Ge, and Wang Yi. Parametric Utilization Bounds for Fixed-Priority Multiprocessor Scheduling. In the proc. of the 26th IEEE International Parallel & Distributed Processing Symposium. 2012.
- Yi Zhang, Nan Guan, Yanbin Xiao, Wang Yi. Implementation and Empirical Comparison of Partitioning-based Multi-core Scheduling. In the proc. of the 6th IEEE International Symposium on Industrial Embedded Systems (SIES11), 2011.
- A. Bastoni, B. Brandenburg, and J. Anderson, " Is Semi-Partitioned Scheduling Practical?", Proceedings of the 23rd Euromicro Conference on Real-Time Systems, 2011.

Outline

Introduction

Semi-Partitioned EDF Scheduling

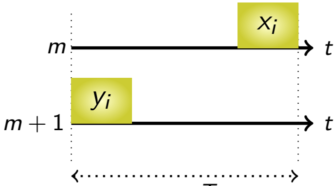
Semi-Partitioned Static-Priority Scheduling

Approaches

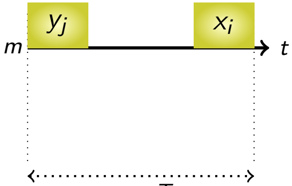
- Use resource reservation for task migration.
  - Suppose that $T_{\min}$ is the minimum period among all the tasks.
  - By a user-designed parameter κ, we divide time into slots with length $S = \frac{T_{\min}}{\kappa}$.
  - We can use the first-fit approach by splitting a task into 2 subtasks, in which one is executed on processor m and the other is executed on processor $m+1$.
  - Execution of a split task is only possible in the reserved time window in the time slot.
  - Applying first-fit algorithm, by taking *SEP* as the upper bound of utilization on a processor.
    - If a task does not fit, split this task into two subtasks and allocate a new processor, one is assigned on the processor under consideration, and the other is assigned on the newly allocated processor.

Reservation for EDF

For each time slot, we will reserve two parts.



$$S = \frac{T_{\min}}{\kappa}$$



$$S = \frac{T_{\min}}{\kappa}$$

If a task τ_i is split, the task can be served only within these two pre-defined time slots with length x_i and y_i.

A processor can host two split tasks, τ_i and τ_j. τ_i is served at the beginning of the time slot, and τ_j is served at the end.

The schedule is EDF, but if a split task instance is in the ready queue, it is executed in the reserved time region.

Inflation for Reservation

To meet the utilization request of a split task τ_i by splitting $lo\_split(\tau_i)$ portion on processor m and $high\_split(\tau_i)$ portion on processor $m+1$, we should guarantee that

$$\frac{x_i + y_i}{S} \geq lo\_split(\tau_i) + high\_split(\tau_i) = \frac{C_i}{T_i} = U_i.$$

- Assigning only $x_i + y_i = S \cdot U_i$ does not work, since we might just miss the available reserved time slot.
- Let's inflate x_i and y_i by a constant portion f, in which $x_i = S \cdot (f + lo\_split(\tau_i))$ and $y_i = S \cdot (f + high\_split(\tau_i))$.
- To ensure the schedulability, we need

$$\frac{\kappa(x_i + y_i)}{(\kappa+1)S - (x_i + y_i)} \geq \frac{C_i}{T_i}.$$

- The above inequality comes from the worst case that we miss the reserved time slot at the beginning of a time slot, and have to finish before the reserved time slot at the end of a time slot.

How Much to Inflate?

$$\frac{\kappa(x_i+y_i)}{(\kappa+1)S-(x_i+y_i)} \geq \frac{C_i}{T_i}$$

$$\Rightarrow \frac{T_{\min}(2f+U_i)}{T_{\min}+\frac{T_{\min}}{\kappa}-\frac{T_{\min}}{\kappa}(2f+U_i)} \geq U_i$$

$$\Rightarrow \frac{2f+U_i}{1+\frac{1}{\kappa}-\frac{1}{\kappa}(2f+U_i)} \geq U_i$$

$$\Rightarrow f \geq \frac{U_i-U_i^2}{2\kappa+2U_i}.$$

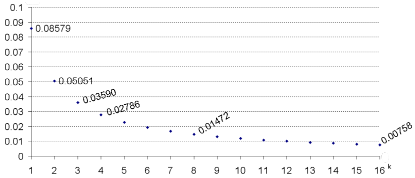


Figure : f versus κ

Therefore, by simple calculus, given U_i, the minimum f happens when $f = \sqrt[2]{\kappa(\kappa+1)} - \kappa$.

Algorithm

We can assign all the tasks τ_i with $U_i > SEP$ on a dedicated processor. So, we only consider tasks with $U_i \leq SEP$.

```
m ← 1, U_m ← 0;
for i = 1 to N, where N = |T| do
    if C_i/T_i + U_m ≤ SEP then
        assign task τ_i on processor m;
        U_m ← U_m + C_i/T_i;
    else
        assign task τ_i on processor m with lo_split(τ_i) set to SEP − U_m
        and on processor m + 1 with high_split(τ_i) set to
        C_i/T_i − (SEP − U_m);
        m ← m + 1 and U_m ← C_i/T_i − (SEP − U_m);
```

- When executing, the reservation to serve τ_i is to set x_i to $S(f + lo\_split(\tau_i))$ and y_i to $S(f + high\_split(\tau_i))$.
- SEP is set as a constant.

Two Split Tasks on a Processor

For split tasks to be schedulable, the following sufficient conditions have to be satisfied

- $lo\_split(\tau_i) + f + high\_split(\tau_i) + f \leq 1$ for any split task τ_i.
- $lo\_split(\tau_j) + f + high\_split(\tau_i) + f \leq 1$ when τ_i and τ_j are assigned on the same processor.

Therefore, the "magic value" SEP

$$SEP \leq 1 - 2f \leq 1 - 2(\sqrt[2]{\kappa(\kappa+1)} - \kappa).$$

However, we still have to guarantee the schedulability of the non-split tasks. It can be shown that the sufficient condition is

$$SEP \leq 1 - 4f \leq 1 - 4(\sqrt[2]{\kappa(\kappa+1)} - \kappa).$$

The proof is omitted here.

Magic Values: f

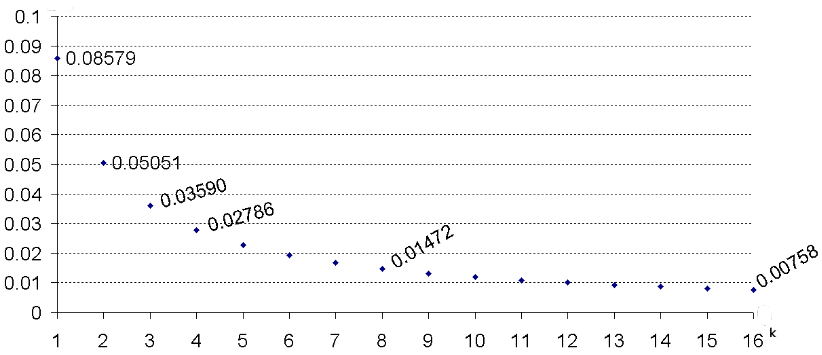


Figure : f versus κ

Magic Values: *SEP*

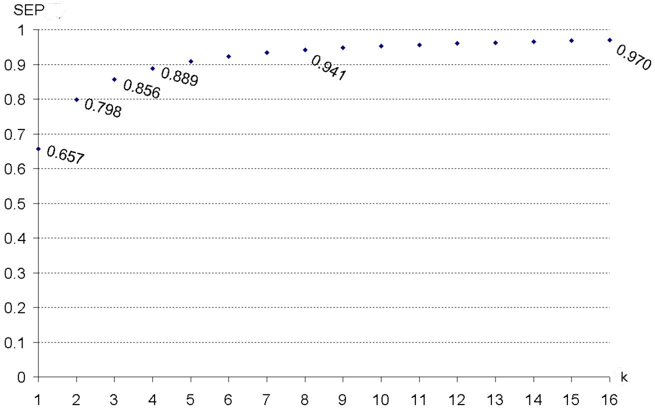


Figure : *SEP* versus κ

Property

Theorem

By taking SEP as $1-4(\sqrt[2]{\kappa(\kappa+1)}-\kappa)$ and $f=\sqrt[2]{\kappa(\kappa+1)}-\kappa$, the above algorithm guarantees to derive feasible schedule if $\sum_{\tau_i \in \mathbf{T}} \frac{C_i}{T_i} \leq M' \cdot SEP$ and $\frac{C_i}{T_i} \leq 1$ for all tasks τ_i.

Outline

Introduction

Semi-Partitioned EDF Scheduling

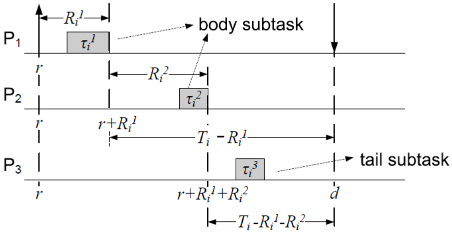
Semi-Partitioned Static-Priority Scheduling

Basic Idea

1. Use Liu and Layland's bound for rate-monotonic scheduling
   - A task set of N implicit-deadline tasks is schedulable (under RM) on one processor if the total utilization is no more than $U_{lub}(RM, N) = N(2^{\frac{1}{N}} - 1) \geq 0.693$.
2. We can start to consider the tasks with the non-increasing order of periods
3. To reduce the utilization, we again pick the processor with the minimum task utilization so far when considering task τ_i
4. If a task cannot fit into the picked processor, we will have to split it into multiple (two or even more) parts.
5. Since we consider periods in an non-increasing order, when τ_i is split, it has higher priority than other tasks that have been considered.
6. Therefore, if τ_i is split and assigned to a processor m and the utilization on processor m after assigning τ_i is at most $U_{lub}(RM, N)$, then τ_i is so far schedulable.

Terminology

- Non-split task: a task that is executed only on one processor.
- Split task: a task τ_i that is executed on k_i processors, where $k_i \geq 2$.
  - There are k_i subtasks, denoted by $\tau_i^1, \tau_i^2, \ldots, \tau_i^{k_i}$, in which none of them will run at the same time.
  - The k_i subtasks have to be synchronized.
  - Subtask τ_i^j is with computation time requirement C_i^j, relative deadline Δ_i^j, and period T_i.
  - Subtask $\tau_i^{k_i}$ is called the *tail subtask* of task τ_i.
  - Subtask $\tau_i^1, \tau_i^2, \ldots, \tau_i^{k_i-1}$ are called *body subtasks* of task τ_i.



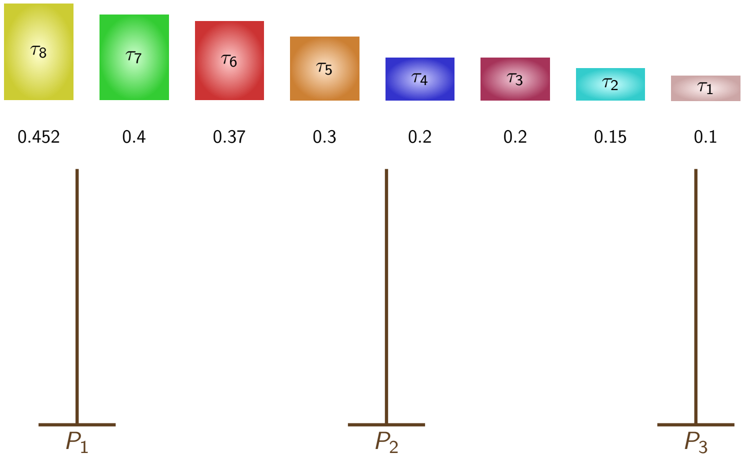
Algorithm: Largest Period First (LPF)

Input: $\mathbf{T}, M$;

1: re-index (sort) tasks such that $T_i \geq T_j$ for $i > j$;
2: initialize utilization U_m on processor m as 0;
3: initialize k_i of task τ_i as 1 and R_i^1 as 0;
4: **for** $i = N$ to 1, where $N = |\mathbf{T}|$ **do**
5:     find m^* with the minimum utilization, i.e., $U_{m^*} = \min_m U_m$;
6:     return "assignment fails" if $U_{m^*} \geq U_{lub}(RM, N)$;
7:     **if** $U_{m^*} + \frac{C_i^{k_i}}{T_i} \leq U_{lub}(RM, N)$ **then**
8:         assign subtask $\tau_i^{k_i}$ onto processor m^*, where $U_{m^*} \leftarrow U_{m^*} + \frac{C_i^{k_i}}{T_i}$;
9:     **else**
10:         split task τ_i further with $C_i^{k_i+1} \leftarrow C_i^{k_i} - (U_{lub}(RM, N) - U_{m^*})T_i$ and $C_i^{k_i} \leftarrow (U_{lub}(RM, N) - U_{m^*})T_i$;
11:         assign subtask $\tau_i^{k_i}$ onto processor m^*, where $U_{m^*} \leftarrow U_{lub}(RM, N)$;
12:         $R_i^{k_i+1} \leftarrow C_i - \sum_{j=1}^{k_i} C_i^j$;
13:         $k_i \leftarrow k_i + 1$ and goto step 5;
14: execute with rate-monotonic scheduling, in which subtask τ_i^j is with offset R_i^j;

An Example for LPF

Let's ignore R_i^j first. $U_{lub}(RM, 8) \approx 0.724$.



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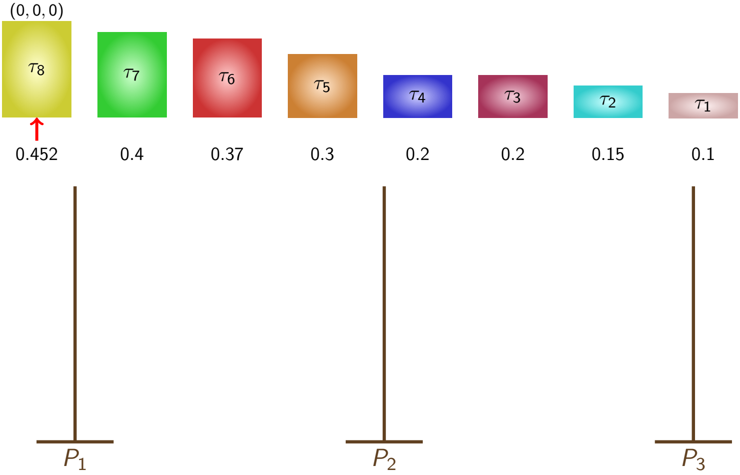

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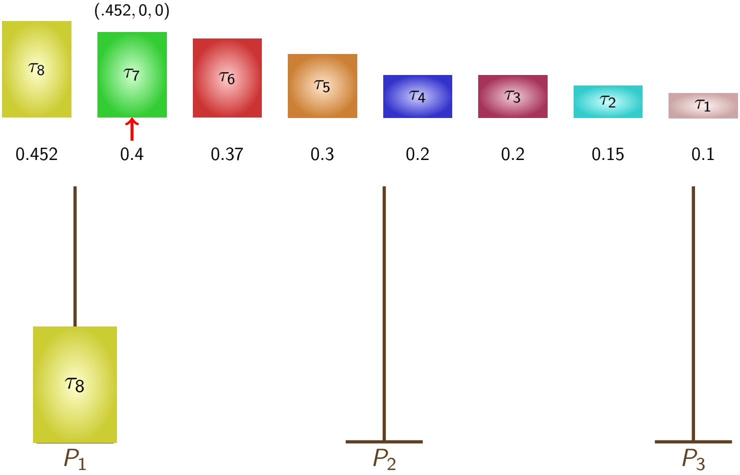

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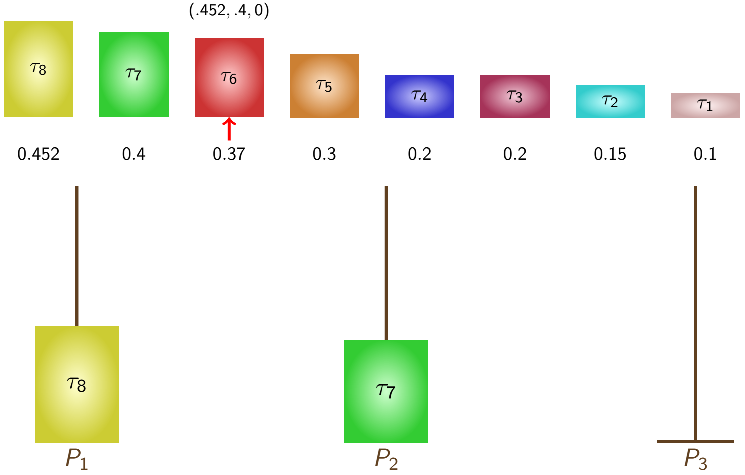

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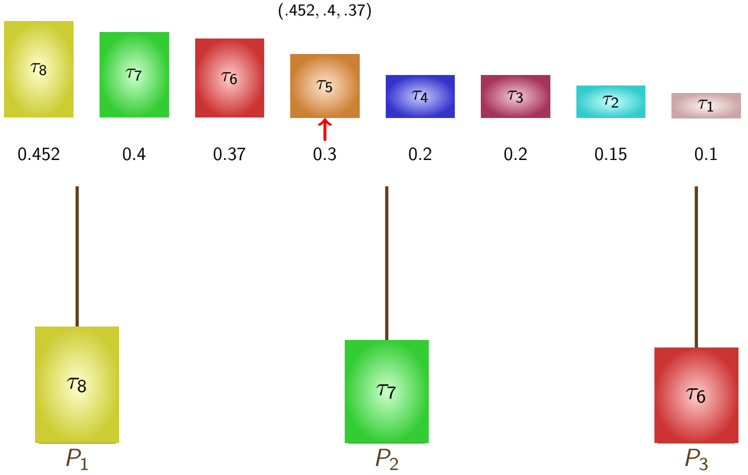

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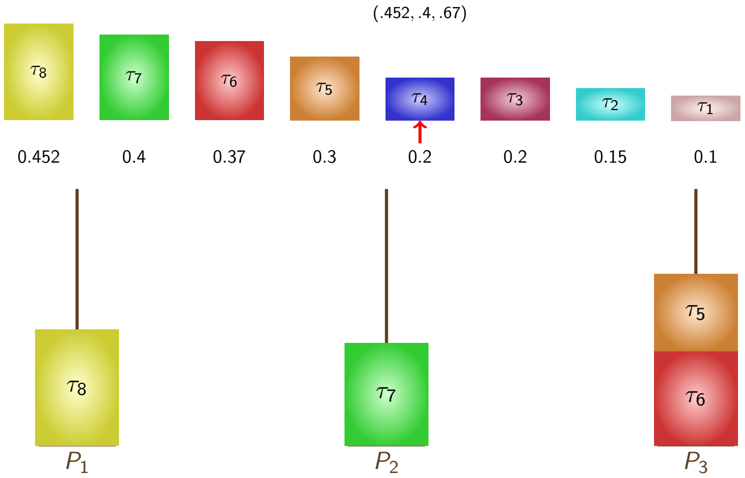

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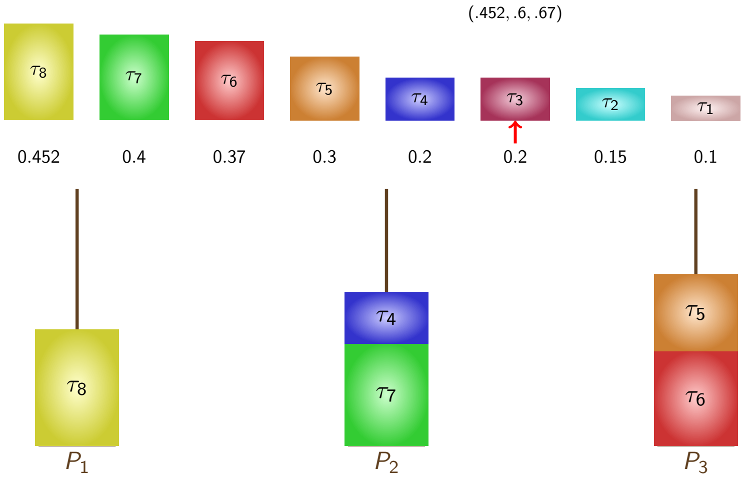

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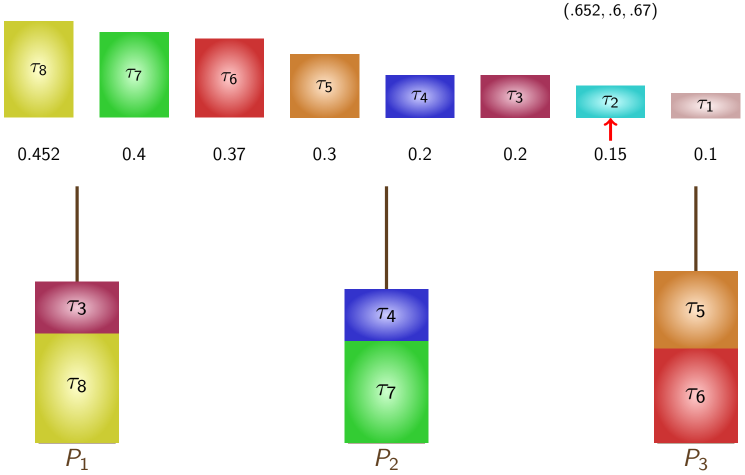

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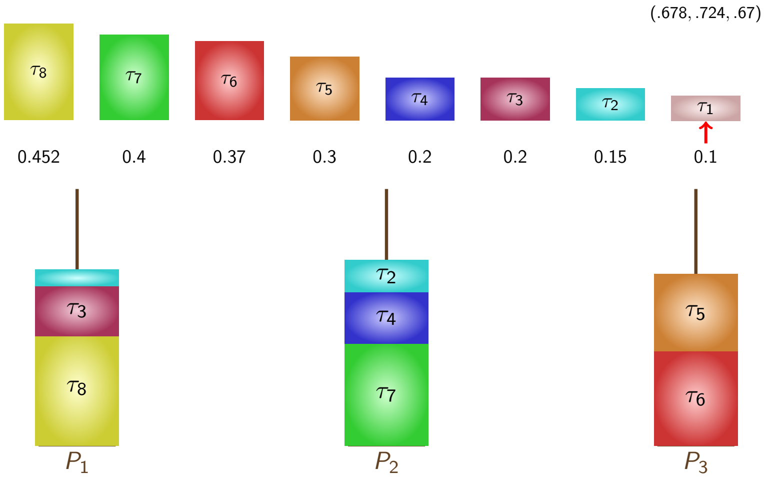

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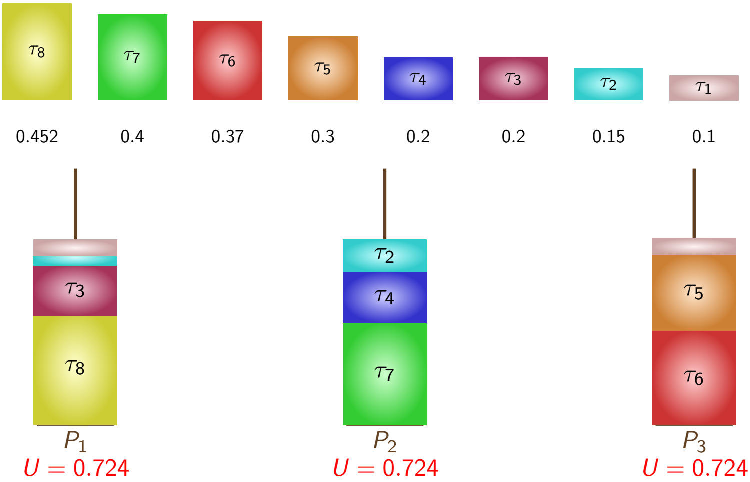


Body Subtasks

Let $m(\tau_i^j)$ be the processor that subtask τ_i^j is assigned onto;

Theorem

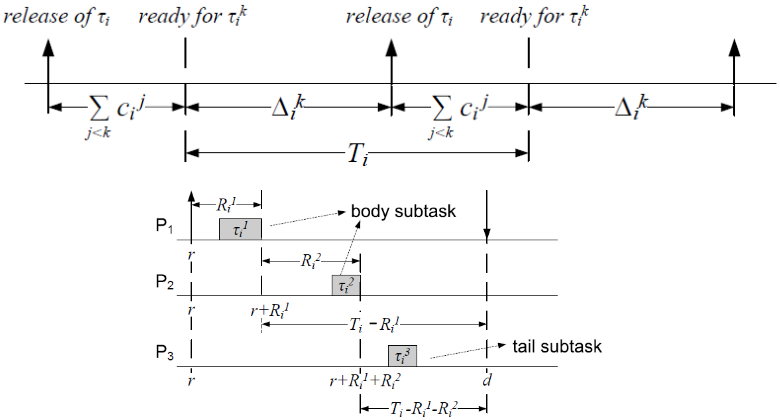
If $j < k_i$, i.e., τ_i^j is a body subtask, τ_i^j is with the highest priority on processor $m(\tau_i^j)$.

Proof

- This simply comes from the algorithm, which considers tasks from the lowest-priority to the highest priority
- When a body subtask is assigned, there is no more (sub)task τ_k^ℓ with $k < i$ going to be assigned on processor $m(\tau_i^j)$ anymore.

How does Scheduling Work?

Let's put R_i^j back.



Schedulability on Good Partitions

Suppose that $\mathbf{T}_m$ is the set of (sub)tasks that are assigned on processor m.

Theorem

If there are only body subtasks or non-split tasks in $\mathbf{T}_m$, then

- the non-split tasks can meet their deadlines and
- the body subtask τ_i^j is with response time C_i^j with deferred release R_i^j.

It should be clear now.

Trouble Maker: Tail Subtasks

- A tail subtask does not have the highest priority...
- It should also be now clear that the offset of a tail subtask $\tau_i^{k_i}$ is equal to $\sum_{j=1}^{k_i-1} C_i^j$.
- Therefore, to meet the requirement with non-overlapping execution of task τ_i, we have to finish $\tau_i^{k_i}$ within $T_i - \sum_{j=1}^{k_i-1} C_i^j = T_i - (C_i - C_i^{k_i})$.
- For simplicity, we use Δ_i to denote $T_i - (C_i - C_i^{k_i})$;
- Suppose that Y_i is the utilization of the tasks that are assigned on the processor $m(\tau_i^{k_i})$ for executing $\tau_i^{k_i}$ and with higher priority than $\tau_i^{k_i}$.
- We would first like to show that $\tau_i^{k_i}$ can finish within Δ_i if

$$\frac{Y_i T_i + C_i^{k_i}}{\Delta_i} \leq U_{lub}(RM, N).$$

Proof for Tail Subtasks

- Again, since rate-monotonic is applied, we only have to consider tasks with periods shorter than task τ_i.
- For notational brevity, let's assume $\tau_i^{k_i}$ is on processor m;
- Let's distinguish two types of tasks on m (by the relationship with Δ_i):

$$\mathbf{T}_m^1 = \{\tau_\ell \mid \tau_\ell \in \mathbf{T}_m, j < i, T_j \geq \Delta_i\}$$
$$\mathbf{T}_m^2 = \{\tau_\ell \mid \tau_\ell \in \mathbf{T}_m, j < i, T_j < \Delta_i\}$$

- By definition, (note that we do not distinguish between split or non-split tasks by defining c_j as the execution time of task τ_j or subtask τ_j^ℓ in $\mathbf{T}_m$):

$$U_{lub}(RM, N) \geq \frac{Y_i T_i + C_i^{k_i}}{\Delta_i} = \frac{(\sum_{\tau_j \in \mathbf{T}_m^1} \frac{c_j}{T_j} + \sum_{\tau_j \in \mathbf{T}_m^2} \frac{c_j}{T_j}) T_i + C_i^{k_i}}{\Delta_i}$$
$$= \sum_{\tau_j \in \mathbf{T}_m^1} \frac{c_j}{T_j} \frac{T_i}{\Delta_i} + \sum_{\tau_j \in \mathbf{T}_m^2} \frac{c_j}{T_j} \frac{T_i}{\Delta_i} + \frac{C_i^{k_i}}{\Delta_i} \geq \sum_{\tau_j \in \mathbf{T}_m^1} \frac{c_j}{\Delta_i} + \sum_{\tau_j \in \mathbf{T}_m^2} \frac{c_j}{T_j} + \frac{C_i^{k_i}}{\Delta_i}.$$

- A more difficult task set (by changing T_j to Δ_i for $\tau_j \in \mathbf{T}_m^1$) finishes $\tau_i^{k_i}$ within Δ_i, so is the original task set.

Issues of the Tail Subtask

- If we are sure that $\frac{Y_i T_i + C_i^{k_i}}{\Delta_i} \leq U_{lub}(RM, N)$, we will be fine.
- However, the above inequality only holds for some cases when the utilization of τ_i is small enough.
- So, the next question is: What is the maximum U_i of a split task τ_i, that enforces $\frac{Y_i T_i + C_i^{k_i}}{\Delta_i} \leq U_{lub}(RM, N)$?
- Let X_i be the total utilization of the tasks on processor m with lower-priority than $\tau_i^{k_i}$.
- By definition, we know that

$$X_i + \frac{C_i^{k_i}}{T_i} + Y_i \leq U_{lub}(RM, N)$$

Issues of the Tail Subtask

As τ_i has k_i subtasks, there are $k_i - 1$ body subtasks assigned on other $k_i - 1$ processors. Therefore,

$$U_{lub}(RM, N) - Y_i \geq X_i + \frac{C_i^{k_i}}{T_i} \geq_1 U_{lub}(RM, N) - \frac{\sum_{j=1}^{k_i-1} C_i^j}{(k_i - 1)T_i} + \frac{C_i^{k_i}}{T_i}$$

$$= U_{lub}(RM, N) - \frac{C_i - C_i^{k_i}}{(k_i - 1)T_i} + \frac{C_i^{k_i}}{T_i} \geq_2 U_{lub}(RM, N) - (\frac{C_i}{T_i} - 2\frac{C_i^{k_i}}{T_i}).$$

$\geq_1$ is because there are $k_i - 1$ processors with utilization lower than or equal to X_i before assigning $\tau_i^{k_i}$. $\geq_2$ is because $k_i \geq 2$. Hence,

$$Y_i \leq (\frac{C_i}{T_i} - 2\frac{C_i^{k_i}}{T_i}).$$

Since $\tau_i^{k_i}$ can finish within Δ_i if $\frac{Y_i T_i + C_i^{k_i}}{\Delta_i} \leq U_{lub}(RM, N)$, we have that if

$$\frac{Y_i T_i + C_i^{k_i}}{\Delta_i} \leq \frac{C_i - C_i^{k_i}}{T_i - (C_i - C_i^{k_i})} = \frac{T_i}{T_i - (C_i - C_i^{k_i})} - 1 \leq U_{lub}(RM, N),$$

$\tau_i^{k_i}$ can finish within Δ_i.

Light and Heavy Tasks

- If task τ_i is with utilization lower than or equal to $\frac{U_{lub}(RM,N)}{1+U_{lub}(RM,N)}$, τ_i is a light task.
- If task τ_i is with utilization higher than $\frac{U_{lub}(RM,N)}{1+U_{lub}(RM,N)}$, τ_i is a heavy task.

Therefore, if τ_i is a light task,

$$\frac{T_i}{T_i - (C_i - C_i^{k_i})} - 1 \leq \frac{\frac{C_i}{T_i}}{1 - \frac{C_i}{T_i}} \leq U_{lub}(RM, N).$$

First Part

Theorem

If all the (split) tasks are light tasks, e.g., $U_i \leq \frac{U_{lub}(RM,N)}{1+U_{lub}(RM,N)} \forall \tau_i \in \mathbf{T}$ (or for every split task τ_i), LPT guarantees the feasibility of the derived schedule when $\frac{\sum_{\tau_i \in \mathbf{T}} U_i}{M} \leq U_{lub}(RM, N)$ and $U_i \leq 1$ for all the tasks τ_i on M homogeneous processors.

Assign Heavy Tasks First (HT-LPT)

Key idea: Pre-assign a HEAVY task such that it is not split.

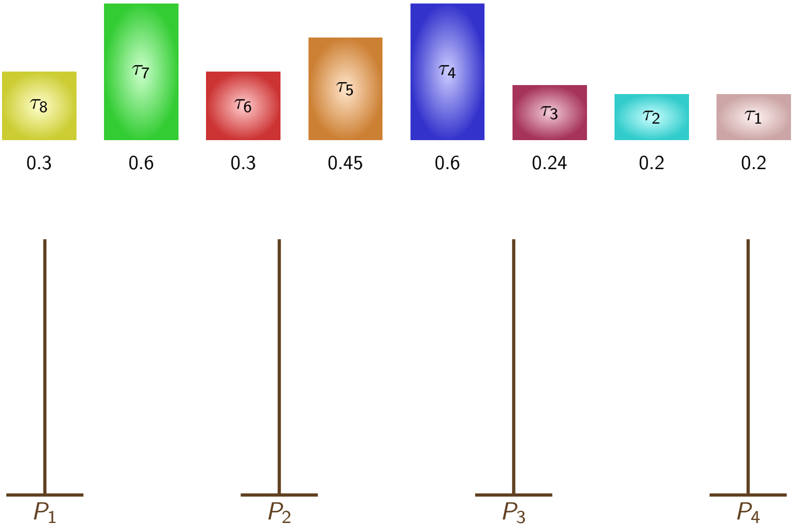
1: pre-assign heavy tasks under some conditions;
2: assign light tasks on processors without pre-assigned tasks as much as possible;
3: assign remaining light tasks to the processors with heavy tasks with a "specific order";

Assign Heavy Tasks First (HT-LPT)

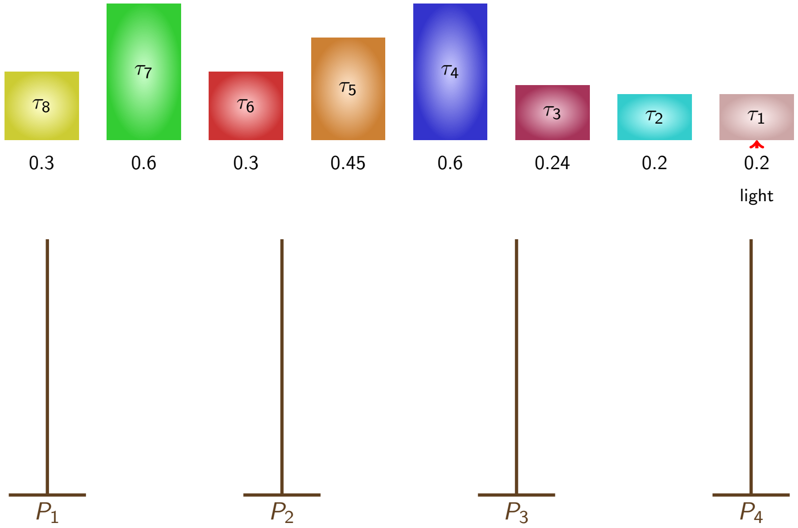
Key idea: Pre-assign a HEAVY task such that it is not split.

$m \leftarrow 1$;
for $i = 1$ to N, where $N = |\mathbf{T}|$ **do**
  if τ_i is not a heavy task **then**
    continue;
  if $\sum_{j=i+1}^{N} \frac{C_j}{T_j} \leq (M - m)U_{lub}(RM, N)$ **then**
    pre-assign task τ_i on processor m;
    $m \leftarrow m + 1$;
let $\mathbf{T}^{pre}$ be the set of tasks that are pre-assigned on $m - 1$ processors after the above loop, and $M^\dagger \leftarrow m - 1$;
run algorithm LPF for deciding $\mathbf{T} \setminus \mathbf{T}^{pre}$ on processor $M^\dagger + 1$ to M;
while there exists task τ_i in $\mathbf{T} \setminus \mathbf{T}^{pre}$ that cannot be assigned on any processor without pre-assigned tasks **do**
  repeatedly assign or split task τ_i as much as possible on the largest-index processor with utilization less than $U_{lub}(RM, N)$ among the first $M^\dagger$ processors;

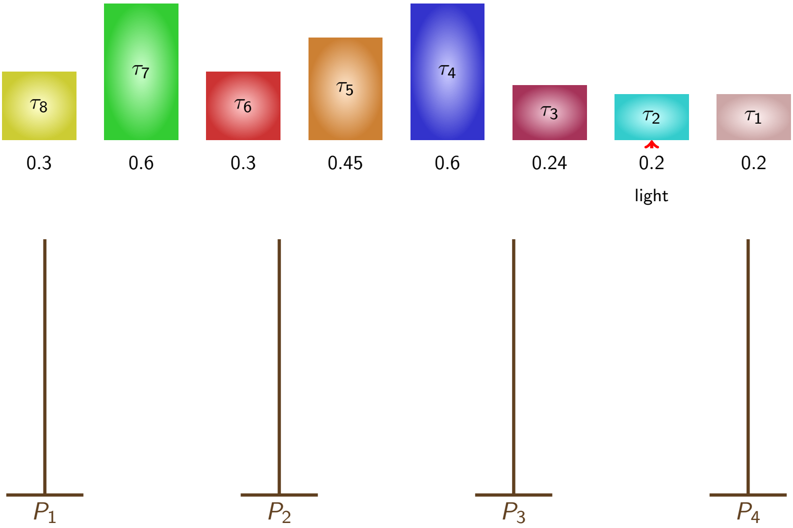
Example of HT-LPT



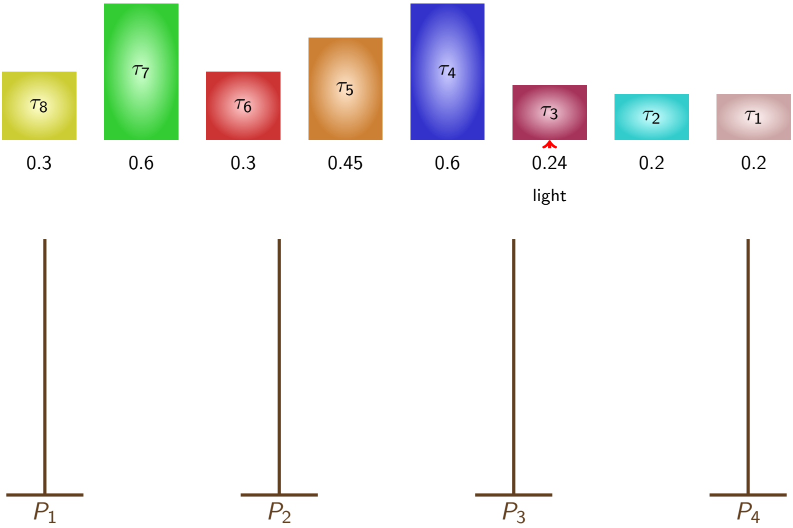
Example of HT-LPT



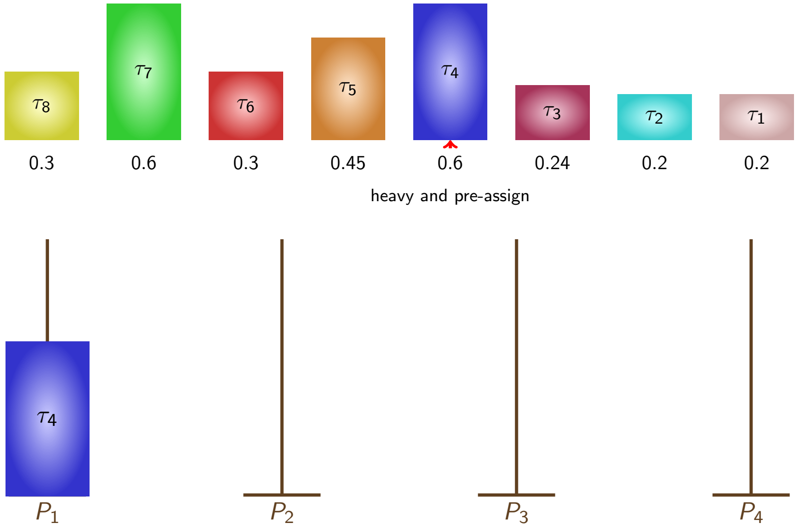
Example of HT-LPT



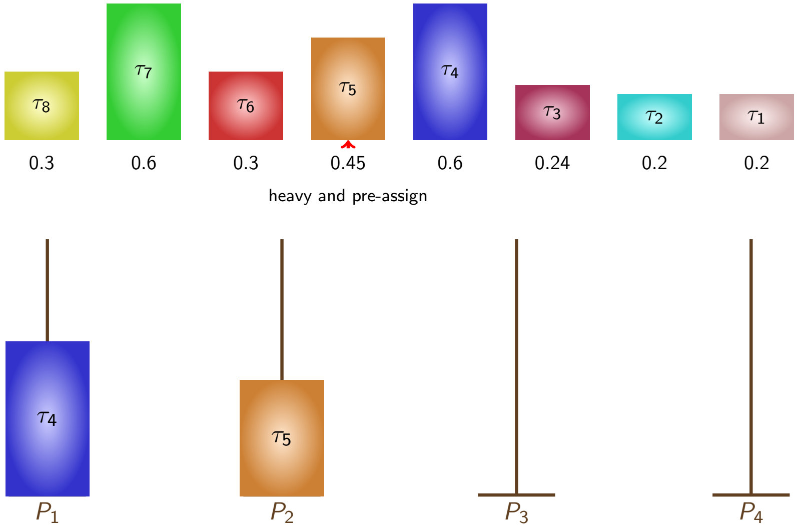
Example of HT-LPT



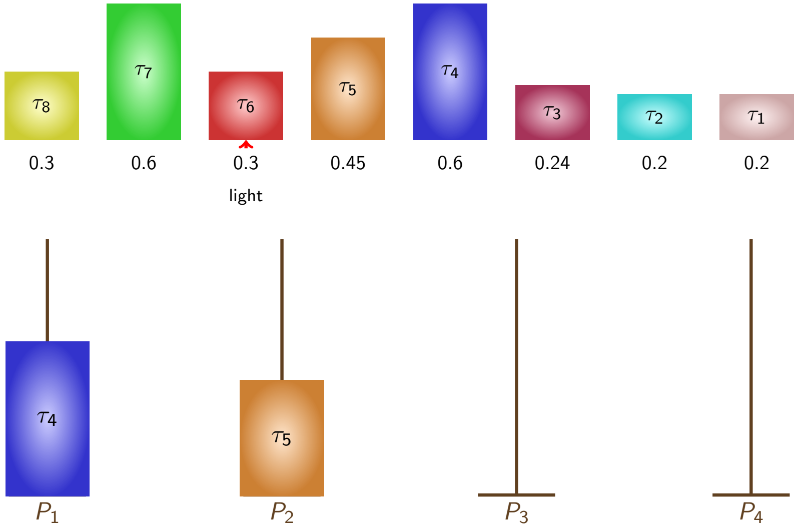
Example of HT-LPT



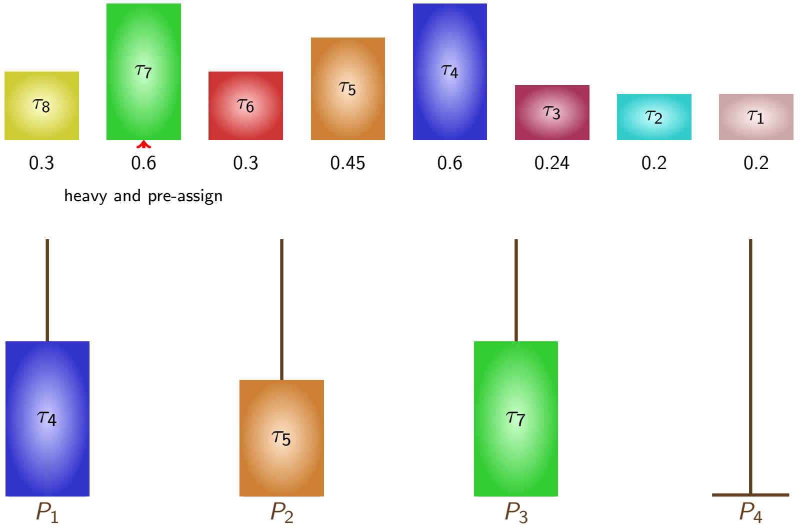
Example of HT-LPT



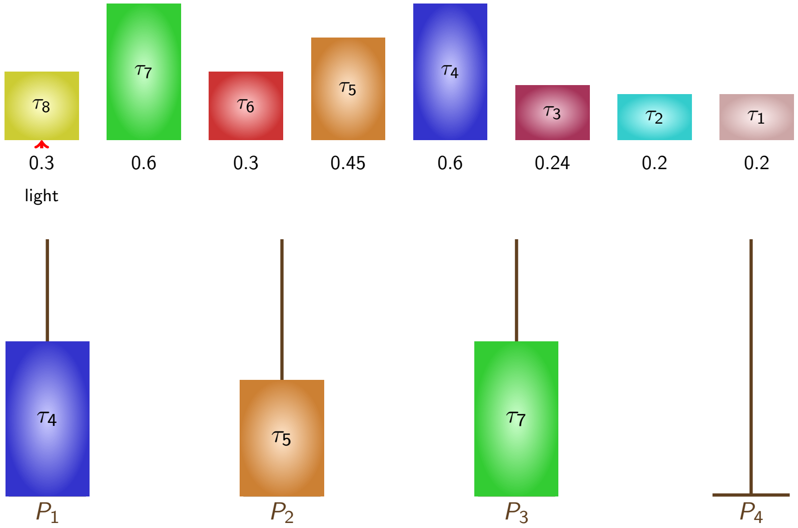
Example of HT-LPT



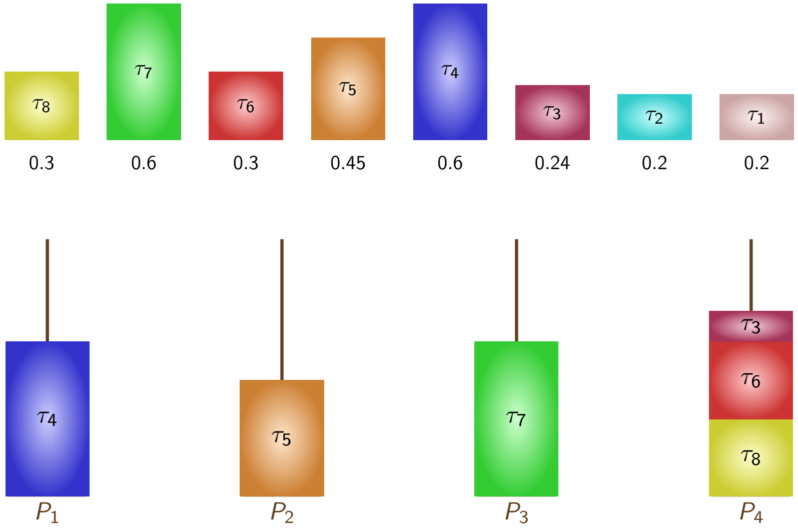
Example of HT-LPT



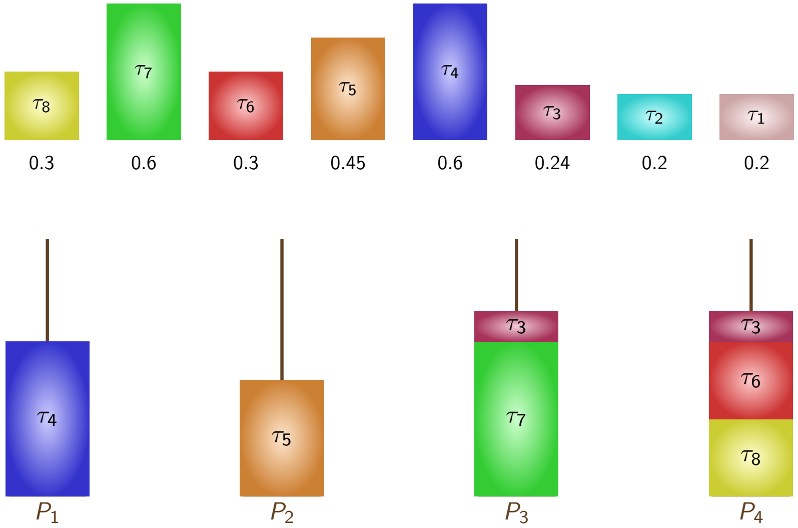
Example of HT-LPT



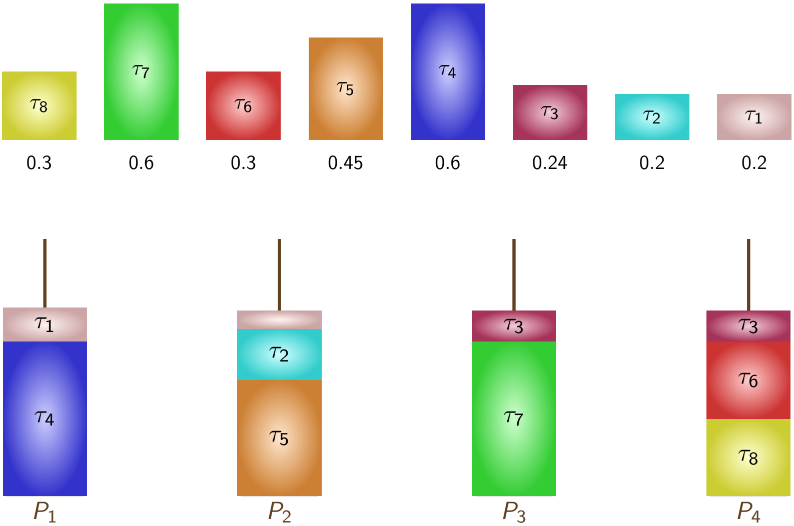
Example of HT-LPT



Example of HT-LPT



Example of HT-LPT



Priority of Heavy Tasks

Let τ_i be a heavy task, and there are η pre-assigned tasks with higher priority than τ_i. Then we know

- If τ_i is a pre-assigned task, it satisfies

$$\sum_{j>i} \frac{C_i}{T_i} \leq (M - \eta + 1) \times U_{lub}(RM, N).$$

- If τ_i is not a pre-assigned task, it satisfies

$$\sum_{j>i} \frac{C_i}{T_i} > (M - \eta + 1) \times U_{lub}(RM, N).$$

Theorem

A pre-assigned task has the lowest-priority on its host processor.

Property of HT-LPT (Proofs are omitted)

Theorem

HT-LPT guarantees the feasibility of the derived schedule when $\frac{\sum_{\tau_i \in \mathbf{T}} U_i}{M} \leq U_{lub}(RM, N)$ and $U_i \leq 1$ for all the tasks τ_i on M homogeneous processors.