Energy Efficiency Analysis for the Single Frequency Approximation (SFA) Scheme

Santiago Pagani and Jian-Jia Chen

LS 12, TU Dortmund

RTCSA - August 2013
Outline

- Introduction
- Motivation and Problem Definition
- Approximation Factor Analysis (energy consumption) of SFA
  - Negligible Leakage Power Consumption
  - Non-negligible Leakage Power Consumption
  - Balanced Task Sets and Non-negligible Overhead for Sleeping
- Simulations
- Conclusions
Outline

• Introduction
Introduction

**Importance of Energy Efficiency:**

- Slow increases of battery capacity.
  - Less Energy Consumption $\Rightarrow$ Prolong Battery Lifetime of Embedded Systems.

- Increasing costs of energy.
  - Less Energy Consumption $\Rightarrow$ Lower Power Bills for Servers.
Introduction

Importance of Energy Efficiency:

- Slow increases of battery capacity.
  - Less Energy Consumption $\Rightarrow$ Prolong Battery Lifetime of Embedded Systems.

- Increasing costs of energy.
  - Less Energy Consumption $\Rightarrow$ Lower Power Bills for Servers.

Outcome for Computing Systems:

- Motivated to move from single-core to multi-core.

- Techniques for power management.
Dynamic Power Management (DPM):

- Technique for putting cores in a low-power mode: idle, sleep, off, etc.
**Introduction**

**Dynamic Power Management (DPM):**
- Technique for putting cores in a low-power mode: idle, sleep, off, etc.

**Dynamic Voltage and Frequency Scaling (DVFS):**
- Technique for scaling the voltage and frequency of cores.
Introduction

Dynamic Power Management (DPM):
• Technique for putting cores in a low-power mode: idle, sleep, off, etc.

Dynamic Voltage and Frequency Scaling (DVFS):
• Technique for scaling the voltage and frequency of cores.
• Per-core DVFS:
  • Individual voltage and frequency for cores.
  • Optimal, but too expensive to manufacture.
Introduction

Dynamic Power Management (DPM):
• Technique for putting cores in a low-power mode: idle, sleep, off, etc.

Dynamic Voltage and Frequency Scaling (DVFS):
• Technique for scaling the voltage and frequency of cores.

• Per-core DVFS:
  • Individual voltage and frequency for cores.
  • Optimal, but too expensive to manufacture.

• Global DVFS:
  • All cores share the same voltage.
  • Energy inefficient.
Introduction

Dynamic Voltage and Frequency Scaling (DVFS):

- Multiple Voltage Islands:
  - Compromise between \textit{Per-core DVFS} and \textit{Global DVFS}.
  - Cores are grouped into \textit{Voltage Islands}.
  - Islands can have different voltages.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{images/dvfs.png}
\caption{Intel’s SCC snapshot}
\end{figure}

Introduction

CMOS Power Model

\[ P(s) = P_{\text{dynamic}}(s) + P_{\text{static}} \]
Introduction

CMOS Power Model

\[ P(s) = P_{\text{dynamic}}(s) + P_{\text{static}} \]

Considering that:

\[ P_{\text{dynamic}}(s) = C_{\text{eff}} V_{dd}^2 s \]

\[ s \propto \frac{(V_{dd} - V_t)^2}{V_{dd}} \]

Santiago Pagani and Jian-Jia Chen (LS 12, TU Dortmund)
Introduction

CMOS Power Model

\[ P (s) = P_{\text{dynamic}} (s) + P_{\text{static}} \]

Considering that:

\[ P_{\text{dynamic}} (s) = C_{\text{eff}} V_{dd}^2 s \]

\[ s \propto \frac{(V_{dd} - V_t)^2}{V_{dd}} \]

We can approximate to:

\[ P (s) = \alpha s^\gamma + \beta \]
Introduction

CMOS Power Model

\[ P(s) = P_{\text{dynamic}}(s) + P_{\text{static}} \]

Considering that:

\[ P_{\text{dynamic}}(s) = C_{\text{eff}} V_{dd}^2 s \]

\[ s \propto \frac{(V_{dd} - V_t)^2}{V_{dd}} \]

We can approximate to:

\[ P(s) = \alpha s^\gamma + \beta \]

**Figure:** \( \alpha = 1.76 \text{ Watts/GHz}^3 \), \( \gamma = 3 \) and \( \beta = 0.5 \text{ Watts} \)}
**Introduction**

**Energy Consumption**

\[
E(s) = (\alpha s^\gamma + \beta) \frac{\Delta c}{s}
\]

**Critical Frequency:**

\[
s_{\text{crit}} = \sqrt[\gamma]{\frac{\beta}{(\gamma - 1) \alpha}}
\]

**Figure:** \(\alpha = 1.76 \frac{\text{Watts}}{\text{GHz}^3}, \gamma = 3, \beta = 0.5 \text{ Watts and } \Delta c = 10^9 \text{ cycles}\)
Outline

- Motivation and Problem Definition
Motivation

In each voltage island (or Global DVFS), for energy minimization:

What voltage/frequency policy should be used?
Motivation

In each voltage island (or Global DVFS), for energy minimization:

**What voltage/frequency policy should be used?**

**Single Frequency Approximation (SFA) Scheme:**

- Use the lowest voltage/frequency, satisfying the timing constraints.
- Is the *simplest* and *most intuitive* strategy.
Motivation

In each voltage island (or Global DVFS), for energy minimization:

**What voltage/frequency policy should be used?**

*Single Frequency Approximation (SFA) Scheme:*

- Use the lowest voltage/frequency, satisfying the timing constraints.
- Is the *simplest* and *most intuitive* strategy.

\[
\text{Core } 4: \quad s_u = \max \{ s_{\text{crit}}, w_4 \}
\]

![Diagram showing Core 1, Core 2, Core 3, and Core 4 with varying s_u values over time t.](image-url)
Motivation

PROs of SFA:

- Linear time complexity.
- Significantly reduces the management overhead.
- No frequency alignment between cores ⇒ Any uni-core DPM technique can be adopted individually in each core.

CONs of SFA:

- SFA might consume more energy than another DVFS schedule.

How much more?

Santiago Pagani and Jian-Jia Chen
(LS 12, TU Dortmund)
Motivation

**PROs of SFA:**

- Linear time complexity.
- Significantly reduces the management overhead.
- No frequency alignment between cores $\implies$ Any uni-core DPM technique can be adopted individually in each core.

**CONs of SFA:**

- SFA might consume more energy than another DVFS schedule.

*How much more?*
Problem Definition

- For real-time tasks, already partitioned into task sets $T_1, T_2, \ldots, T_M$.
- Task sets ordered by their cycle utilizations: $w_1 \leq w_2 \leq \cdots \leq w_M$.
- Considering partitioned scheduling.
- Using Earliest-Deadline-First (EDF) algorithm.

**Objective:** Provide *theoretical analysis* to show the effectiveness of SFA for energy minimization.

$$AF_{SFA} = \max \frac{E_{SFA}}{E_{OPT}} \leq \max \frac{E_{SFA}}{E^*}$$
Outline

- Approximation Factor Analysis (energy consumption) of SFA
  - Negligible Leakage Power Consumption
Negligible Leakage Power Consumption

Energy Consumption for SFA (when $\beta = 0$):

- We execute at (single frequency) $s_u = w_M$.
- The cycle utilization distribution does not matter.

$$E_{SFA}^{\beta=0} (w_M) = \alpha L (w_M^{\gamma-1}) \sum_{i=1}^{M} w_i$$

---

Negligible Leakage Power Consumption

**Energy Consumption for SFA (when $\beta = 0$):**

- We execute at (single frequency) $s_u = w_M$.
- The cycle utilization distribution does not matter.

\[
E_{\text{SFA}}^{\beta=0} (w_M) = \alpha L (w_M^{\gamma-1}) \sum_{i=1}^{M} w_i
\]

**Lower Bound Energy Consumption (when $\beta = 0$):**

- Unroll periodic tasks in a hyper-period $\Rightarrow$ frame-based tasks.
- Use the results from Yang et al. \(^1\):

\[
E_{\beta=0}^* = \alpha L \left[ \sum_{i=1}^{M} (w_i - w_{i-1}) \sqrt{M - i + 1} \right]^{\gamma}
\]

Critical Cycle Utilization Distribution: Minimizes the lower bound of energy consumption, for a fixed $w_M$ and $\sum_{i=1}^{M} w_i$.

- $w_1 = w_2 = \cdots = w_{M-1} = \text{Average}(w_1, w_2, \ldots, w_{M-1})$
- Utilization Ratio: $0 \leq \delta = \frac{\text{Average}(w_1, w_2, \ldots, w_{M-1})}{w_M} \leq 1$
Negligible Leakage Power Consumption

Approximation factor of SFA when $\beta = 0$:

$$ AF_{SFA}^{\beta=0} \leq h(\delta) = \frac{1 - \delta + \delta M}{(1 - \delta + \delta \sqrt{M})^\gamma} \leq h(\delta^*) $$

$h(\delta)$ for $\gamma = 3$: 

\[h(\delta)\] for $\gamma = 3$: 

\[
\begin{array}{c}
\text{M=32} \\
\end{array}
\]
Negligible Leakage Power Consumption

Approximation factor of SFA when $\beta = 0$:

$$AF_{SFA}^{\beta=0} \leq h(\delta) = \frac{1 - \delta + \delta M}{(1 - \delta + \delta \sqrt{M})^\gamma} \leq h(\delta^*)$$

$h(\delta)$ for $\gamma = 3$: 

![Graph showing $h(\delta)$ for $\gamma = 3$.]
Negligible Leakage Power Consumption

Approximation factor of SFA when $\beta = 0$:

$$AF_{SFA}^{\beta=0} \leq h(\delta) = \frac{1 - \delta + \delta M}{(1 - \delta + \delta \sqrt{M})^\gamma} \leq h(\delta^*)$$

$h(\delta)$ for $\gamma = 3$: 

![Graph showing $h(\delta)$ for different values of $M$.](image)
Negligible Leakage Power Consumption

Approximation factor of SFA when $\beta = 0$ (function of $M$):

Note: $AF^\beta=0_{SFA}$ only depends on the values of $\gamma$ and $M$. 

Santiago Pagani and Jian-Jia Chen
(LS 12, TU Dortmund)
Outline

• Approximation Factor Analysis (energy consumption) of SFA
  • Non-negligible Leakage Power Consumption
Non-negligible Leakage Power Consumption

We approximate the Lower Bound Energy Consumption:

\[ P(s) = \begin{cases} 
\alpha s^\gamma & \text{if } w_M > s_{\text{dyn}} \\
\alpha s_{\text{crit}}^\gamma + \beta & \text{if } w_M \leq s_{\text{dyn}} 
\end{cases} \]
Non-negligible Leakage Power Consumption

We approximate the Lower Bound Energy Consumption:

\[
P(s) = \begin{cases} 
\alpha s^\gamma & \text{if } w_M > s_{dyn} \\
\alpha s_{crit} + \beta & \text{if } w_M \leq s_{dyn} 
\end{cases}
\]

\[
E^*(w_M) \text{ [Joule]}
\]

\[
w_M \text{ [GHz]}
\]

Santiago Pagani and Jian-Jia Chen (LS 12, TU Dortmund)
Non-negligible Leakage Power Consumption

We approximate the Lower Bound Energy Consumption:

\[
P(s) = \begin{cases} 
\alpha s^\gamma & \text{if } w_M > s_{\text{dyn}} \\
\alpha s_{\text{crit}}^\gamma + \beta s_{\text{min}} & \text{if } w_M \leq s_{\text{dyn}}
\end{cases}
\]

\[E^*(w_M) \text{ [Joule]}\]

\[w_M \text{ [GHz]}\]
Non-negligible Leakage Power Consumption

The approximation factor depends on how we choose $s_{\text{dyn}}$:
Non-negligible Leakage Power Consumption

Approximation factor of SFA when $\beta \neq 0 \Rightarrow A_{F_{SFA}} \leq \frac{\gamma^{-1}}{1 + [\gamma h(\delta^*)]^{\gamma-1}}$

Note: $A_{F_{SFA}}$ only depends on the values of $\gamma$ and $M$. 

Santiago Pagani and Jian-Jia Chen
(£S 12, TU Dortmund)
Outline

• Approximation Factor Analysis (energy consumption) of SFA

• Balanced Task Sets and Non-negligible Overhead for Sleeping
Balanced Task Sets & Non-negligible Sleeping Overhead

Balanced Task Sets:
If $\delta \geq 0.5$ (e.g., using LTF) $\Rightarrow AF_{SFA} (\delta \geq 0.5) < AF_{SFA}$

---

Balanced Task Sets & Non-negligible Sleeping Overhead

**Balanced Task Sets:**
If $\delta \geq 0.5$ (e.g., using LTF) $\Rightarrow AF_{\text{SFA}} (\delta \geq 0.5) < AF_{\text{SFA}}$

**Non-negligible Overhead for Sleeping:**

- SFA can be combined with any uni-core DPM solution.
- For example, with Left-To-Right (LTR)$^2$ algorithm:

$$AF_{\text{SFA-LTR}} = AF_{\text{SFA}} + 1$$

---

Outline

• Simulations
Simulation Results

For negligible overhead for sleeping:

- Power parameters modelled from SCC.
- Discrete Frequencies:
  - [0.1 GHz; 3.0 GHz]
  - Steps of 0.1 GHz
- \( W_M \):
  - [0.2 GHz; 3.0 GHz]
  - Steps of 20 MHz
- \( L = 1, 2, \ldots, 5 \) seconds.
Outline

- Conclusions
Conclusions

• SFA: state-of-the-art energy efficient scheduling for periodic tasks.

• Approximation factor of SFA for energy efficiency:
  • Considered cases: negligible leakage, non-negligible leakage, balanced task sets, and combinations with DPM.
  • Bounded by $\gamma$ and $M$ (for all cases).
  • Simulations show a small gap compared with our analysis (for the worst-case).

• SFA is an acceptable scheme based on the worst-case analysis.

• The analysis for SFA for fixed task sets is a cornerstone for task partitioning. Further work considering SFA and task partitioning will be published in RTSS 2013.³

Thank you!
Thank you!

Questions?
Thank you!

Questions?
Extensions for Practical Systems

**Systems with Discrete Frequencies:**

- Available frequencies \( \{f_1, f_2, \ldots, f_F\} \).

Approximation factor of SFA for discrete frequencies \(\Rightarrow\)

\[ \text{AF}_{\text{SFA}} \cdot \theta_{\text{max}} \]

\[ \theta_{\text{max}} = \max_{1<i\leq F} \frac{P(f_i) \cdot f_{i-1}}{P(f_{i-1}) \cdot f_i} \]
Extensions for Practical Systems

**Systems with Discrete Frequencies:**

- Available frequencies \( \{ f_1, f_2, \ldots, f_F \} \).

Approximation factor of SFA for discrete frequencies \( \Rightarrow \)
\[
AF_{SFA} \cdot \theta_{\text{max}}
\]

\[
\theta_{\text{max}} = \max_{1 < i \leq F} \frac{P(f_i) \cdot f_{i-1}}{P(f_{i-1}) \cdot f_i}
\]

For example:

- If \( \alpha = 1.76 \frac{\text{Watts}}{\text{GHz}}^3 \), \( \beta = 0.5 \) Watts, \( \gamma = 3 \)

- Available frequencies \( \{0.1 \text{ GHz}, 0.2 \text{ GHz}, \ldots, 3.0 \text{ GHz} \} \)

\( \Rightarrow \) \( \theta_{\text{max}} = 1.14 \)
Extensions for Practical Systems

Systems with Multiple Voltage Islands:

- Given mapping of task partitions in every island.
- Using SFA in each individual island.

\[ \Rightarrow \quad \text{AF}_{\text{SFA}}^{V\text{-islands}} = \frac{\sum_{j=1}^{V} E_{\text{SFA}j}}{\sum_{j=1}^{V} E_{\text{OPT}j}} \leq \frac{\sum_{j=1}^{V} \text{AF}_{\text{SFA}} \cdot E_{j}^*}{\sum_{j=1}^{V} E_{j}^*} = \text{AF}_{\text{SFA}} \]
Simulation Setup

Experimental results on SCC:\(^4\):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>301.48</td>
<td>0.70</td>
<td>25.38</td>
</tr>
<tr>
<td>0.75</td>
<td>368.82</td>
<td>0.80</td>
<td>37.26</td>
</tr>
<tr>
<td>0.85</td>
<td>569.45</td>
<td>0.91</td>
<td>50.76</td>
</tr>
<tr>
<td>0.94</td>
<td>742.96</td>
<td>1.00</td>
<td>70.73</td>
</tr>
<tr>
<td>1.04</td>
<td>908.92</td>
<td>1.05</td>
<td>91.25</td>
</tr>
<tr>
<td>1.14</td>
<td>1077.11</td>
<td>1.10</td>
<td>110.15</td>
</tr>
<tr>
<td>1.23</td>
<td>1223.37</td>
<td>1.14</td>
<td>125.27</td>
</tr>
<tr>
<td>1.32</td>
<td>1303.79</td>
<td>1.21</td>
<td>161.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.28</td>
<td>201.40</td>
</tr>
</tbody>
</table>

Hardware parameters modelled from SCC:

- \(\alpha = 1.76 \frac{\text{Watts}}{\text{GHz}^3}\), \(\beta = 0.5\) Watts, \(\gamma = 3\) and \(s_{\text{crit}} = 0.52\) GHz.
- Available frequencies: \(\{0.1\ \text{GHz}, 0.2\ \text{GHz}, \ldots, 3.0\ \text{GHz}\}\).

---

Simulation Setup

Maximum Cycle Utilization $w_M$ (stepped by 20 MHz):

• (a) From 0.2 GHz to 1.3 GHz.

• (b) From 0.2 GHz to 3.0 GHz.

Hyper-periods (for every $w_M$):

• $L = 1, 2, \ldots, 5$ seconds.

Cycle Utilization Distribution:

• (1) *Critical Utilization Distribution* with $\delta = \delta^*$ (worst-case).

• (2) *Critical Utilization Distribution* with $\delta = 0.5$ (balanced task sets).

• (3) 100 different random utilization distributions.
Detailed Simulation Results

For negligible overhead for sleeping:

\[
\frac{E_{SFA}}{E^*} \text{ peak} \quad \text{Random (3 GHz)}
\]
\[
\frac{E_{SFA}}{E^*} \text{ peak} \quad \text{Random (1.3 GHz)}
\]
\[
\frac{E_{SFA}}{E^*} \text{ peak} \quad \delta = 0.5 \quad (3 \text{ GHz})
\]
\[
\frac{E_{SFA}}{E^*} \text{ peak} \quad \delta = 0.5 \quad (1.3 \text{ GHz})
\]
\[
\frac{E_{SFA}}{E^*} \text{ peak} \quad \delta = \delta^* \quad (3 \text{ GHz})
\]
\[
\frac{E_{SFA}}{E^*} \text{ peak} \quad \delta = \delta^* \quad (1.3 \text{ GHz})
\]
\[
\theta_{max} \cdot AF_{SFA} \quad \delta = 0.5
\]
\[
\theta_{max} \cdot AF_{SFA} \quad \delta = \delta^*
\]
Detailed Simulation Results

For non-negligible overhead for sleeping:

\[ \frac{E_{SFA-LTR}}{E^*} \text{ peak} \]

\( \delta = 0.5 \) (3 GHz)
\( \delta = 0.5 \) (1.3 GHz)
\( \delta = \delta^* \) (3 GHz)
\( \delta = \delta^* \) (1.3 GHz)

\[ \theta_{\text{max}} \cdot AF_{SFA-LTR} \delta = 0.5 \]
\[ \theta_{\text{max}} \cdot AF_{SFA-LTR} \delta = \delta^* \]