Basic Task Models in Real-Time Systems and Applications

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Fundamentals

- Algorithm:
- Program:
- Process/job/task:

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  - It is the logical procedure to solve a certain problem
  - It is informally specified a sequence of elementary steps that an "execution machine" must follow to solve the problem
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  - It is the implementation of an algorithm in a programming language
  - It can be executed several times with different inputs
- Process/job/task:
  - An instance of a program that given a sequence of inputs produces a set of outputs

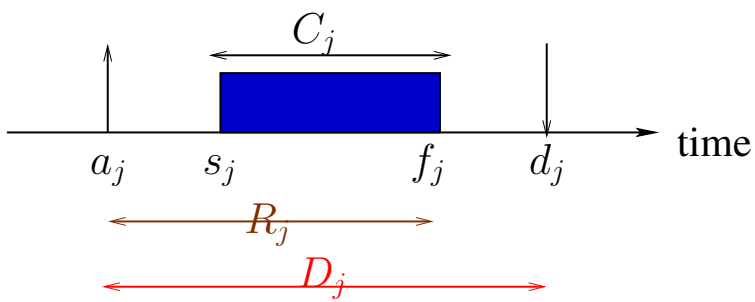
Operating System

An operating system is a program that

- acts as an intermediary between a user of a computer and the computer hardware by providing interfaces
- provides an "abstraction" of the physical machine (for example, a file, a virtual page in memory, etc.)
- manages the access to the physical resources of a computing machine
- makes the computer system convenient to use
- executes user programs and makes solving user problems easier
- and more .....

Timing parameters of a job J_j

- Arrival time (a_j) or release time (r_j) is the time at which the job becomes ready for execution
- Computation (execution) time (C_j) is the time necessary to the processor for executing the job without interruption (= WCET).
- Absolute deadline (d_j) is the time at which the job should be completed.
- Relative deadline (D_j) is the time length between the arrival time and the absolute deadline.
- Start time (s_j) is the time at which the job starts its execution.
- Finishing time (f_j) is the time at which the job finishes its execution.
- Response time (R_j) is the time length at which the job finishes its execution after its arrival, which is $f_j - a_j$.

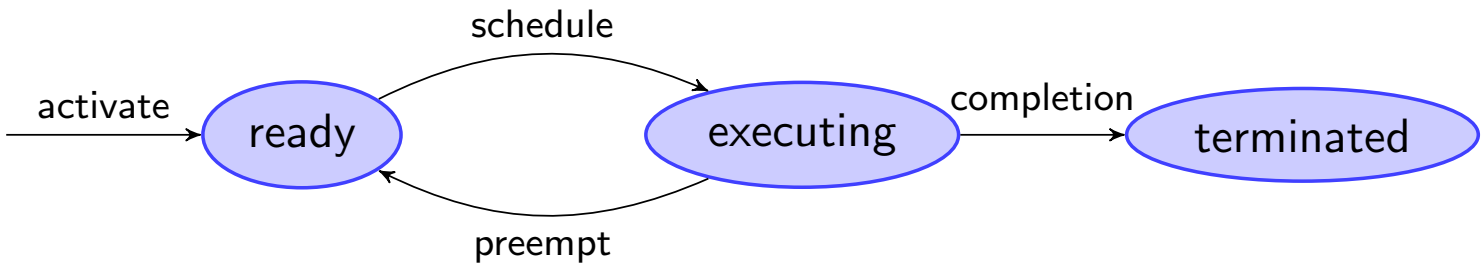


Multi-Tasking (Recap)

- The execution entities (tasks, processes, threads, etc.) are competing from each other for shared resources
- Scheduling policy is needed
  - When to schedule an entity?
  - Which entity to schedule?
  - How to schedule entities?

Scheduling Concepts

- **Scheduling Algorithm**: determines the order that jobs execute on the processor
- Jobs (a simplified version) may be in one of three states:

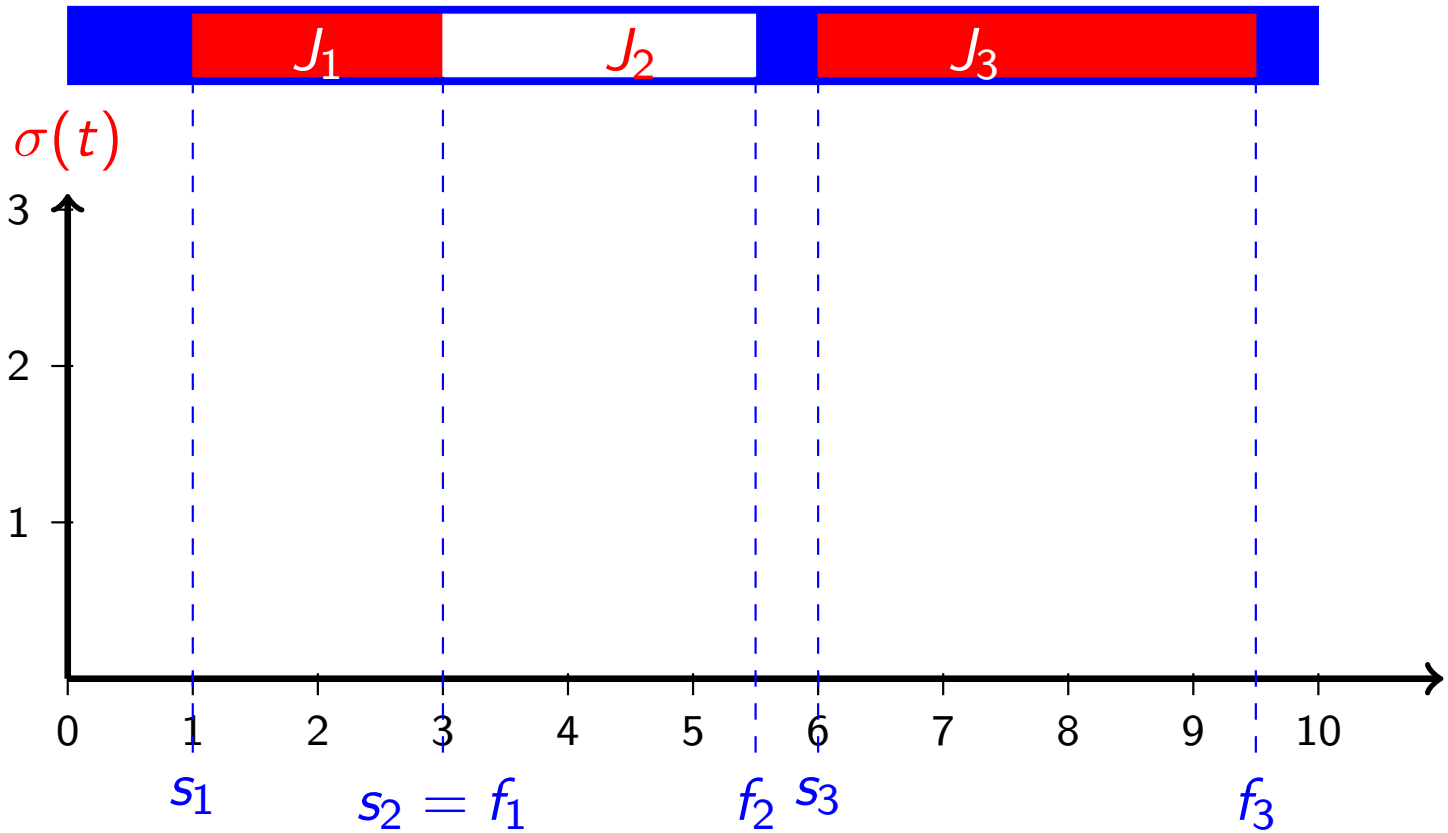


Schedules for a set of jobs $\{J_1, J_2, \ldots, J_N\}$

- A schedule is an assignment of jobs to the processor, such that each job is executed until completion.
- A schedule can be defined as an integer step function $\sigma : \mathbb{R} \to \mathbb{N}$, where $\sigma(t) = j$ denotes job J_j is executed at time t, and $\sigma(t) = 0$ denotes the system is idle at time t.
- If $\sigma(t)$ changes its value at some time t, then the processor performs a context switch at time t.
- Non-preemptive scheduling: there is only one interval with $\sigma(t) = j$ for every J_j, where t is covered by the interval.
- Preemptive scheduling: there could be more than one interval with $\sigma(t) = j$.

Scheduling Concept: Non-preemptive

Schedule: $\sigma : \mathbb{R} \rightarrow \mathbb{N}$ function of processor time to jobs



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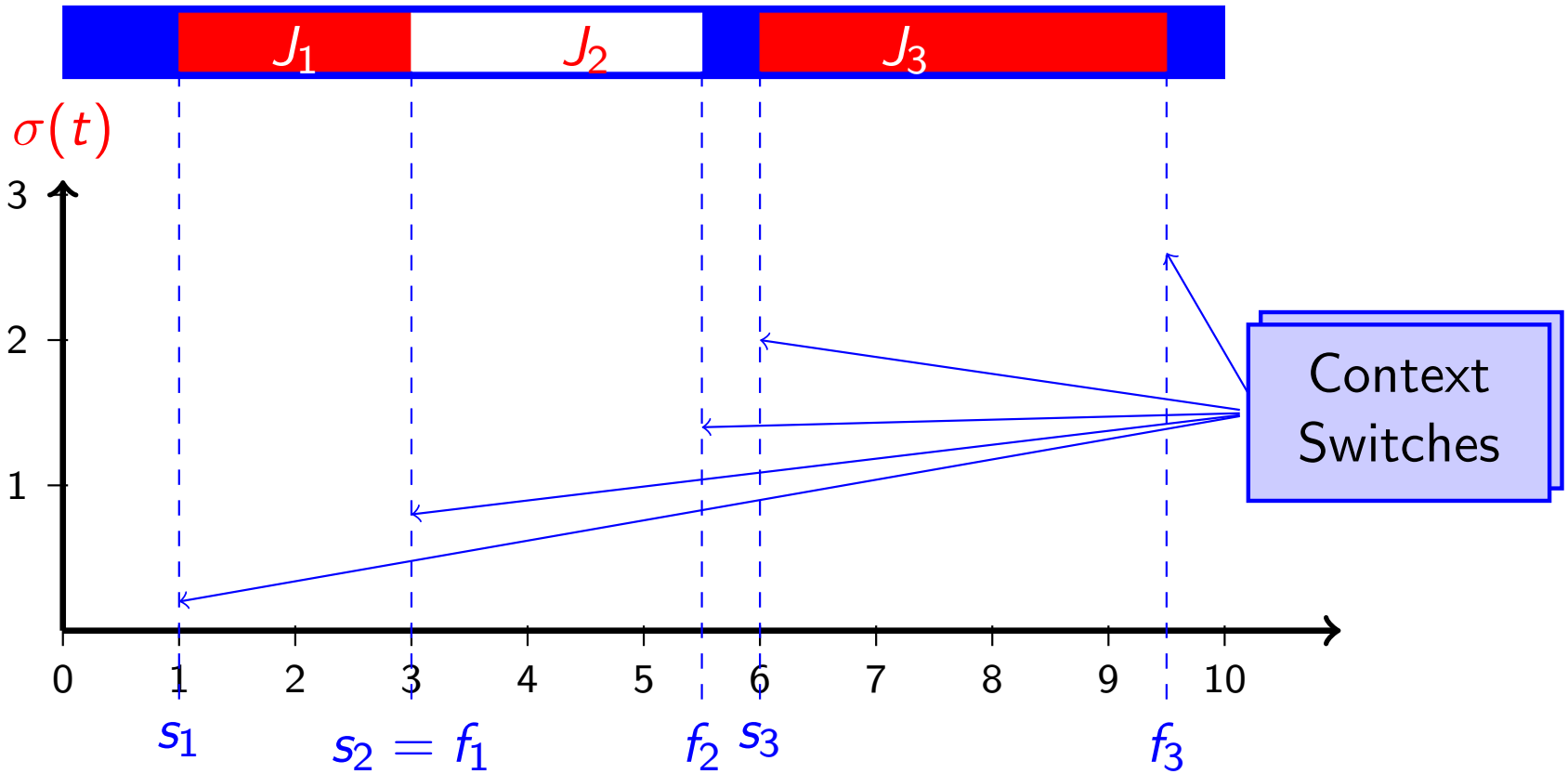

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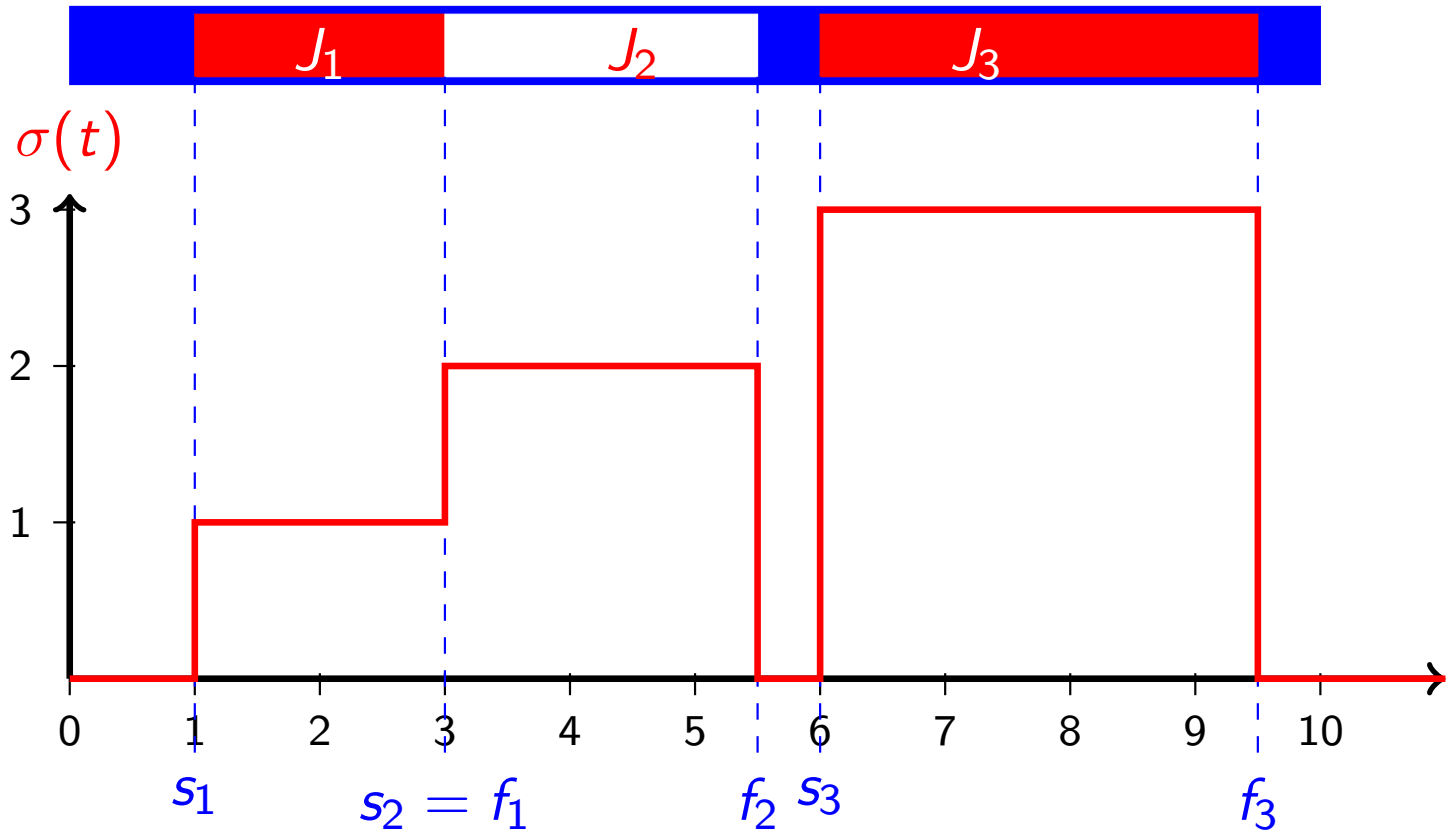

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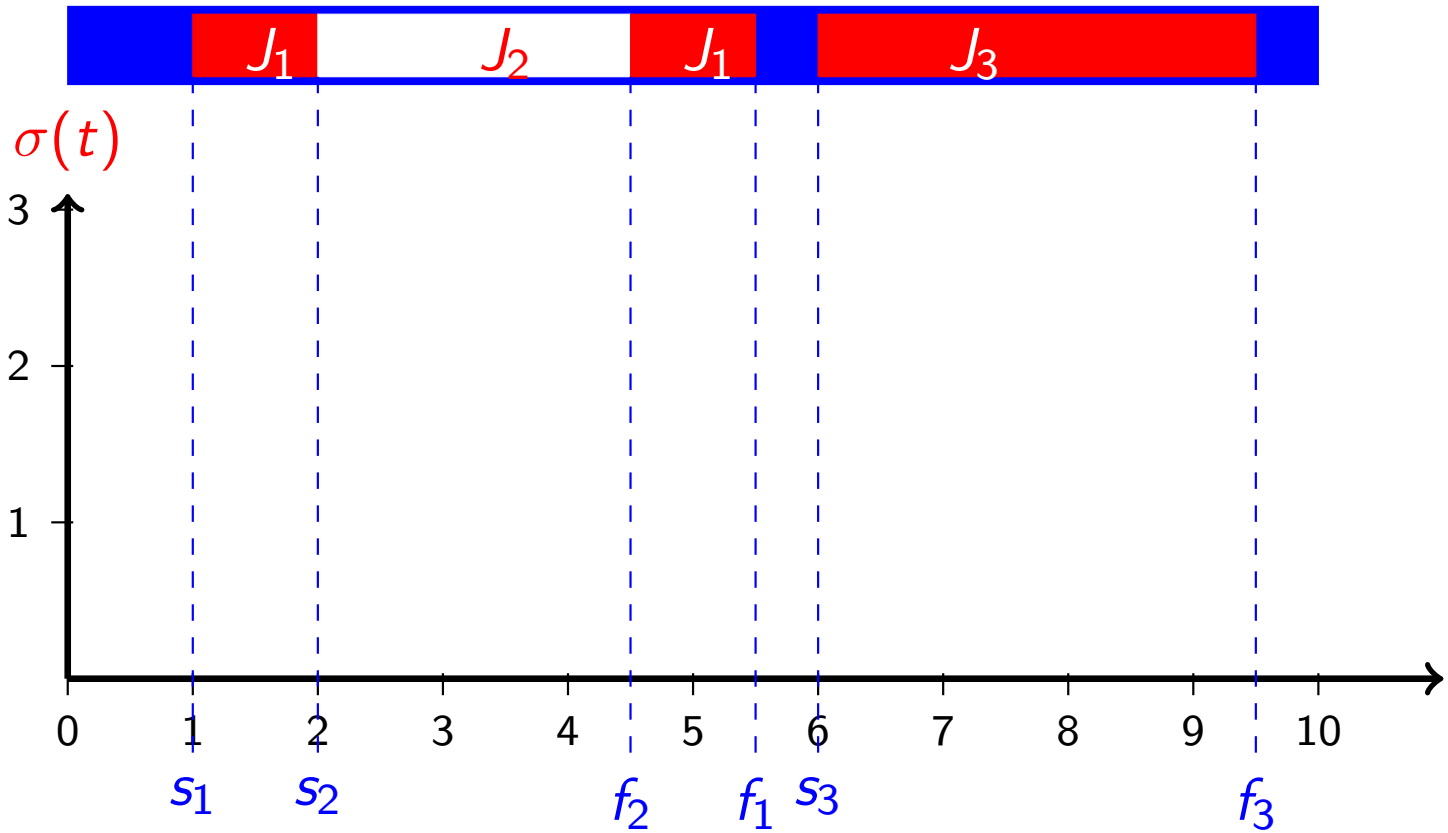

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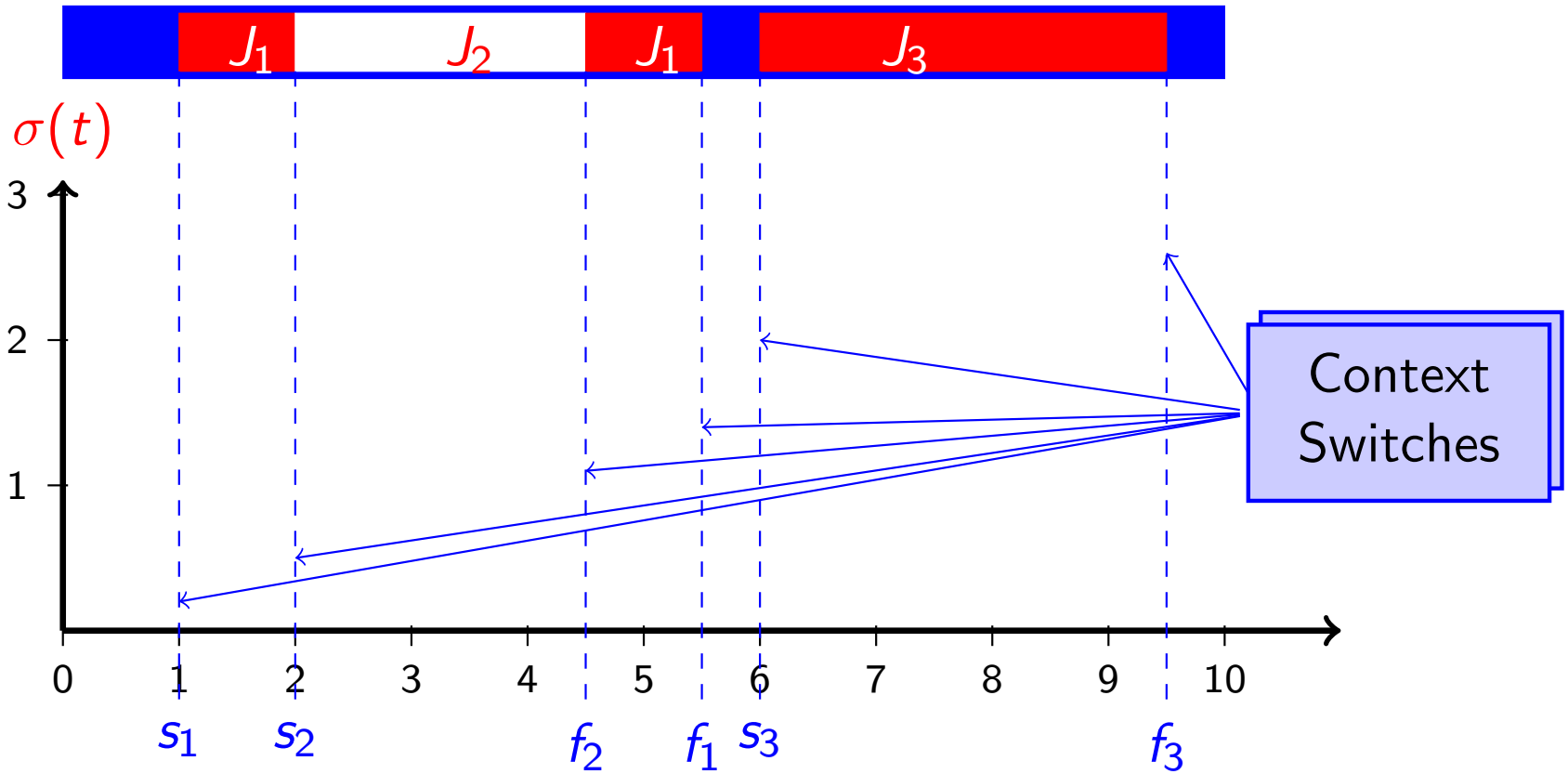

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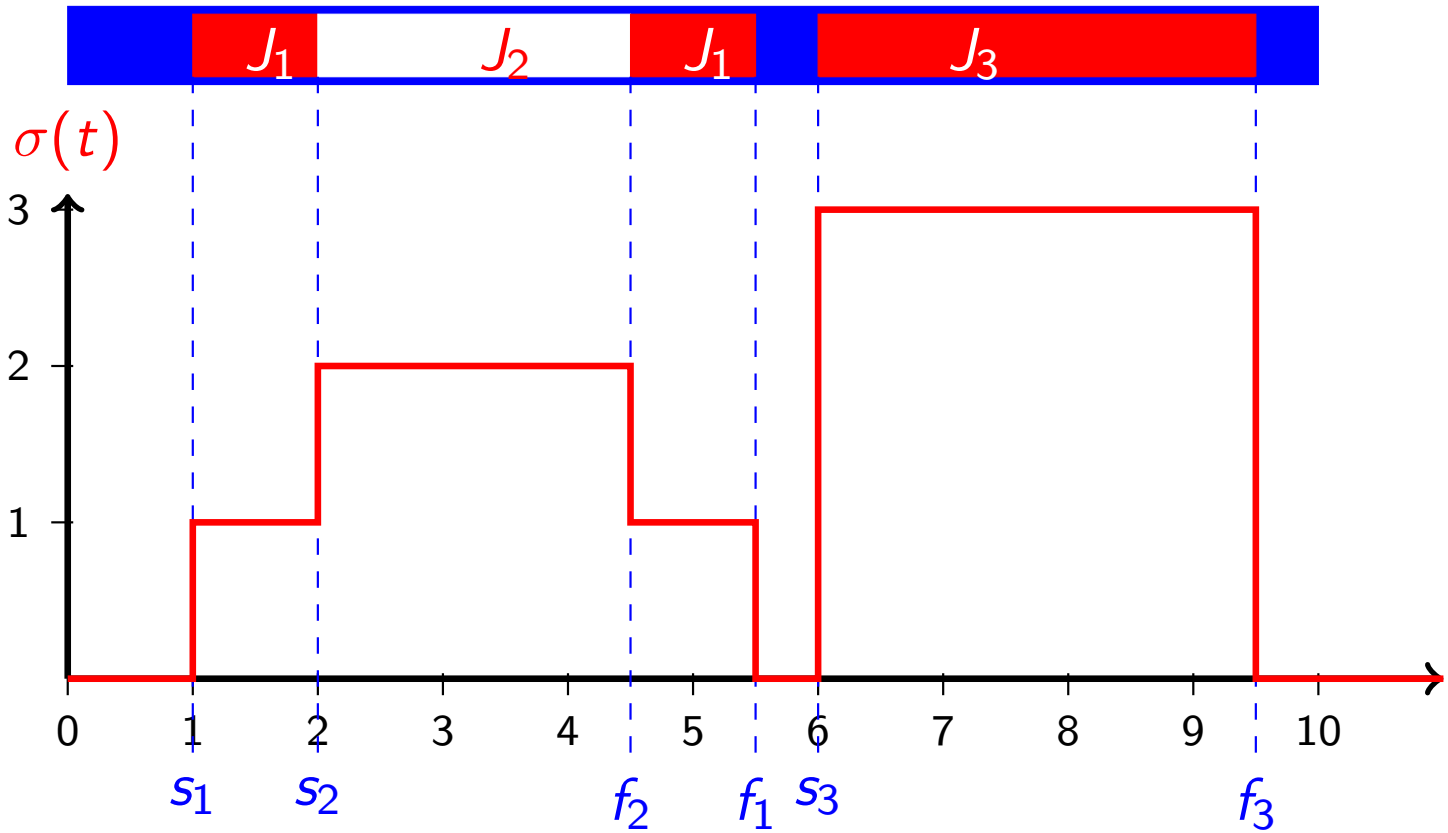
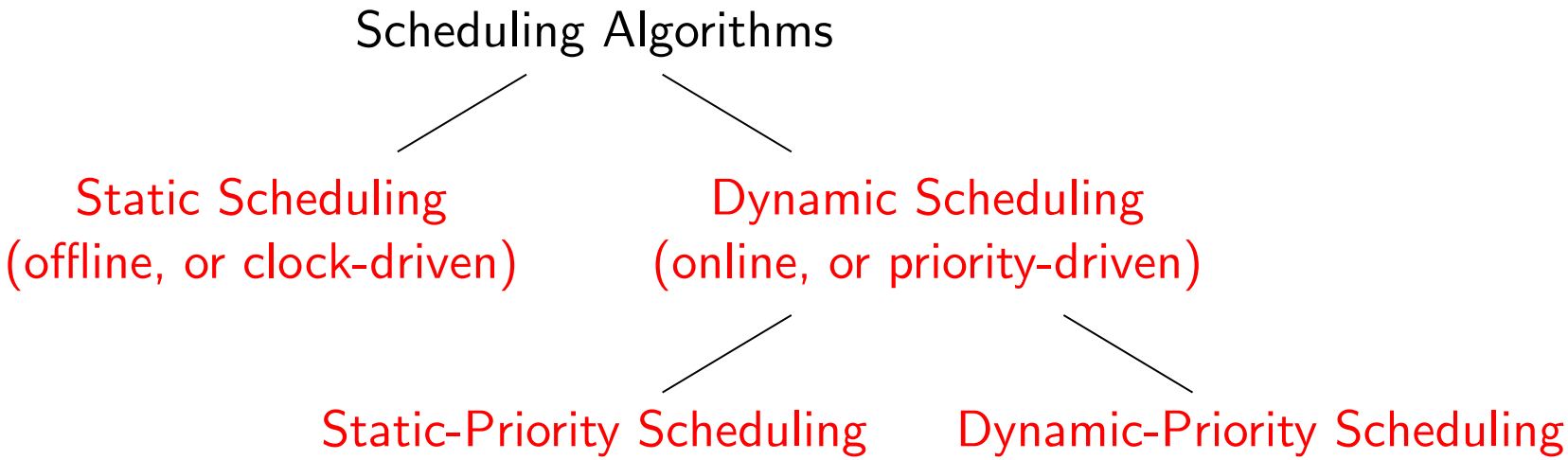


Feasibility of Schedules and Schedulability

- A schedule is feasible if all jobs can be completed according to a set of specified constraints.
- A set of jobs is schedulable if there exists a feasible schedule for the set of jobs.
- A scheduling algorithm is optimal if it always produces a feasible schedule when one exists (under any scheduling algorithm).

Scheduling Algorithms



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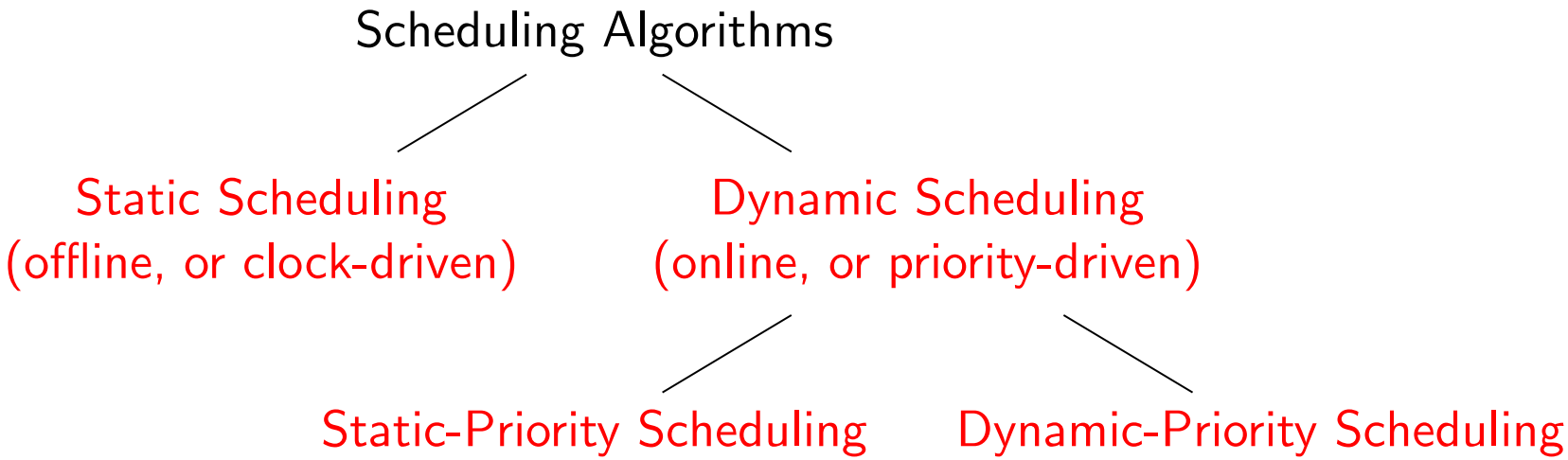

- Preemptive vs. Non-preemptive
- Guarantee-Based vs. Best-Effort
- Optimal vs. Non-optimal

Static/offline scheduling

Scheduling taking a priori knowledge about arrival times, execution times, and deadlines into account. Dispatcher allocates processor when interrupted by timer. Timer controlled by a table generated at design time.

Time	Action	WCET
10	*Start Task*$_1$	12
17	send M5	
22	*Start Task*$_3$	
38	*Start Task*$_2$	20
47	send M3	

Time-Triggered Systems

In an entirely time-triggered system, the temporal control structure of all tasks is established a priori by off-line support-tools. This temporal control structure is encoded in a Task-Descriptor List (TDL) that contains the cyclic schedule for all activities of the node. This schedule considers the required precedence and mutual exclusion relationships among the tasks such that an explicit coordination of the tasks by the operating system at run time is not necessary...

The dispatcher is activated by the synchronized clock tick. It looks at the TDL, and then performs the action that has been planned for this instant [Kopetz].

The disadvantage is that the response to sporadic events may be poor.

An Example: Shortest-Job-First (SJF)

- At any moment, the system executes the job with the *shortest* remaining time among the jobs in the ready queue.

	J_1	J_2	J_3	J_4	J_5
a_j	0	2	8	10	15
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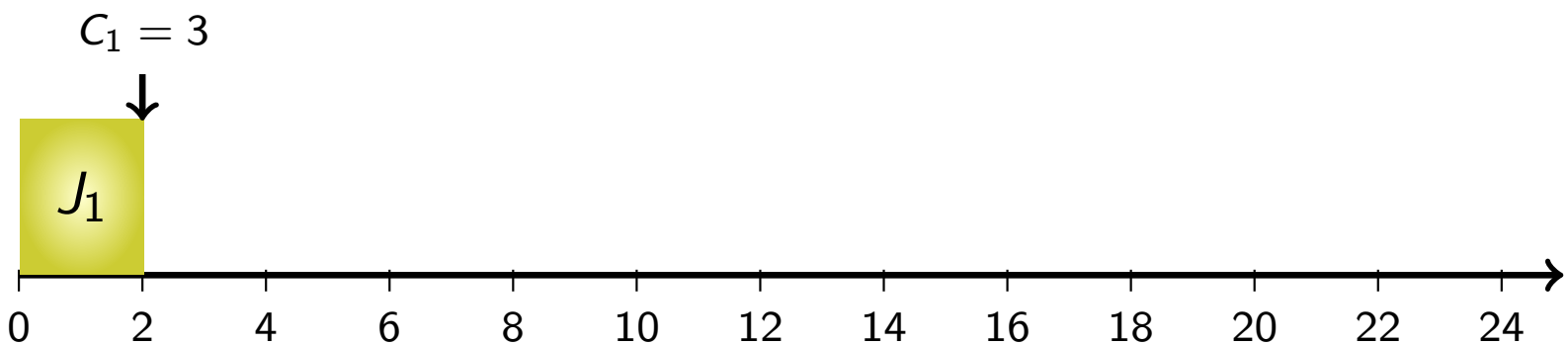

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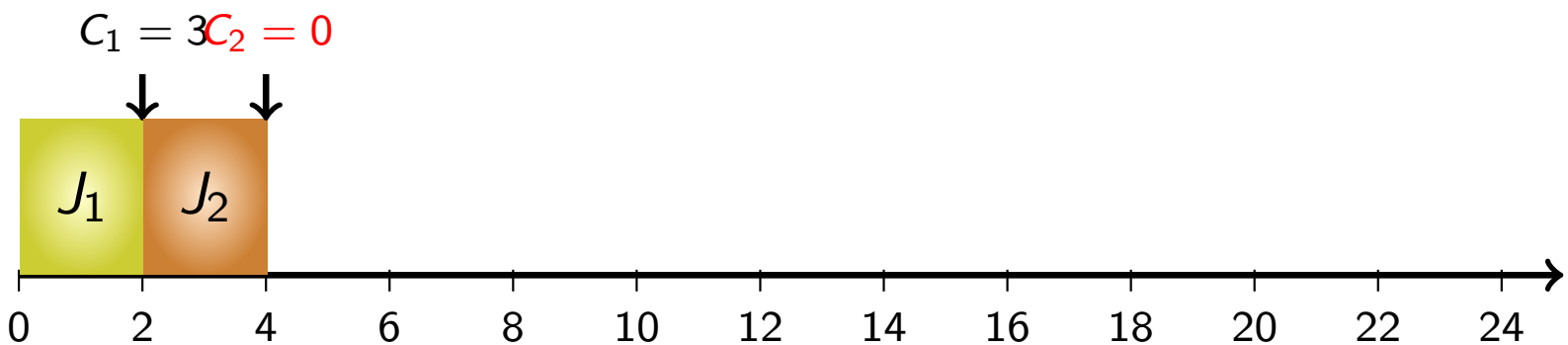

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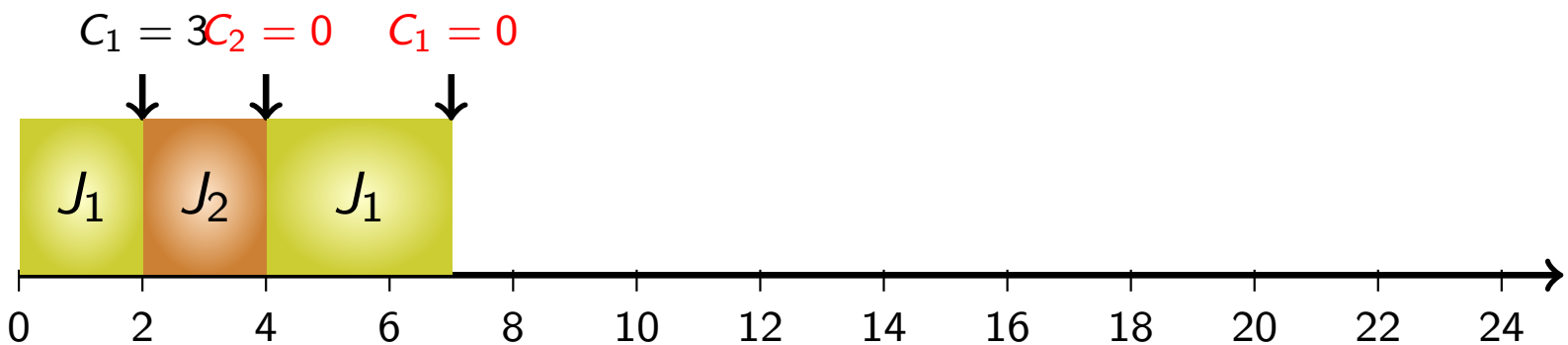

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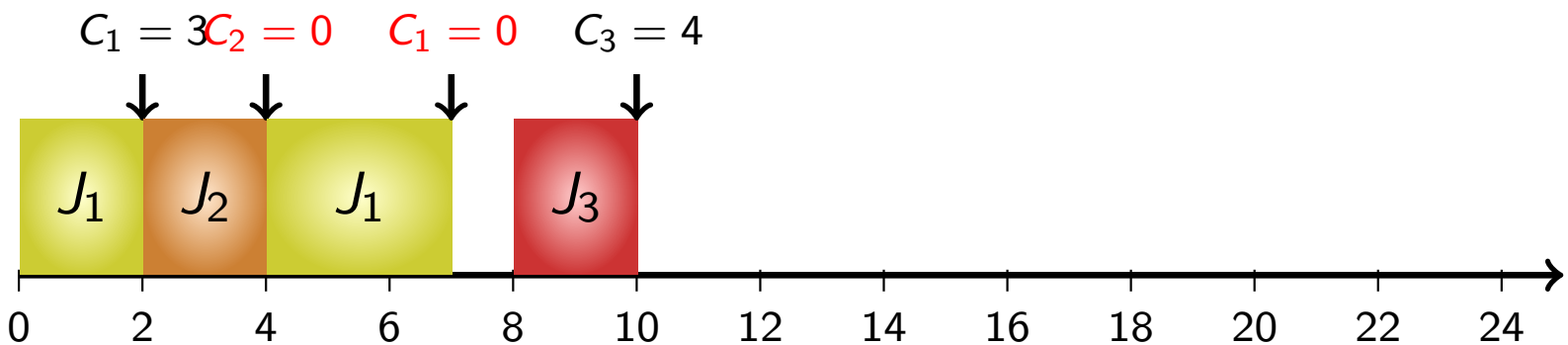

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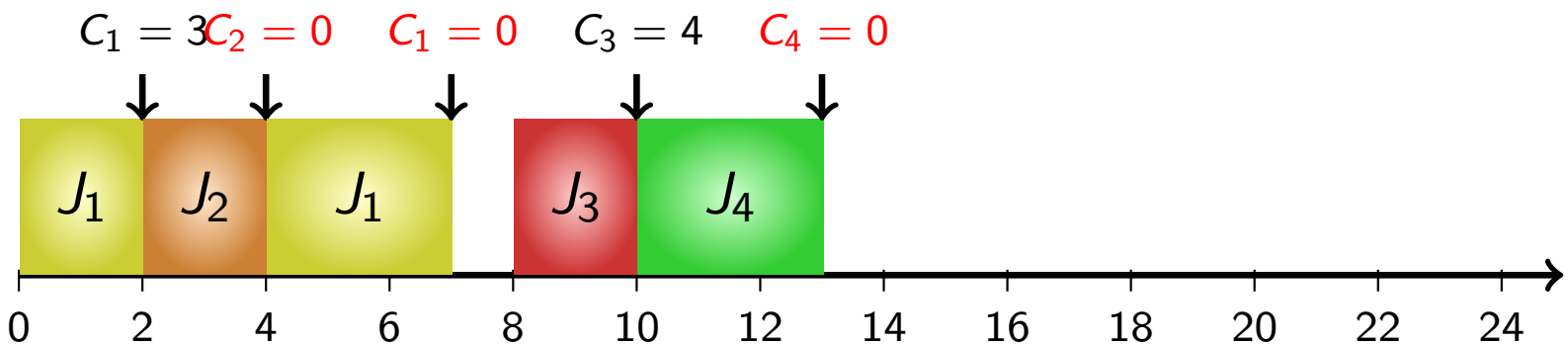

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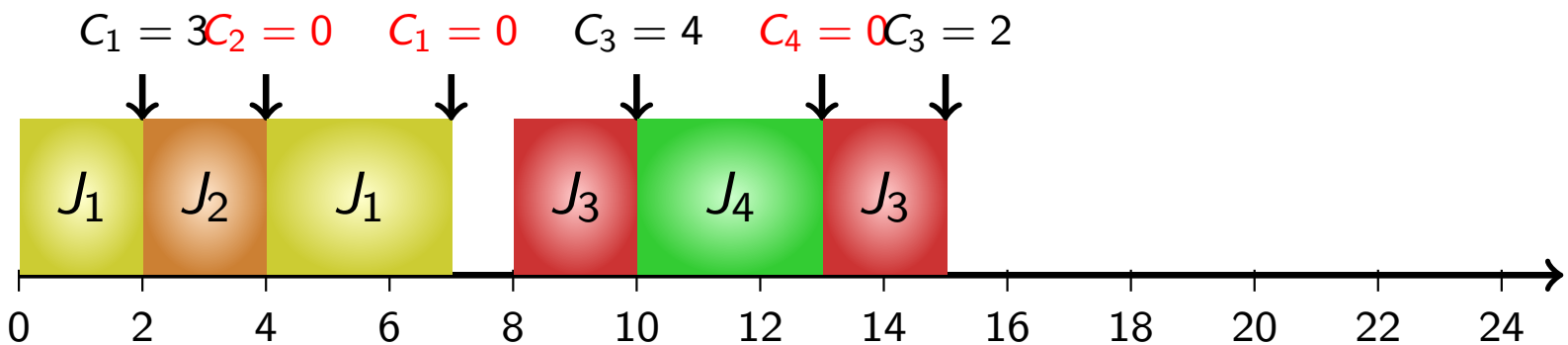

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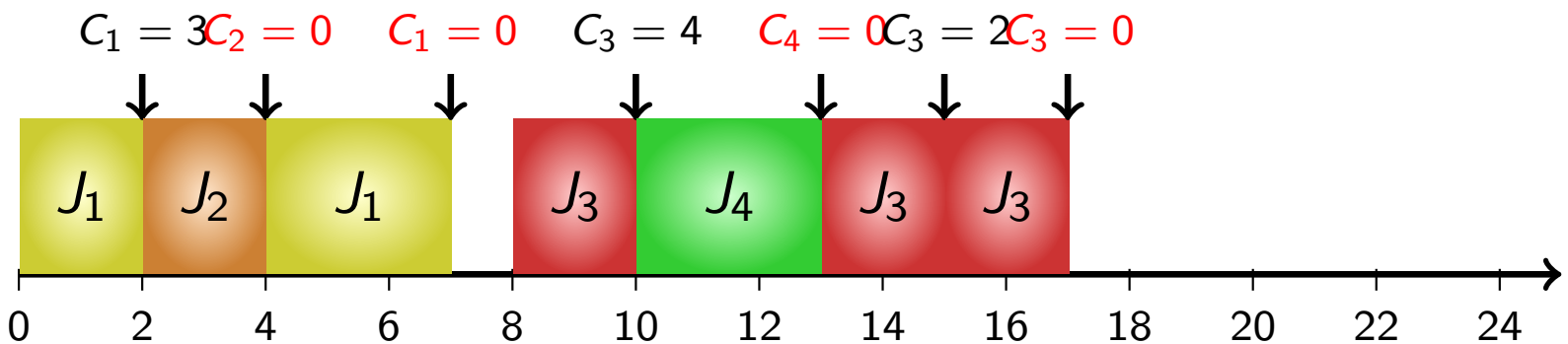

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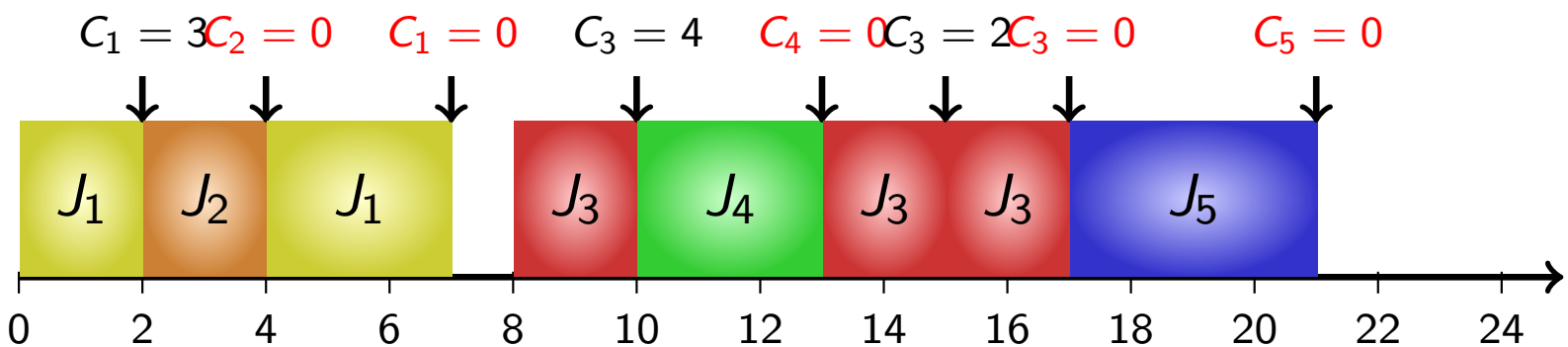

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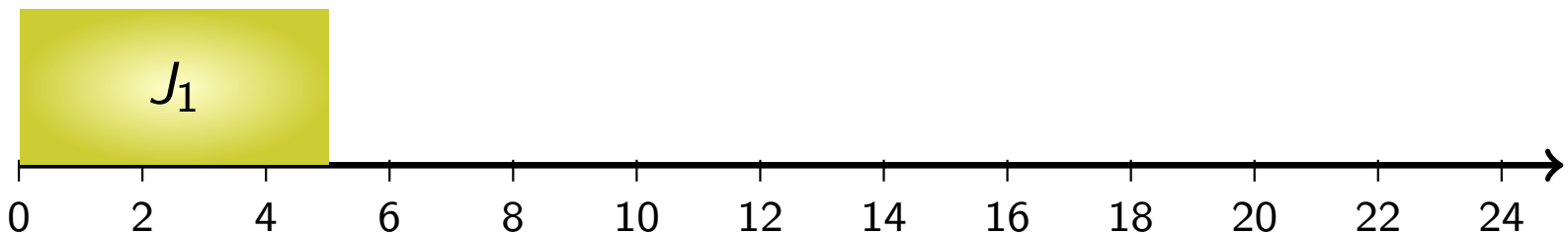

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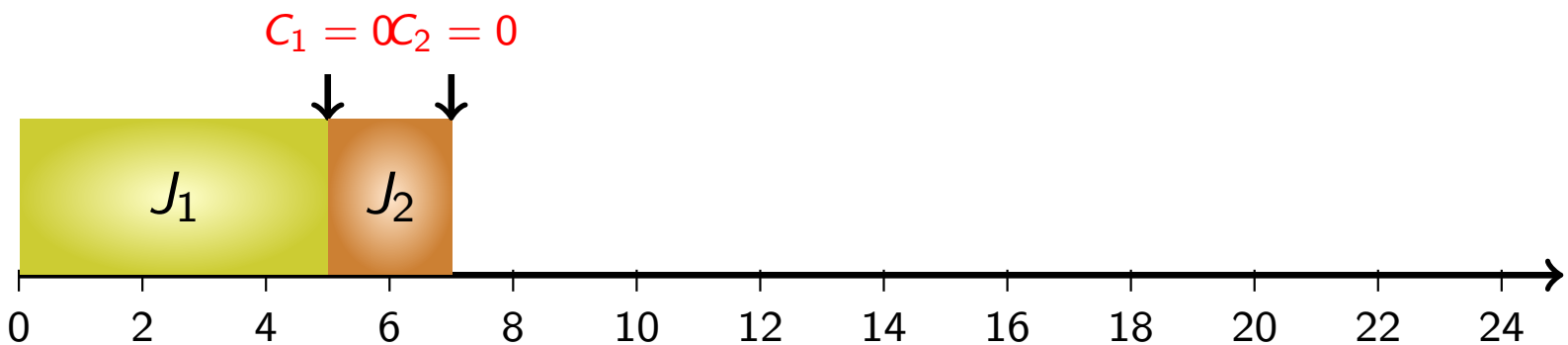

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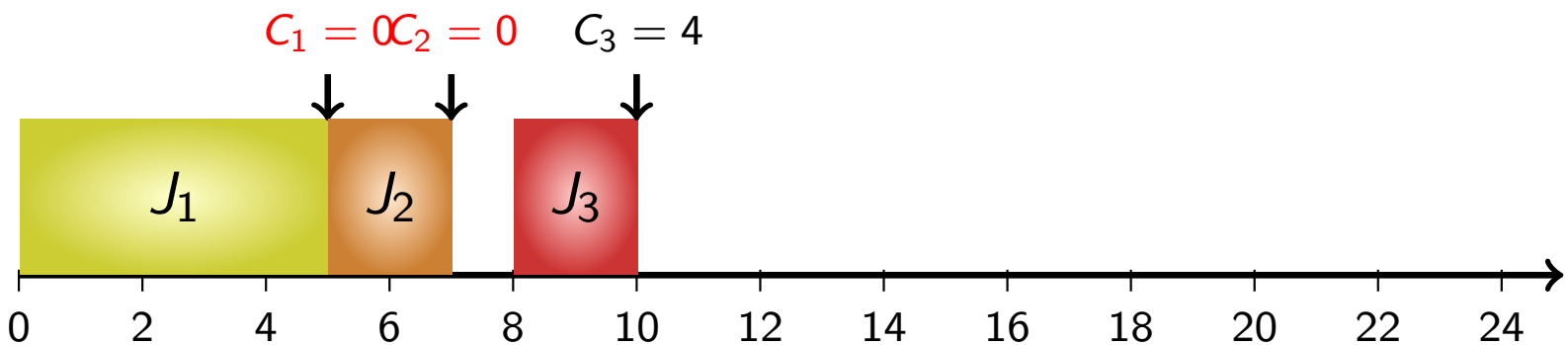

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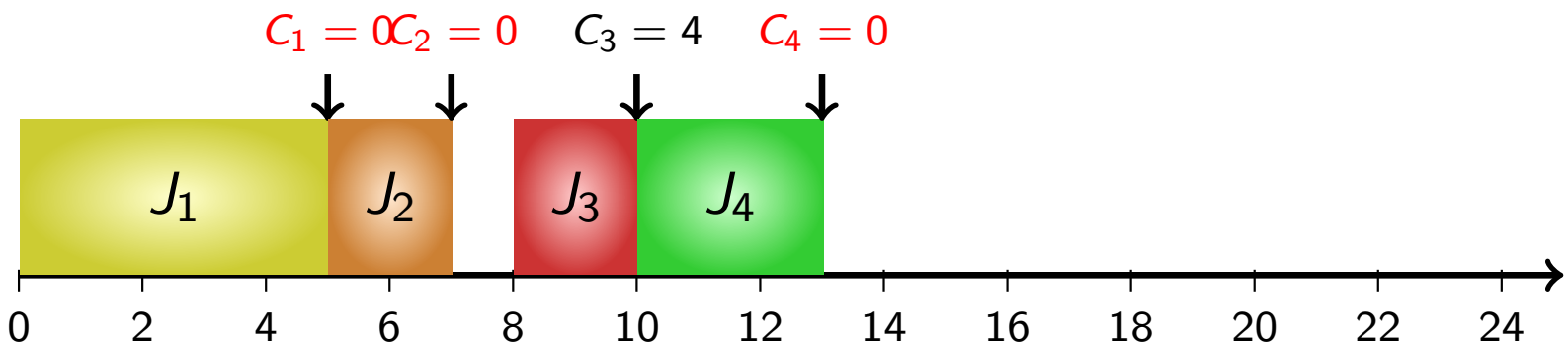

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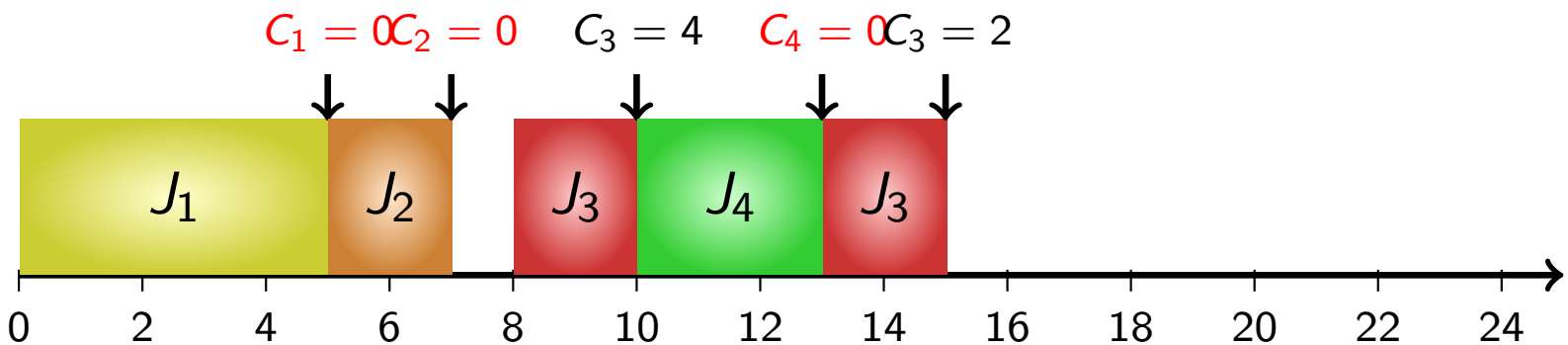

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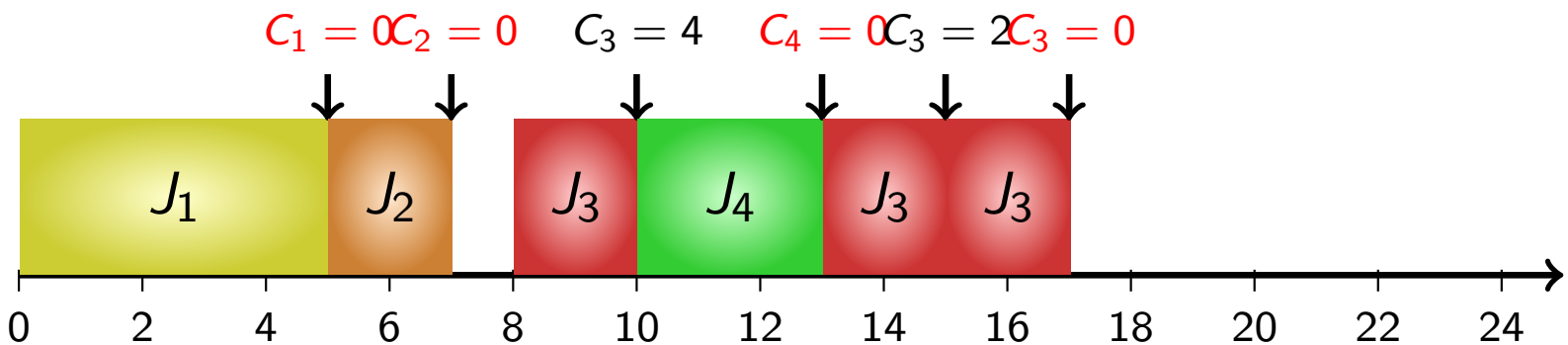

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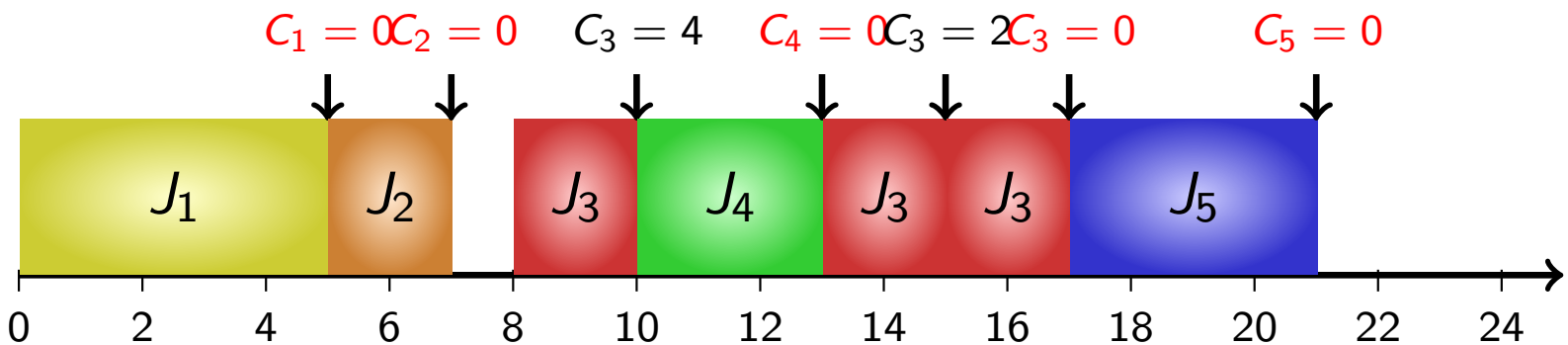

Evaluating A Schedule

For a job J_j:

- Lateness L_j: delay of job completion with respect to its deadline.

$$L_j = f_j - d_j$$

- Tardiness E_j: the time that a job stays active after its deadline.

$$E_j = \max\{0, L_j\}$$

- Laxity (or Slack Time)(X_j): The maximum time that a job can be delayed and still meet its deadline.

$$X_j = d_j - a_j - C_j$$

Metrics of Scheduling Algorithms (for Jobs)

Given a set $\mathbb{J}$ of n jobs, common metrics are to minimize

- Average response time:

$$\sum_{J_j \in \mathbb{J}} \frac{f_j - a_j}{|\mathbb{J}|}$$

- Makespan (total completion time):

$$\max_{J_j \in \mathbb{J}} f_j - \min_{J_j \in \mathbb{J}} a_j$$

- Total weighted response time:

$$\sum w_j (f_j - a_j)$$

- Maximum latency:

$$L_{\max} = \max_{J_j \in \mathbb{J}} (f_j - d_j)$$

- Number of late jobs:

$$N_{late} = \sum_{J_j \in \mathbb{J}} miss(J_j),$$

where $miss(J_j) = 0$ if $f_j \leq d_j$, and $miss(J_j) = 1$ otherwise.

Hard/Soft Real-Time Systems

- Hard Real-Time Systems
  - If any hard deadline is ever missed, then the system is incorrect
  - The tardiness for any job must be 0
  - **Examples**: Nuclear power plant control, flight control
- Soft Real-Time Systems
  - A soft deadline may occasionally be missed
  - Various definitions for "occasionally"
    - minimize the number of tardy jobs, minimize the maximum lateness, etc.
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We mostly consider hard real-time systems in this course.

Recurrent Task Models

- When jobs (usually with the same computation requirement) are released recurrently, these jobs can be modeled by a recurrent task
- Periodic Task τ_i:
  - A job is released exactly and periodically by a period T_i
  - A phase ϕ_i indicates when the first job is released
  - A relative deadline D_i for each job from task τ_i
  - (ϕ_i, C_i, T_i, D_i) is the specification of periodic task τ_i, where C_i is the worst-case execution time.

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- Sporadic Task τ_i:
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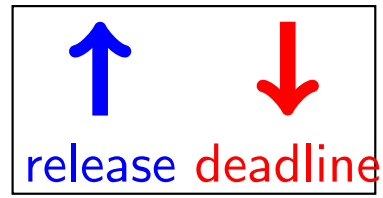
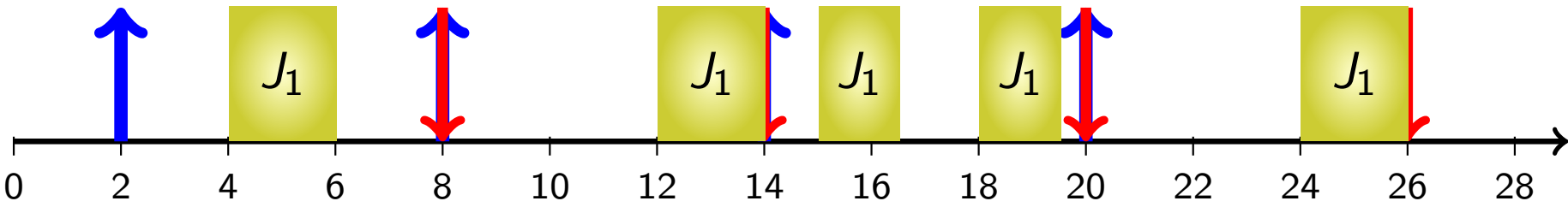
- Aperiodic Task: Identical jobs released arbitrarily (we will revisit this part in Real-Time Calculus).

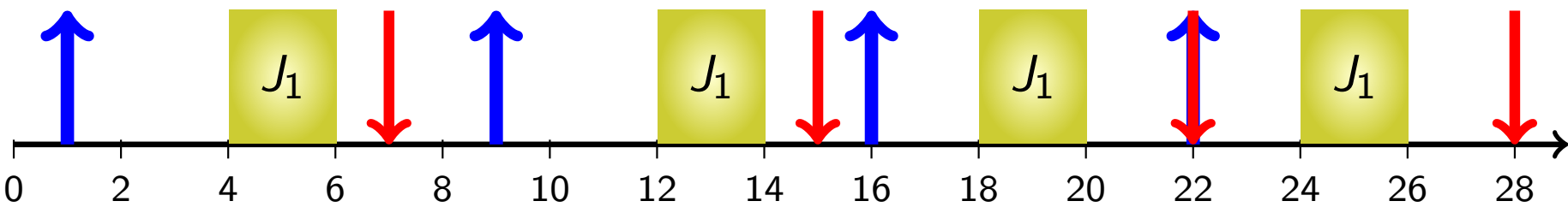
Examples of Recurrent Task Models

Periodic task: $(\phi_i, C_i, T_i, D_i) = (2, 2, 6, 6)$





Sporadic task: $(C_i, T_i, D_i) = (2, 6, 6)$



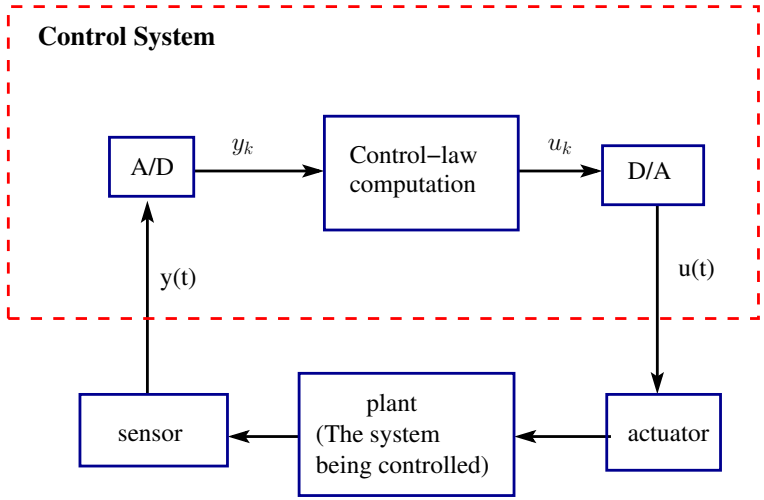
Example: Sporadic Control System

Pseudo-code for this system

while (true)

- start := get the system tick;
- perform analog-to-digital conversion to get y;
- compute control output u;
- output u and do digital-to-analog conversion;
- end := get the system tick;
- $timeToSleep := T - (end - start)$;
- sleep $timeToSleep$;

end while



Example: Periodic Control System

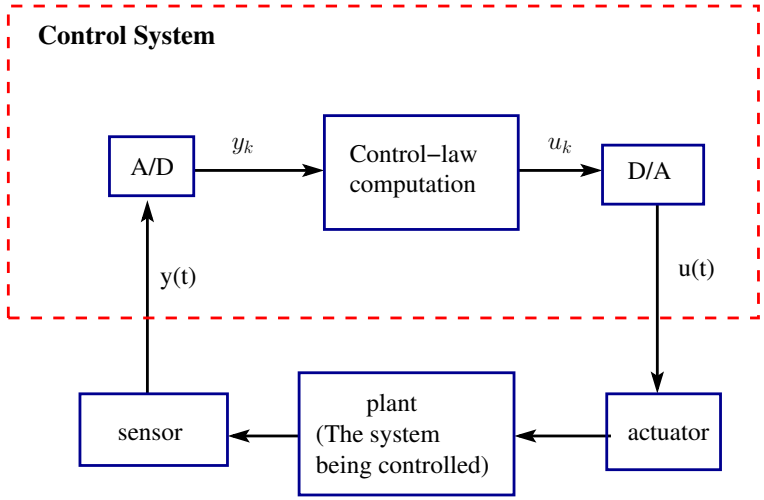
Pseudo-code for this system

set timer to interrupt periodically with period T;

at each timer interrupt
do

- perform analog-to-digital conversion to get y;
- compute control output u;
- output u and do digital-to-analog conversion;

od



Evaluating A Schedule for Tasks

For a job J_j:

- Lateness L_j: delay of job completion with respect to its deadline.

$$L_j = f_j - d_j$$

- Tardiness E_j: the time that a job stays active after its deadline.

$$E_j = \max\{0, L_j\}$$

- Laxity (or Slack Time)(X_j): The maximum time that a job can be delayed and still meet its deadline.

$$X_j = d_j - a_j - C_j$$

For a task τ_i:

- Lateness L_i: maximum latency of jobs released by task τ_i
- Tardiness E_i: maximum tardiness of jobs released by task τ_i
- Laxity X_i: $D_i - C_i$;

Relative Deadline <=> Period

For a task set, we say that the task set is with

- implicit deadline when the relative deadline D_i is equal to the period T_i, i.e., $D_i = T_i$, for every task τ_i,
- constrained deadline when the relative deadline D_i is no more than the period T_i, i.e., $D_i \leq T_i$, for every task τ_i, and
- arbitrary deadline when the relative deadline D_i could be larger than the period T_i for some task τ_i.

Some Definitions for Periodic Tasks

- The jobs of task τ_i are denoted $J_{i,1}, J_{i,2}, \ldots\ldots$
- Synchronous system: Each task has a phase of 0.
- Asynchronous system: Phases are arbitrary.
- Hyperperiod: Least common multiple (LCM) of T_i.
- Task utilization of task τ_i: $u_i = \frac{C_i}{T_i}$.
- System utilization: $\sum_{\tau_i} u_i$.

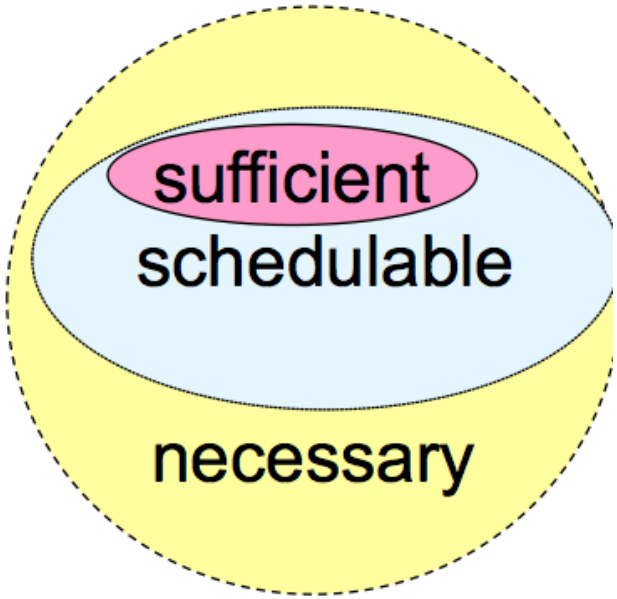
Feasibility and Schedulability for Recurrent Tasks

- A schedule is feasible if all the jobs of all tasks can be completed according to a set of specified constraints.
- A set of tasks is schedulable if there exists a feasible schedule for the set of tasks.
- A set of tasks is schedulable by a scheduling algorithm if the schedule is always feasible.
- A scheduling algorithm is optimal if it always produces a feasible schedule when one exists (under any scheduling algorithm).

Different Tests

The issue for timing analysis is on how to analyze the schedulability.

- *Sufficient Test*: If A holds, then the task set is schedulable (by a scheduling algorithm).
- *Necessary Test*: If the task set is schedulable (by a scheduling algorithm), then B holds.
- *Exact Test*: the task set is schedulable (by a scheduling algorithm) if and only if A^* holds.



Graham's Scheduling Algorithm Classification

- Classification: $a|b|c$
  - a: machine environment
    (e.g., uniprocessor, multiprocessor, distributed, ...)
  - b: task and resource characteristics
    (e.g., preemptive, independent, synchronous, ...)
  - c: performance metric and objectives
    (e.g., $L_{\max}$, sum of finish times, ...)
- Examples:
  - $1|\text{non-prem}|L_{\max}$
  - $M||C_{\max}$

Earliest Due Date Algorithm

Theorem

$1|\text{sync}|L_{\max}$: Given a set of n independent jobs that arrive synchronously (release time is 0), any algorithm that executes tasks in order of non-decreasing deadlines is optimal with respect to minimizing the maximum lateness.
Denoted as Earliest Due Date (EDD) Algorithm [Jackson, 1955]

Proof

Let σ be the schedule for J produced by scheduling algorithm A. We can transform A to EDD schedule A' without increasing $L_{\max}$. Details are in the textbook by Buttazzo [Theorem 3.1].

Optimality of EDF

Theorem

Given a set of n independent aperiodic tasks (jobs) with arbitrary arrival times, if the aperiodic task set is feasible on a single processor then any algorithm that executes tasks with earliest deadline (among the set of active tasks) is guaranteed to meet all tasks' deadlines.

- Several proofs of optimality exist: Liu and Layland (1973), Horn (1974), and Dertouzos (1974).
- Similar to Jackson Algorithm proof of optimality, but need to account for preemption.

Monotonicity of Scheduling Algorithms

A good scheduling algorithm should be monotonic

- If a scheduling algorithm derives a feasible schedule, it should also guarantee the feasibility with
  - less execution time of a task/job,
  - less number of tasks/jobs, or
  - more number of processors/machines.

Why is Real-Time Scheduling Hard?

Single-processor (Eisenbrand and Rothvoß, in RTSS 2008)

Fixed-Priority Real-Time Scheduling: Response Time Computation Is $\mathcal{NP}$-Hard

Multiprocessor (Graham 1976)

Changing the priority order, increasing the number of processors, reducing execution times, or weakening precedence constraints can result in a deadline miss.

Many Cases

Scheduling problems in multiprocessor systems are usually $\mathcal{NP}$-Hard.

Fundamentals: Computational Complexity

- $\mathcal{NP}$-Complete for a problem Π:
  - If Π can be solved in polynomial time by using a non-deterministic Turning machine, the problem is said in the computational complexity class $\mathcal{NP}$.
  - Π is $\mathcal{NP}$-Complete if Π is in $\mathcal{NP}$ and any problem in the $\mathcal{NP}$ class can reduce to Π in polynomial time (or log space).
  - Π is $\mathcal{NP}$-hard if any problem in the $\mathcal{NP}$ class can reduce to Π in polynomial time (or log space).

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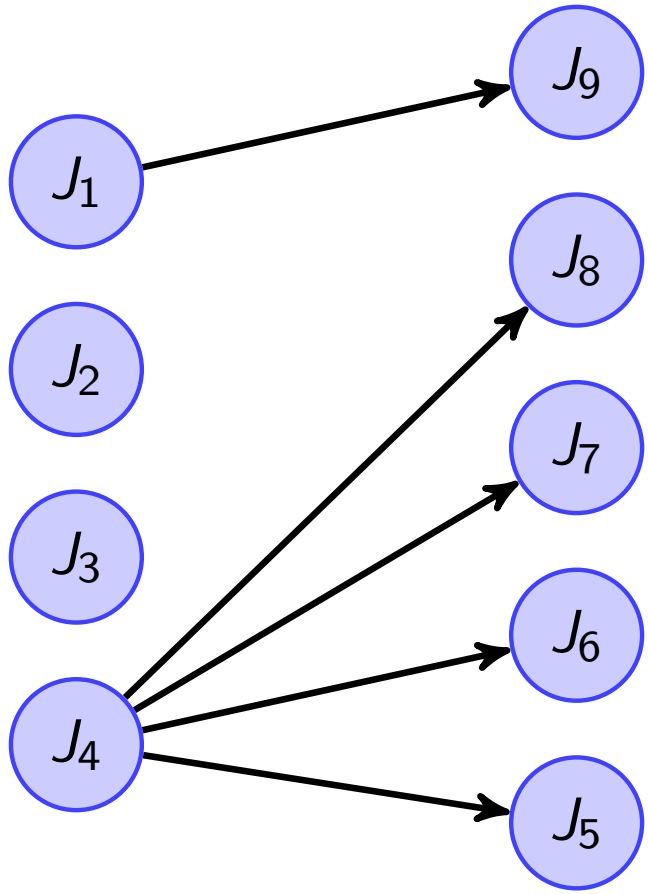
  - Π is $\mathcal{NP}$-hard if any problem in the $\mathcal{NP}$ class can reduce to Π in polynomial time (or log space).
- More intuitively (informally)
  - The computing machines we have developed so far are deterministic Turning machines.
  - If Π can be solved in polynomial time by using a deterministic Turning machine, the problem is said in the computational complexity class $\mathcal{P}$.
  - If a problem is $\mathcal{NP}$-Complete or $\mathcal{NP}$-hard, there is no efficient (polynomial-time) algorithm to derive optimal/feasible solutions unless $\mathcal{P} = \mathcal{NP}$.

Multiprocessor Anomalies

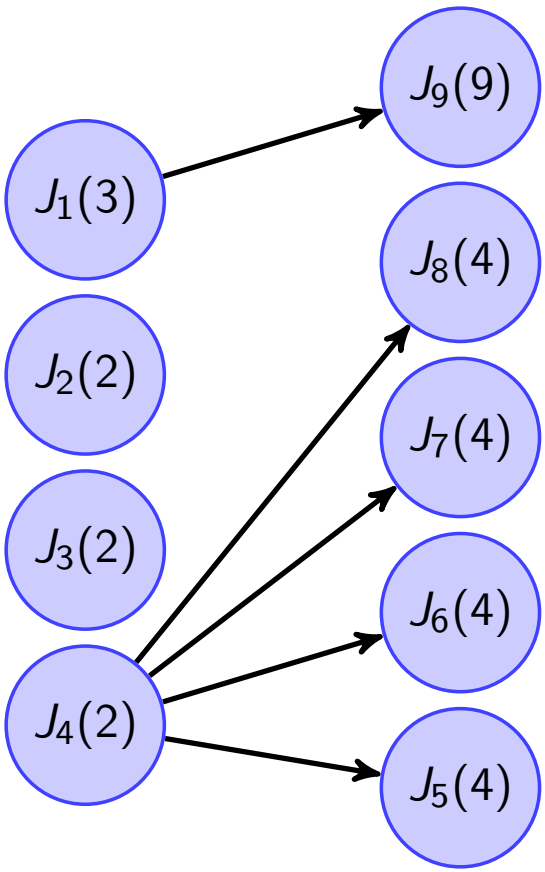
- Partitioned scheduling (Each task/job is on a processor)
  - As most partitioning algorithms are not optimal, a system might become infeasible with
    - Less execution time of a task/job
    - Less number of tasks/jobs
    - More number of processors/machines
- Global scheduling
  - As most priority-assignment algorithms are not optimal, a system might become infeasible with
    - Less execution time of a task/job
    - Less number of tasks/jobs
    - More number of processors/machines

Precedence Constraints

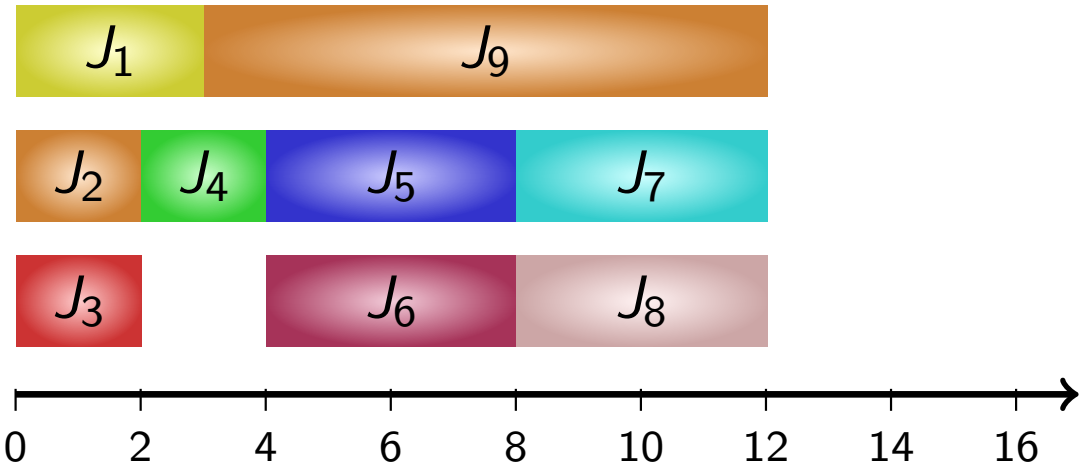
Jobs (and tasks) may have to execute in a pre-specified order.



Multiprocessor Anomaly: Case 1



On 3 processors



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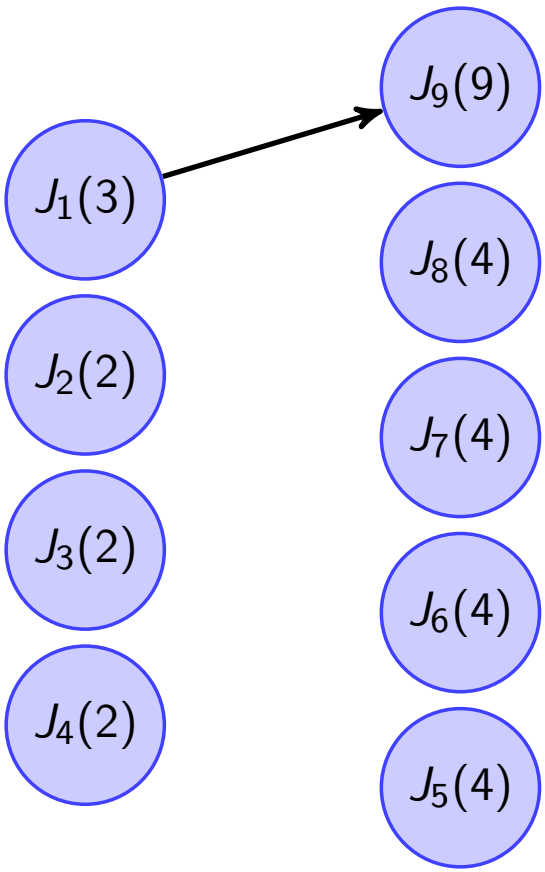
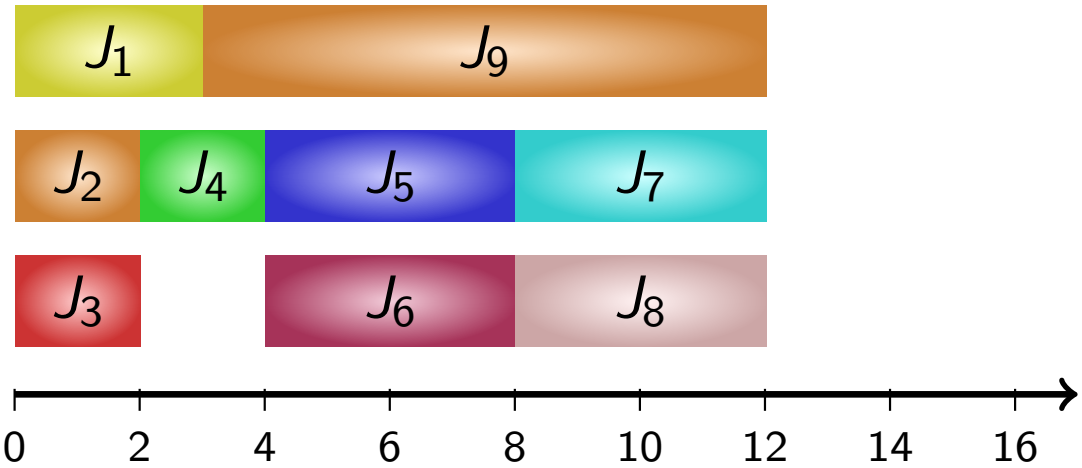
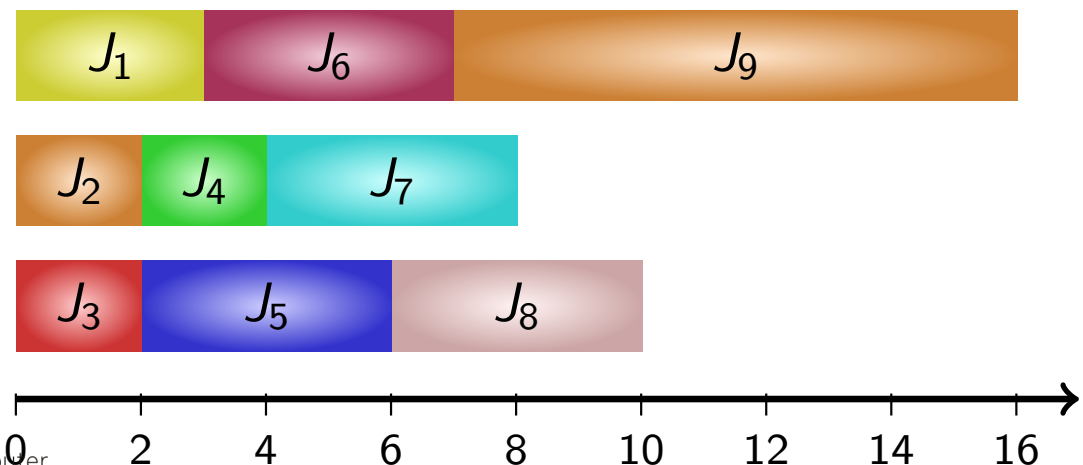


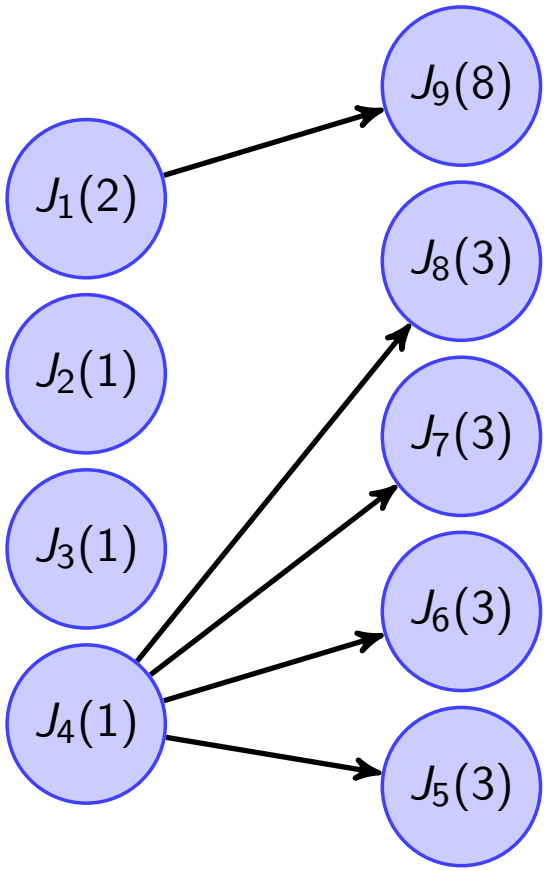
On 3 processors



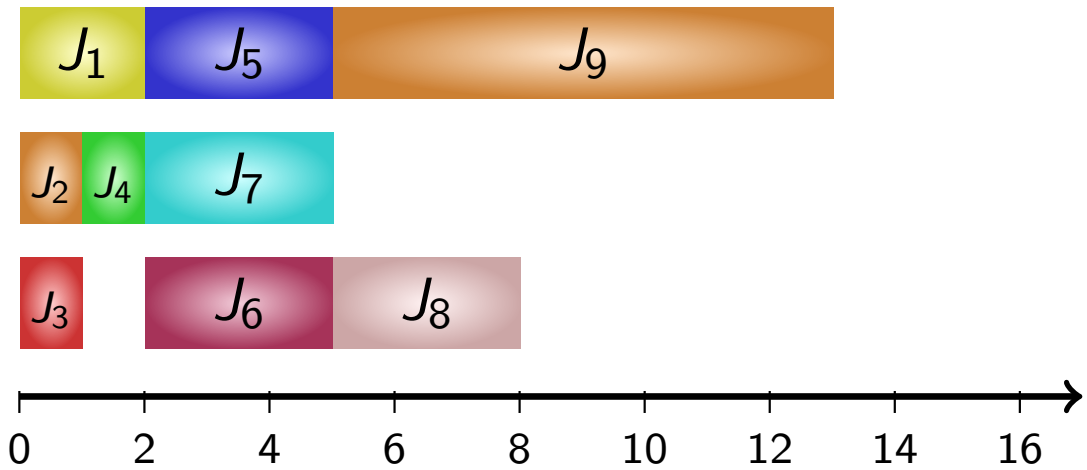
Removing the precedence constraints on J_4...



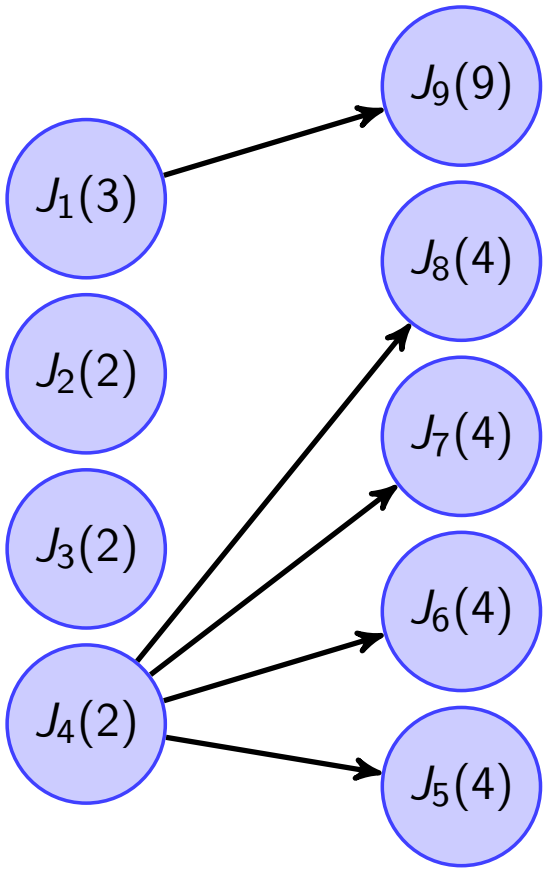
Multiprocessor Anomaly: Case 2



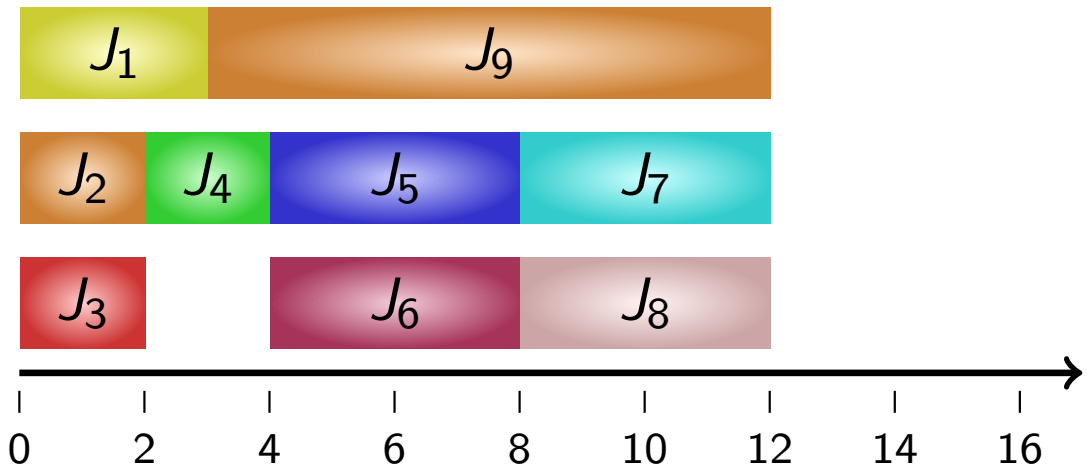
Reduce the execution time by 1, and schedule on 3 processors



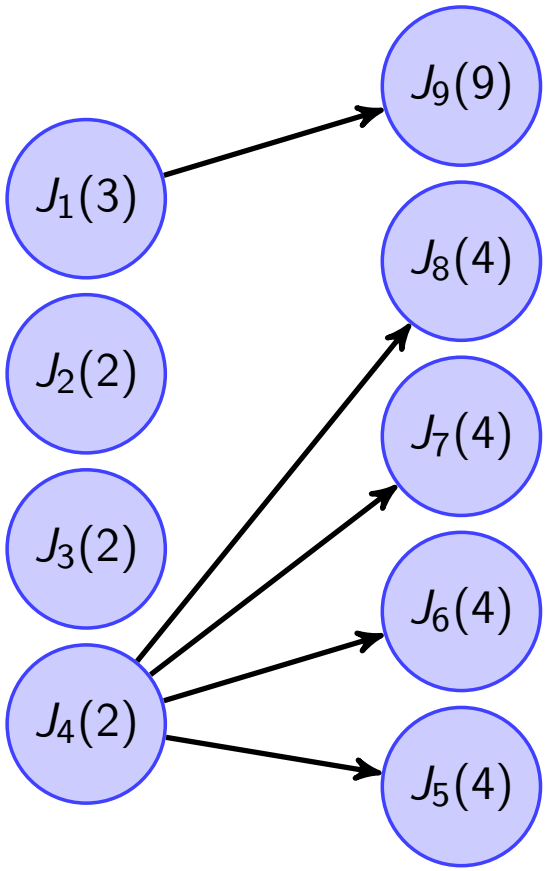
Multiprocessor Anomaly: Case 3



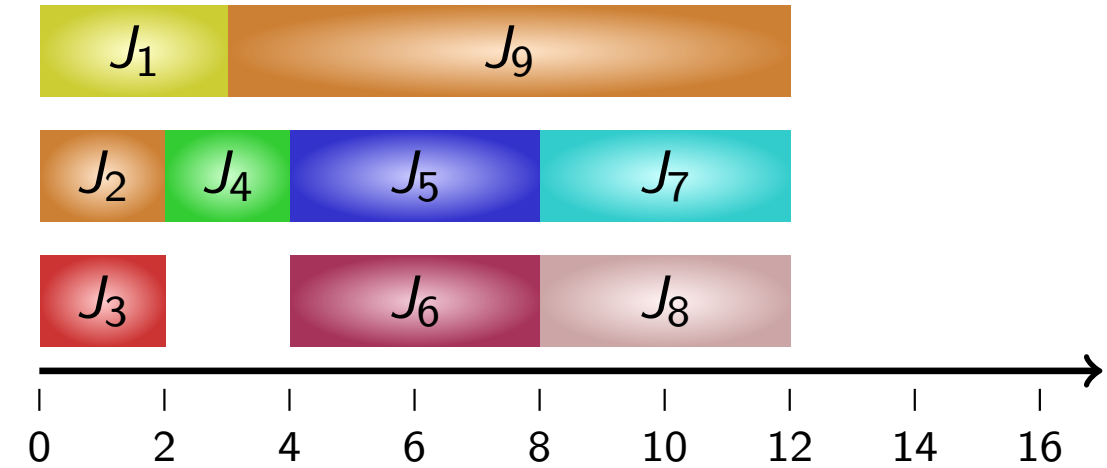
On 4 processors



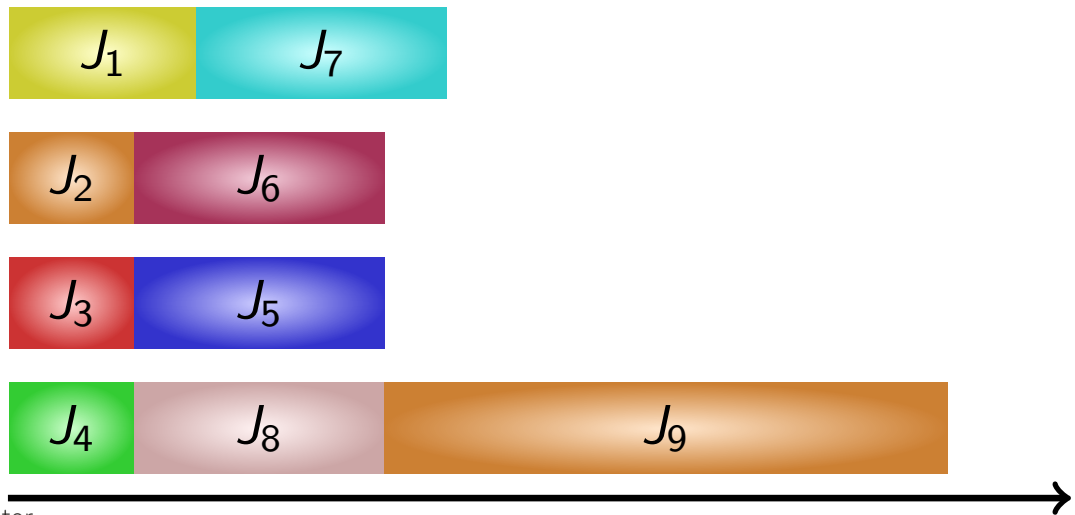
Multiprocessor Anomaly: Case 3



On 4 processors



Use 4 processors



Note about the Material in the Course

- For the rest of the course, if the context is not emphasized, we will focus on the design and analysis for scheduling algorithms with
  - preemptive scheduling and
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- We will have two sessions, discussing about
  - non-preemptive scheduling or
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The material this week has covered the corresponding contents in Chapters 2 and 3 in Buttazzo's textbook.