Unified Utilization-Based Schedulability Analysis for Fixed-Priority Real-Time Systems

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Outline

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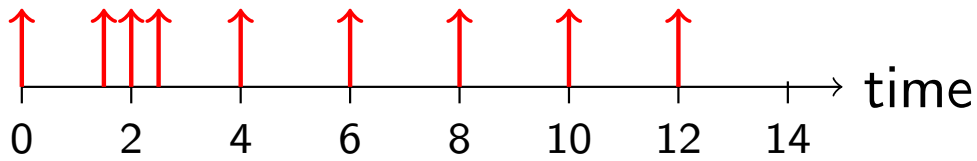
Example 1: Arrival Curve Model



Thiele et al. 2000.

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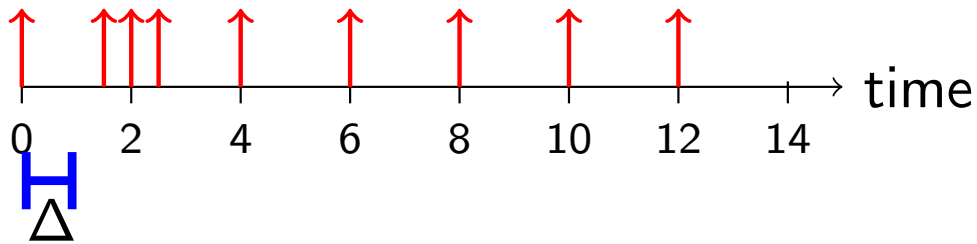

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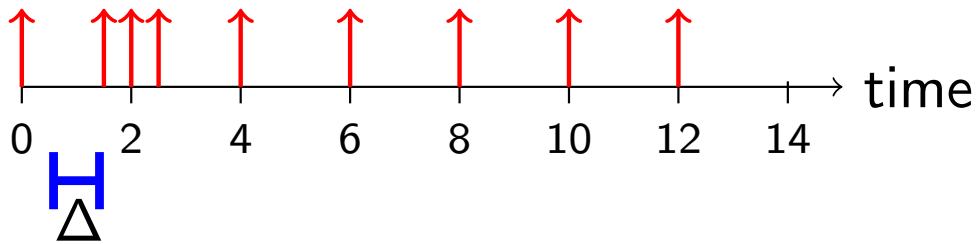

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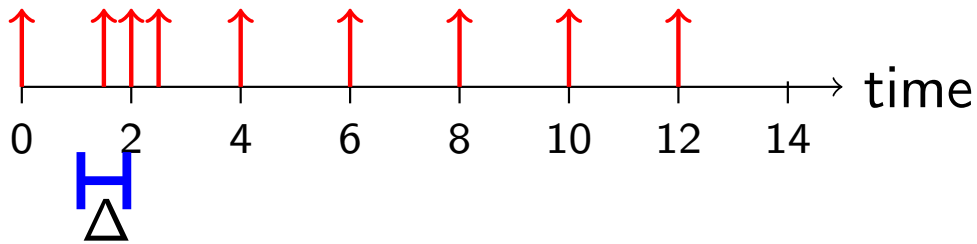

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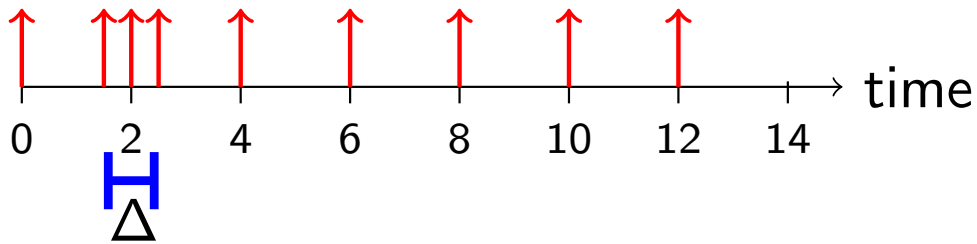

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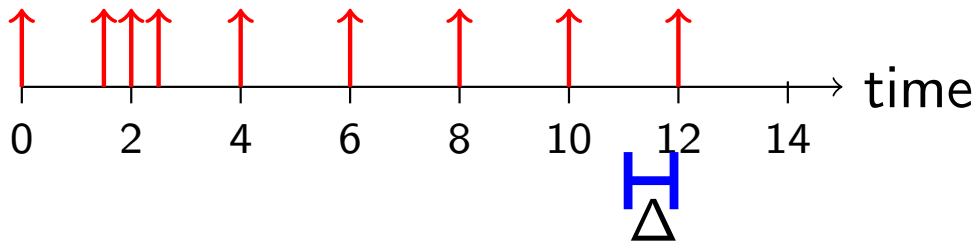

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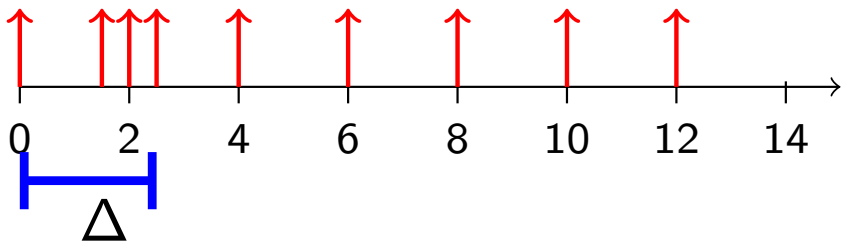

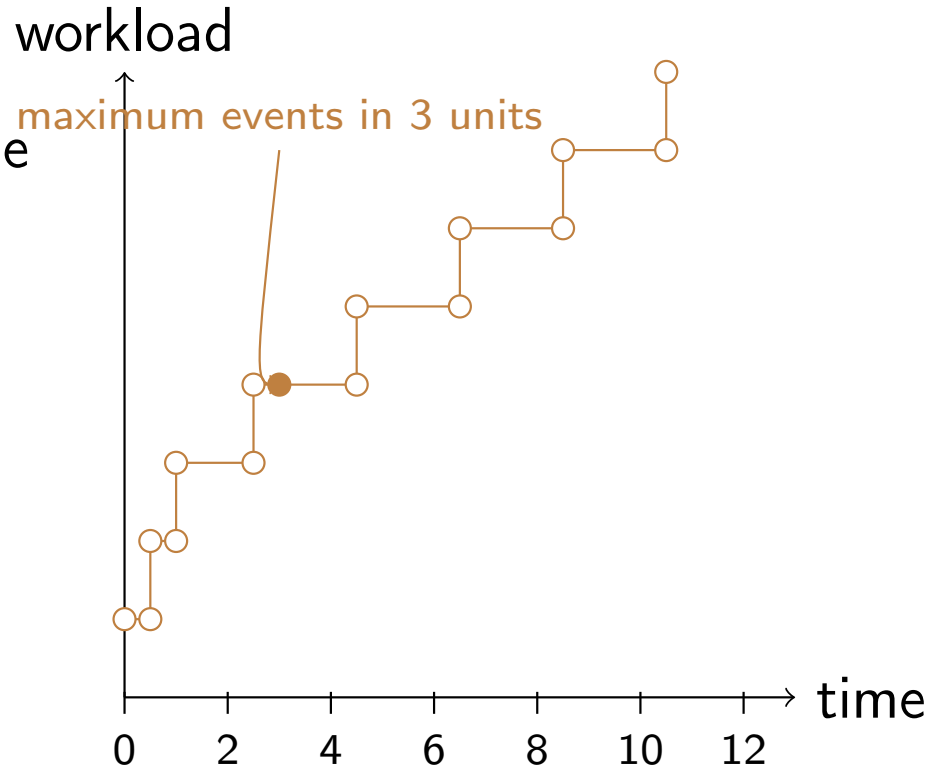

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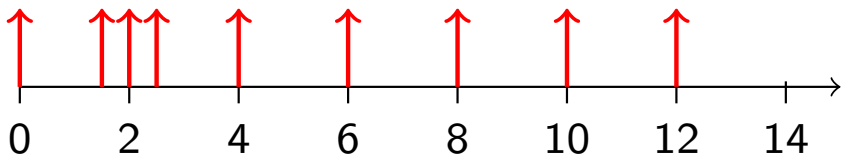

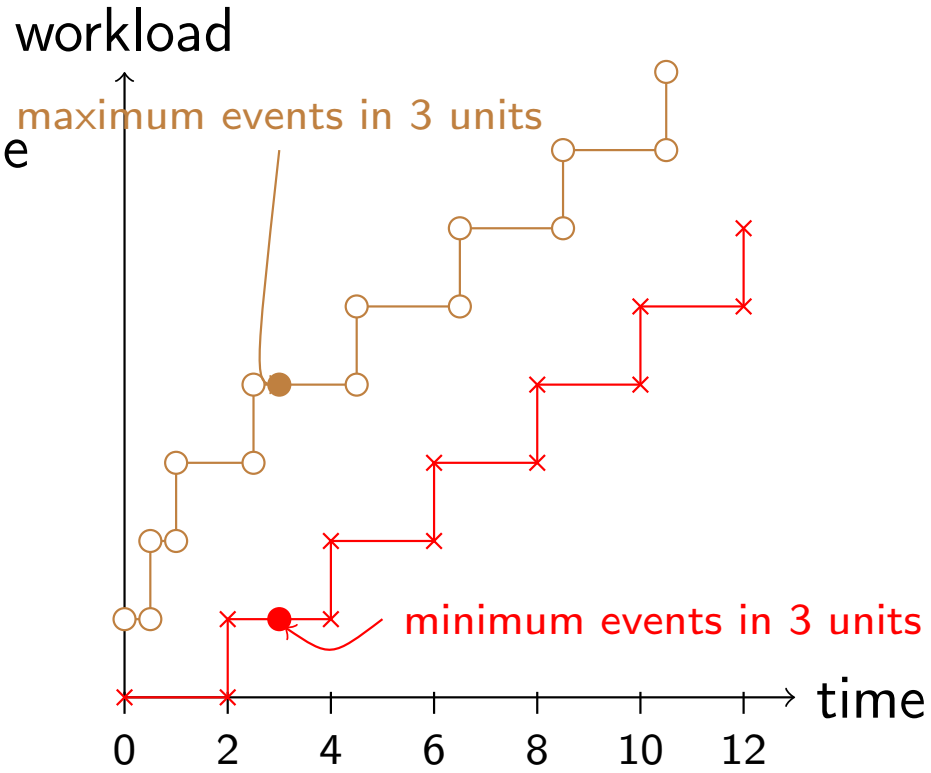

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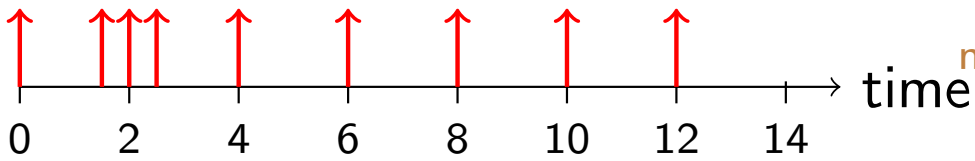
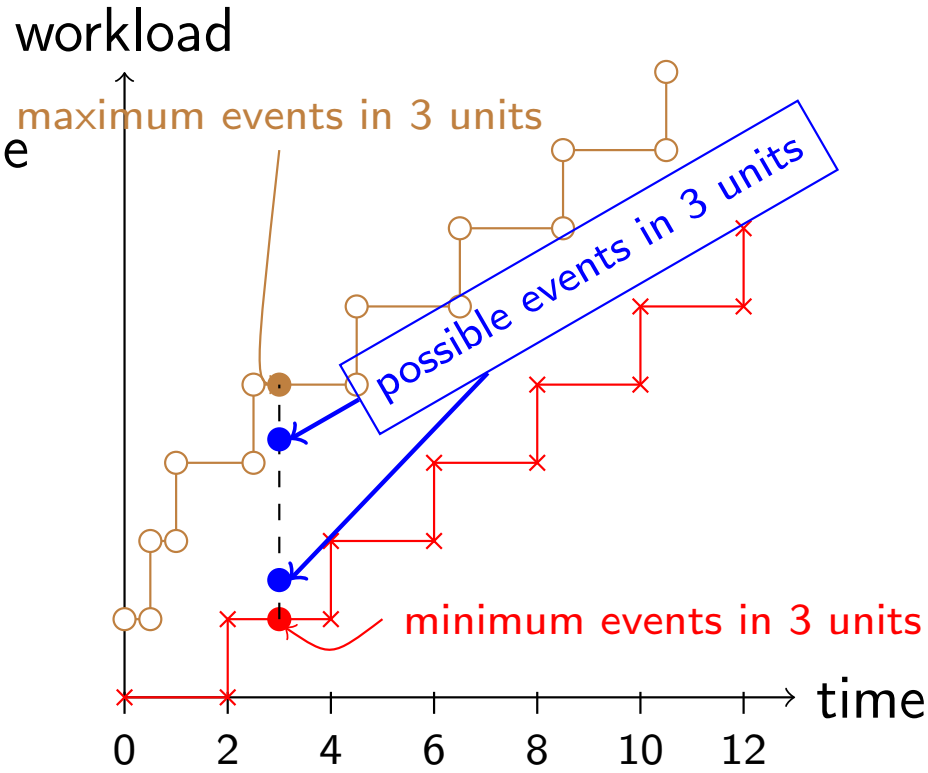
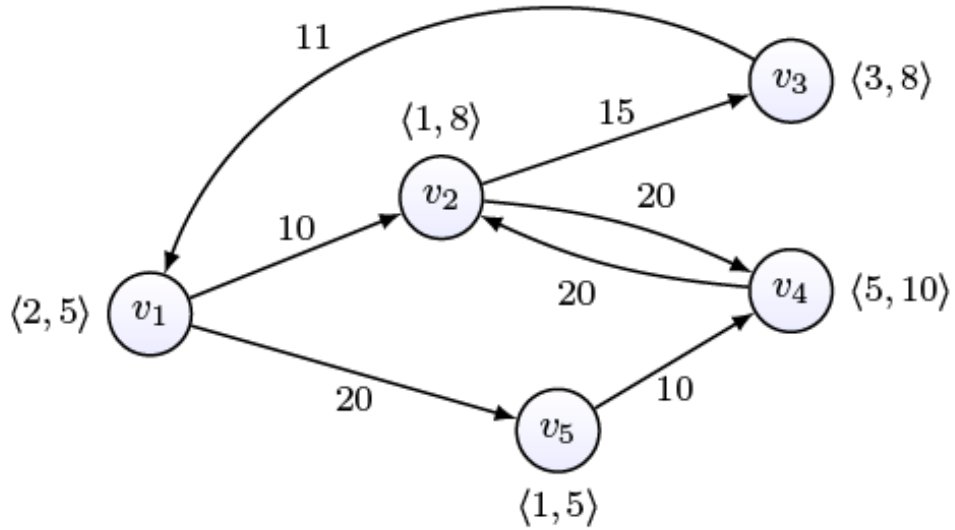




Thiele et al. 2000.

Example 2: Digraph Task Model



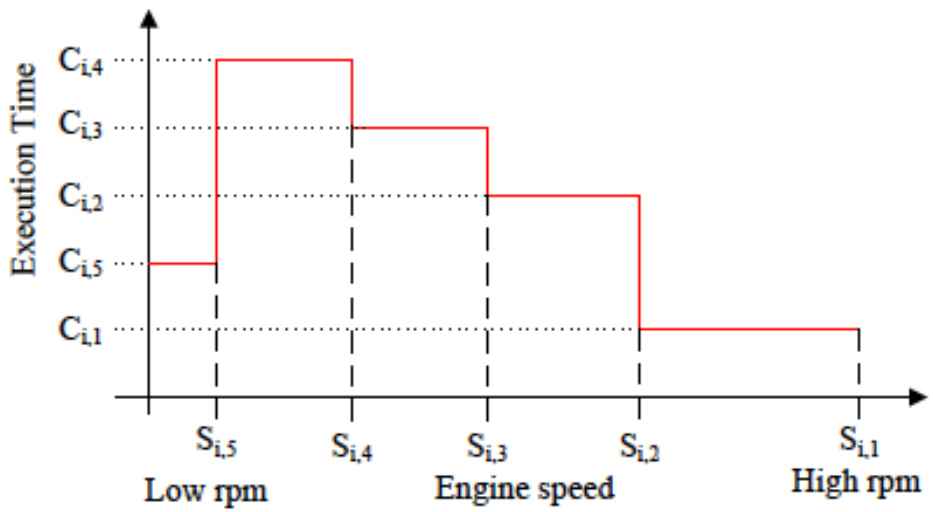
Stigge et al. in RTAS'11

- Each vertex specifies the execution modes (Worst-case Execution Time, Relative Deadline)
- Each edge specifies the minimal inter-invocation time of the next mode

Example 3: Variable Rate Behavior (VRB) Task Model

- Each mode has only one specified inter-invocation time, and can stay in the same mode forever

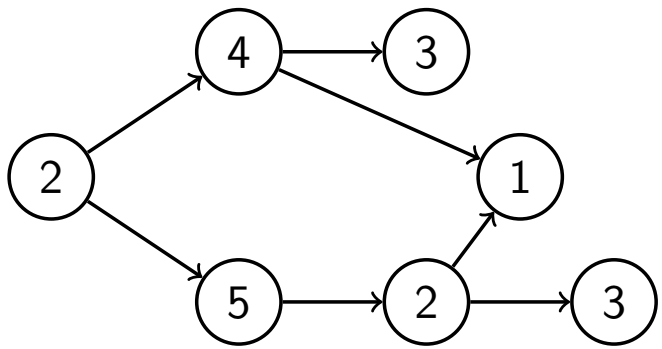
```
TASK(Variant_execution)(){
    f1();
    if(rpm < 3000) {
        f2();
    }
    f3();
}
```



Davis et al. in RTAS'14

Example 4: Tasks with DAG Structures

- Each task τ_i is a sporadic task:
  - period T_i
  - relative deadline $D_i = T_i$
  - Each task is characterized by a directed acyclic graph (DAG)



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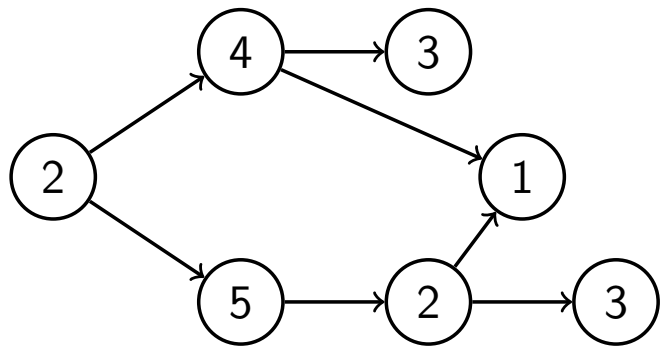
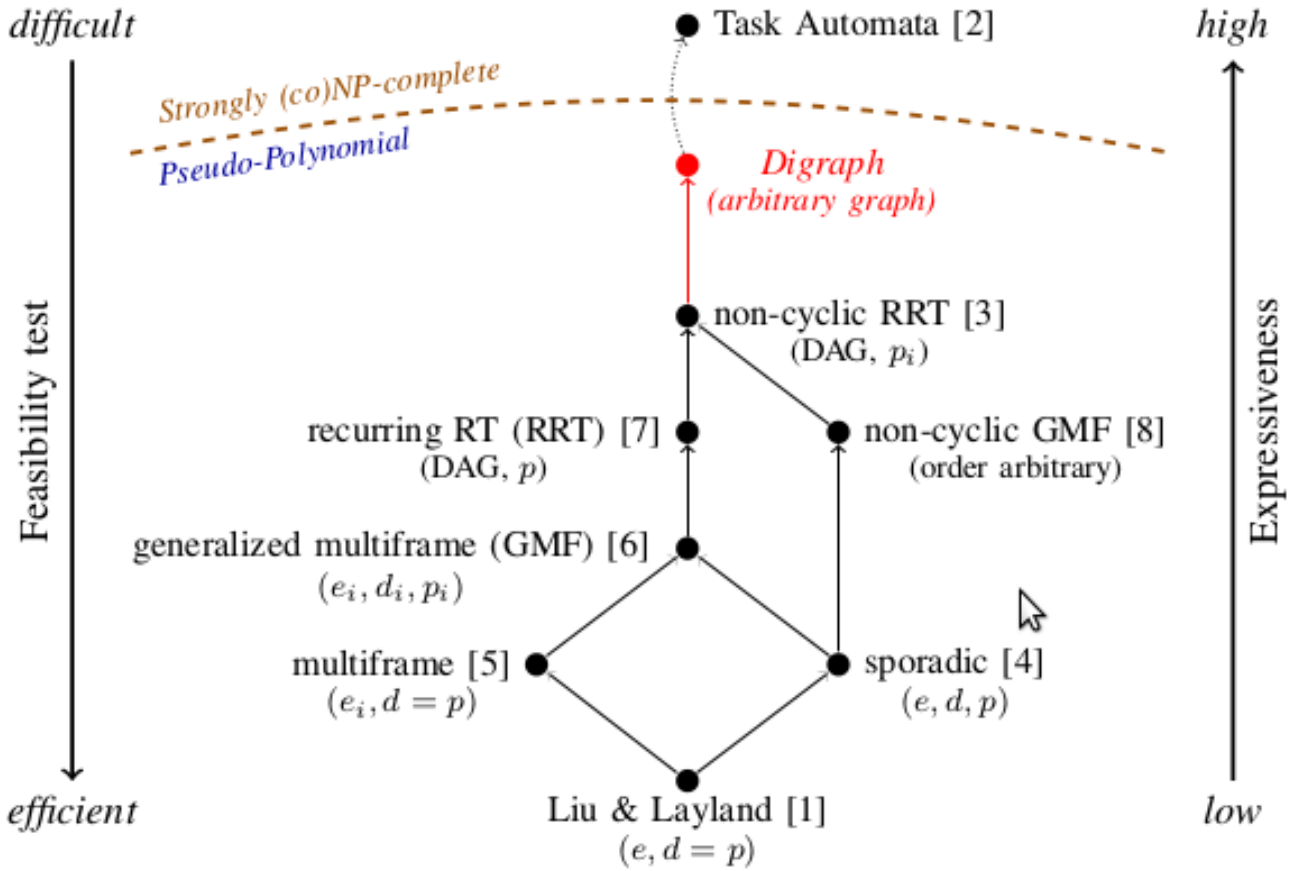


- Almost new topic for multiprocessor systems in Real-Time since 2009 with some nice results for traditional sporadic task models, but not more than that

Expressiveness versus Efficiency

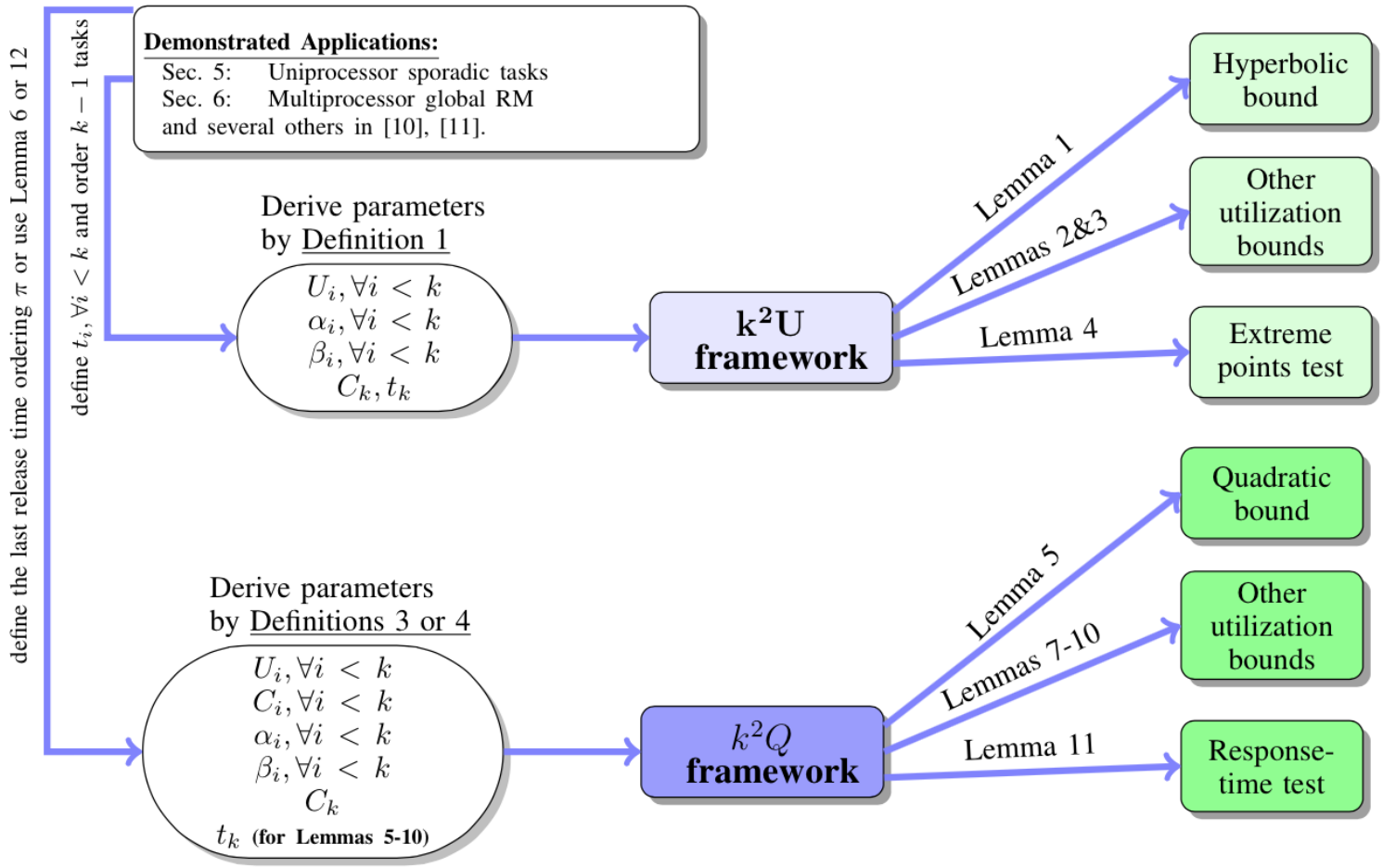


Stigge et al. in RTAS'11

Recent Results Towards Flexible Models

- Linear-time schedulability test framework
  - Two applicable versions, depending upon the needs
  - Basically handles the above problems very well with low time complexity
  - The first evidence to translate *exponential-time schedulability tests* to *linear-time tests* with test quality guarantees

Framework (illustrated)



Chen et al. CoRR 2015

Outline

Schedulability Test of RM Scheduling

The time-demand function $W_i(t)$ of the task τ_i is defined as follows:

$$W_i(t) = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{t}{T_j} \right\rceil C_j.$$

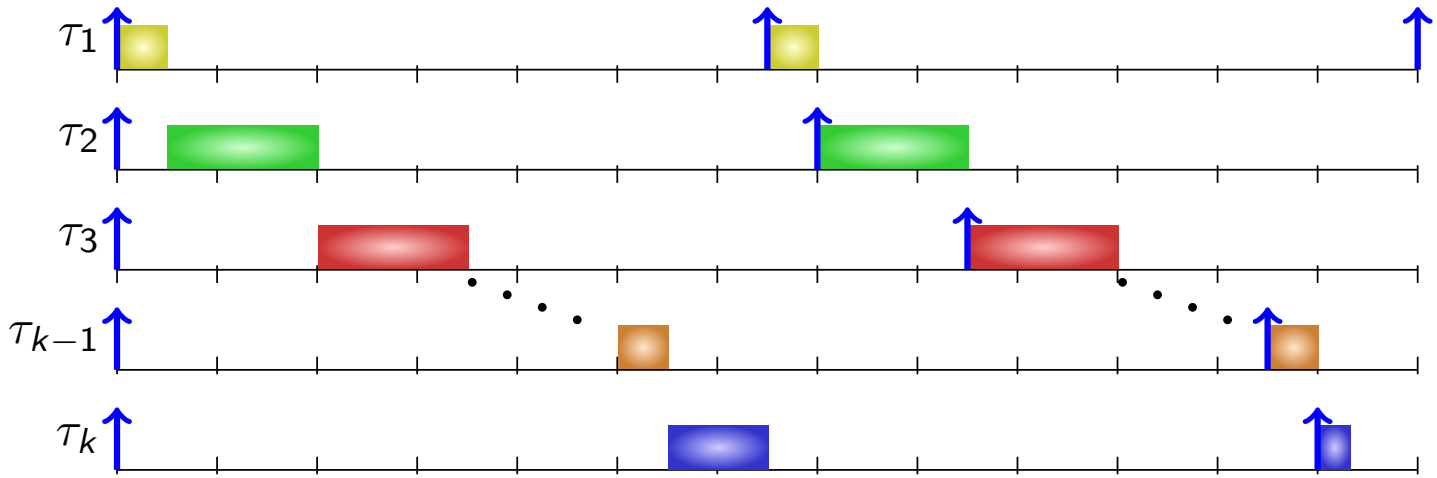
Theorem

A system $\mathcal{T}$ of periodic, independent, preemptable tasks is schedulable on one processor by a fixed-priority scheduling algorithm A if

$$\forall \tau_i \in \mathcal{T} \ \exists t \text{ with } 0 < t \leq D_i \ \textit{ and } \ W_i(t) \leq t.$$

(Liu and Layland-Bound) Structure

The non-schedulability of task τ_k also implies the following structure if $2T_1 \geq T_k$:



$$C_k + \sum_{i=1}^{k-1} C_i + \sum_{i=0}^{j-1} C_i > T_j, \forall j = 1, 2, \ldots, k-1,$$

$$C_k + 2\sum_{i=1}^{k-1} C_i > T_k,$$

where C_0 is defined as 0 for brevity.

k-Point Effective Schedulability Test: k^2U

Suppose that $\{t_1, t_2, \ldots t_k\}$ are given.

Definition

A k-point effective schedulability test is a sufficient test by verifying the existence of $t_j \in \{t_1, t_2, \ldots t_k\}$ with $t_1 \leq t_2 \leq \cdots \leq t_k$ such that

$$C_k + \sum_{i=1}^{k-1} \alpha_i t_i U_i + \sum_{i=1}^{j-1} \beta_i t_i U_i \leq t_j, \tag{1}$$

where $C_k > 0$, $\alpha_i > 0$, $U_i > 0$, and $\beta_i > 0$ are dependent upon the setting of the task models and task τ_i.

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where $C_k > 0$, $\alpha_i > 0$, $U_i > 0$, and $\beta_i > 0$ are dependent upon the setting of the task models and task τ_i.

For the previous example, $\alpha_i = 1$ and $\beta_i = 1$.

Key Lemma

Lemma

[Lemma 1] For a given k-point effective schedulability test of a scheduling algorithm, in which $0 < \alpha_i \leq \alpha \neq \infty$, *and* $0 < \beta_i \leq \beta \neq \infty$ *for any* $i = 1, 2, \ldots, k-1$, $0 < t_k \neq \infty$, *task* τ_k *is schedulable by the scheduling algorithm if the following condition holds*

$$\frac{C_k}{t_k} \leq \frac{\frac{\alpha}{\beta} + 1}{\prod_{j=1}^{k-1}(\beta U_j + 1)} - \frac{\alpha}{\beta}. \tag{2}$$

A Sketched Proof

The unschedulability implies that $C_k > C_k^*$, where C_k^* is defined in the following optimization problem:

$$\begin{aligned} \text{infimum} \quad & C_k^* \\ \text{such that} \quad & C_k^* + \sum_{i=1}^{k-1} \alpha t_i^* U_i + \sum_{i=1}^{j-1} \beta t_i^* U_i > t_j^*, \qquad \forall j = 1, 2, \ldots, k \\ & t_j^* \geq 0, \qquad \forall j = 1, 2, \ldots, k, \end{aligned}$$

where $t_1^*, t_2^*, \ldots, t_{k-1}^*$ and C_k^* are variables, α, β are constants, and t_k^* is defined as t_k.

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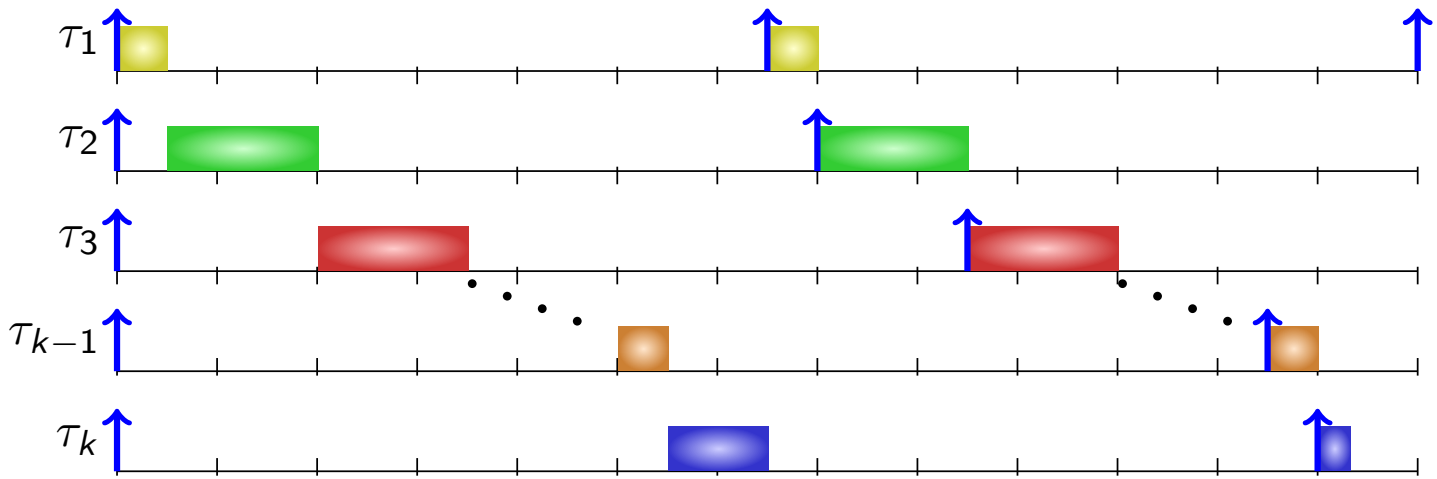
where $t_1^*, t_2^*, \ldots, t_{k-1}^*$ and C_k^* are variables, α, β are constants, and t_k^* is defined as t_k.

The above linear programming gives the minimum C_k^* to be unschedulable. Therefore, if $C_k \leq C_k^*$, task τ_k is guaranteed to be schedulable.

Hyperbolic Bound for Sporadic Task Systems



Let t_i be $\left\lfloor \frac{T_k}{T_i} \right\rfloor$. Therefore, we have $\alpha_i = 1$ and $\beta_i \leq 1$.

Theorem

[Bini and Buttazzo, ECRTS 2001] Task τ_k is schedulable by RM on a uniprocessor system if

$$\prod_{i=1}^{k}(U_i + 1) \leq 2.$$

Direct Implications

Lemma

[Lemma 2] Task τ_k is schedulable by the scheduling algorithm if

$$\frac{C_k}{t_k} + \sum_{i=1}^{k-1} U_i \leq \frac{(k-1)((\alpha+\beta)^{\frac{1}{k}} - 1) + ((\alpha+\beta)^{\frac{1}{k}} - \alpha)}{\beta}. \quad (3)$$

Lemma

[Lemma 3] Task τ_k is schedulable by the scheduling algorithm if

$$\beta \sum_{i=1}^{k-1} U_i \leq \ln\left(\frac{\frac{\alpha}{\beta}+1}{\frac{C_k}{t_k}+\frac{\alpha}{\beta}}\right). \quad (4)$$

Liu and Layland Bound

Theorem

[Liu and Layland, JACM 1973] Task τ_k is schedulable by RM on a uniprocessor system if

$$\sum_{i=1}^{k} U_i \leq k(2^{\frac{1}{k}} - 1)$$

k	utilization bound	k	utilization bound
2	0.828	3	0.779
4	0.756	5	0.743
6	0.734	7	0.728
8	0.724	9	0.720
10	0.717	∞	$0.693 = ln2$

More Precise Approximation

Lemma

[Lemma 4] For a given k-point effective schedulability test of a fixed-priority scheduling algorithm, task τ_k is schedulable by the scheduling algorithm, in which $0 < \alpha_i \neq \infty$ and $0 < \beta_i \neq \infty$ for any $i = 1, 2, \ldots, k-1$, $0 < t_k \neq \infty$, if the following condition holds

$$0 < \frac{C_k}{t_k} \leq 1 - \sum_{i=1}^{k-1} \frac{U_i(\alpha_i + \beta_i)}{\prod_{j=i}^{k-1}(\beta_j U_j + 1)}. \tag{5}$$

k-Point Effective Schedulability Test: k^2Q

Definition

[Last Release Time Ordering] Let π be the last release time ordering assignment as a bijective function $\pi : hp(\tau_k) \rightarrow \{1, 2, \ldots, k-1\}$ to define the last release time ordering of task $\tau_j \in hp(\tau_k)$ in the window of interest.

Definition

A k-last-release effective schedulability test under a given ordering π of the $k-1$ higher priority tasks is a sufficient schedulability test of a fixed-priority scheduling policy by verifying the existence of $t_1 \leq t_2 \leq \cdots \leq t_{k-1} \leq t_k$ such that

$$C_k + \sum_{i=1}^{k-1} \alpha_i t_i U_i + \sum_{i=1}^{j-1} \beta_i C_i \leq t_j, \tag{6}$$

where $C_k > 0$, for $i = 1, 2, \ldots, k-1$, $\alpha_i > 0$, $U_i > 0$, $C_i \geq 0$, and $\beta_i > 0$ are dependent upon the setting of the task models and task τ_i.

Key Lemma

Lemma

[Lemma 5] For a given k-last-release effective schedulability test, in which $0 < \alpha_i \neq \infty$, *and* $0 < \beta_i \neq \infty$ *for any* $i = 1, 2, \ldots, k-1$, $0 < t_k \neq \infty$, *and* $\sum_{i=1}^{k-1} \beta_i C_i \leq t_k$, *task* τ_k *is schedulable by the fixed-priority scheduling algorithm if*

$$\frac{C_k}{t_k} \leq 1 - \sum_{i=1}^{k-1} \alpha_i U_i - \frac{\sum_{i=1}^{k-1}(\beta_i C_i - \alpha_i U_i(\sum_{\ell=i}^{k-1} \beta_\ell C_\ell))}{t_k}. \quad (7)$$

Key Lemma

Lemma

[Lemma 5] For a given k-last-release effective schedulability test, in which $0 < \alpha_i \neq \infty$, *and* $0 < \beta_i \neq \infty$ *for any* $i = 1, 2, \ldots, k-1$, $0 < t_k \neq \infty$, *and* $\sum_{i=1}^{k-1} \beta_i C_i \leq t_k$, *task* τ_k *is schedulable by the fixed-priority scheduling algorithm if*

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The worst-case ordering π of the $k-1$ higher-priority tasks is to order the tasks in a non-increasing order of $\frac{\beta_i C_i}{\alpha_i U_i}$.

A Sketched Proof

The unschedulability implies that $C_k > C_k^*$, where C_k^* is defined in the following optimization problem:

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where $t_1^*, t_2^*, \ldots, t_{k-1}^*$ and C_k^* are variables, α_i, β_i are constants, and t_k^* is defined as t_k.

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The above linear programming gives the minimum C_k^* to be unschedulable. Therefore, if $C_k \leq C_k^*$, task τ_k is guaranteed to be schedulable.

Direct Implications

Suppose that $0 < \beta_i C_i \leq \beta U_i t_k \neq \infty$.

Lemma

[Lemma 7] Task τ_k is schedulable by the scheduling algorithm if

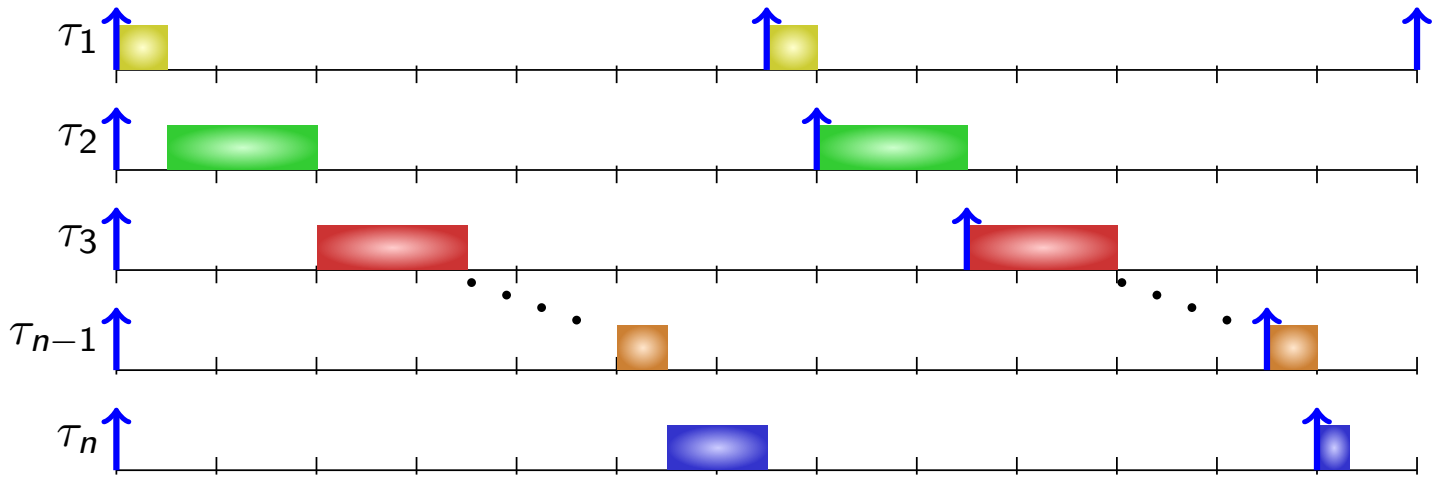
$$\frac{C_k}{t_k} \leq 1 - (\alpha + \beta) \sum_{i=1}^{k-1} U_i + 0.5\alpha\beta \left((\sum_{i=1}^{k-1} U_i)^2 + (\sum_{i=1}^{k-1} U_i^2) \right) \quad (8)$$

or

$$\sum_{i=1}^{k-1} U_i \leq \left(\frac{k-1}{k} \right) \left(\frac{\alpha + \beta - \sqrt{(\alpha+\beta)^2 - 2\alpha\beta(1 - \frac{C_k}{t_k})\frac{k}{k-1}}}{\alpha\beta} \right), \quad (9)$$

which converges to $\frac{\alpha + \beta - \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta\frac{C_k}{t_k}}}{\alpha\beta}$ *when* $k \to \infty$.

Quadratic Bound for Sporadic Task Systems



Let t_i be $\left\lfloor \frac{T_k}{T_i} \right\rfloor$. Therefore, $\alpha_i = 1$ and $\beta_i \leq 1$.

Theorem

Task τ_k is schedulable by RM on a uniprocessor system if

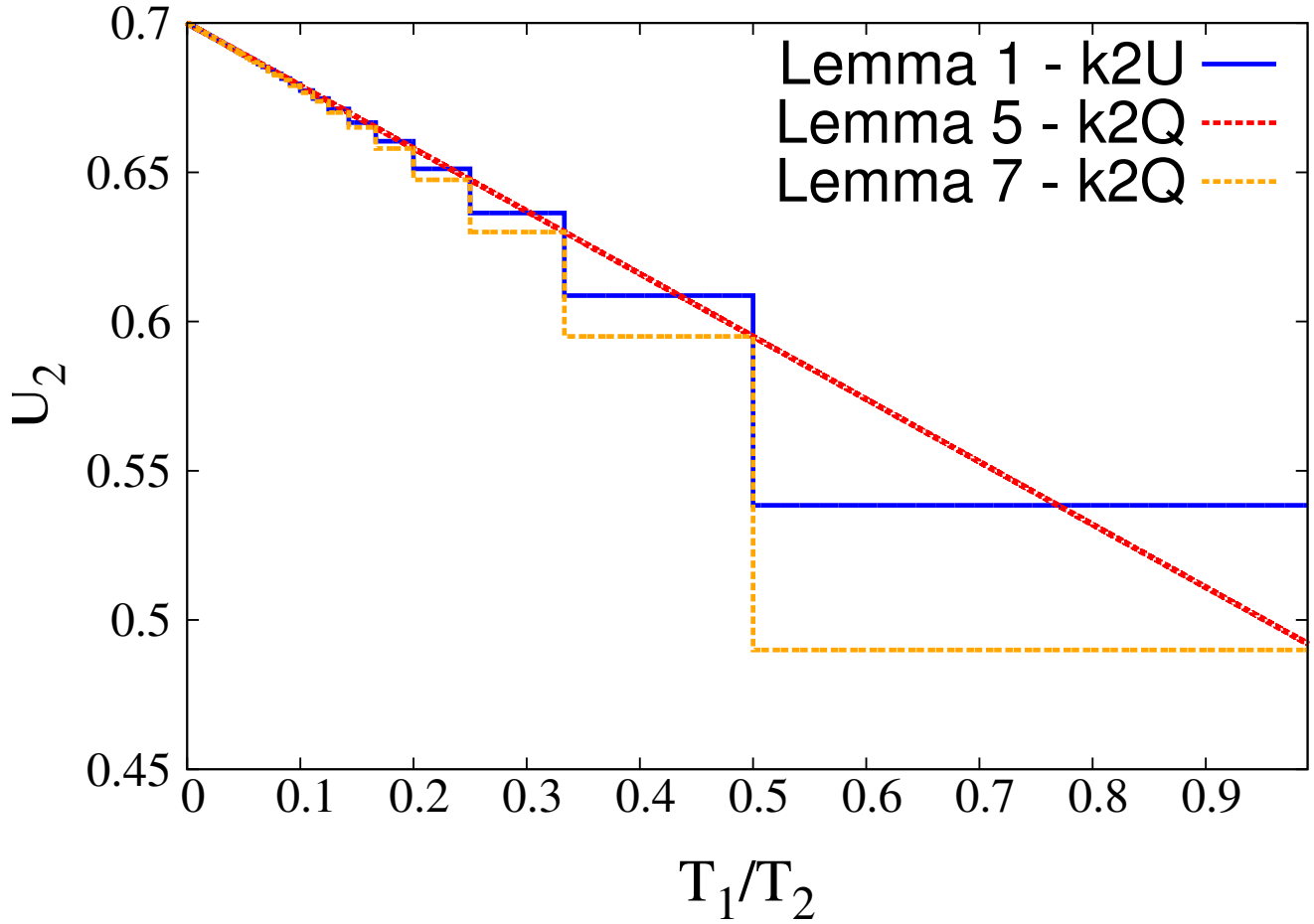
$$0 \leq 1 - U_k - 2\sum_{i=1}^{k-1} U_i + 0.5\left((\sum_{i=1}^{k-1} U_i)^2 + (\sum_{i=1}^{k-1} U_i^2)\right).$$

How to Use the Framework

- Parameters α_i and β_i affect the quality of the schedulability bounds
- Deriving the *good* settings of α_i and β_i is not part of this framework.
  - The framework simply derives the bounds/tests according to α_i and β_i
  - The correctness of α_i and β_i is not verified by the framework.
- The hyperbolic/quadratic bounds or utilization bounds can be automatically derived
  - The other approaches seek for the total utilization bounds
  - They have limited applications and are less flexible.
- After α_i and β_i or their safe upper bounds α and β are derived, the task model is not further referred.

Comparisons

Adopting different tests from k^2U and k^2Q for RM uniprocessor scheduling with $k = 2$.



When to Use Which?

- k^2U (Version 1 above)
  - define any valid k points to obtain the corresponding α_i and β_i
  - more precise if the corresponding exponential-time (pseudo-polynomial-time) test is an exact test
  - may be less precise if the corresponding test requires some pessimism to be constructed, to be shown later
- k^2Q (Version 2 above)
  - define k last release points to obtain the corresponding α_i and β_i
  - has to typically consider the last release ordering
  - less precise if the corresponding exponential-time (pseudo-polynomial-time) test is an exact test
  - may be more precise by starting from the exponential-time test, to be shown later
  - can be generalized for response-time analysis

Outline

Deadline-Monotonic Scheduling

- let $hp(\tau_k)$ be the set of tasks with higher priority than τ_k.
  - $hp_1(\tau_k)$ consists of the higher-priority tasks τ_i with $T_i < D_k$.
  - $hp_2(\tau_k)$ consists of the higher-priority tasks τ_i with $T_i \geq D_k$.
- The schedulability test is equivalent to the verification of

$$\exists 0 < t \leq D_k \qquad C_k + \sum_{\tau_i \in hp_2(\tau_k)} C_i + \sum_{\tau_i \in hp_1(\tau_k)} \left\lceil \frac{t}{T_i} \right\rceil C_i \leq t.$$

$$\Rightarrow \exists 0 < t \leq D_k \qquad C'_k + \sum_{\tau_i \in hp_1(\tau_k)} \left\lceil \frac{t}{T_i} \right\rceil C_i \leq t.$$

- Apply k^2U or k^2Q to get the utilization-based schedulability tests, by setting $\alpha_i = 1$ and $0 < \beta_i \leq 1$.

Deadline-Monotonic Scheduling (cont.)

The schedulability condition of task τ_k by using k^2U is

$$\left(\frac{C'_k}{D_k} + 1\right) \prod_{\tau_j \in hp_1(\tau_k)} (U_j + 1) \leq 2.$$

The schedulability condition of task τ_k by using k^2Q is

$$\frac{C'_k}{D_k} \leq 1 - 2\sum_{i=1}^{k-1} U_i + 0.5\left((\sum_{i=1}^{k-1} U_i)^2 + (\sum_{i=1}^{k-1} U_i^2)\right).$$

- It can be proved that the speed-up factor of DM is 1.76322, compared to EDF.

Uniprocessor Non-Preemptive (NP) Scheduling

Let $\widehat{C_k} = C_k + B_k + \sum_{\tau_i \in hp_2(\tau_k)} C_i$, where B_k is $\left\{\max_{\tau_i \in lp(\tau_k)} C_i\right\}$. The schedulability condition of task τ_k by using $k^2 U$ is

$$\left(\frac{\widehat{C_k'}}{D_k} + 1\right) \prod_{\tau_j \in hp_1(\tau_k)} (U_j + 1) \leq 2 \tag{10}$$

Theorem

Suppose that $\gamma = \max_{\tau_k} \left\{\max_{\tau_i \in lp(\tau_k)} \left\{\frac{C_i}{C_k}\right\}\right\}$. *A task set can be feasibly scheduled by RM-NP if*

$$U_{sum} \leq \begin{cases} \frac{\gamma}{\gamma+1} + ln\left(\frac{2}{1+\gamma}\right) & \text{if } \gamma \leq 1 \\ \frac{1}{1+\gamma} & \text{if } 1 < \gamma \end{cases} \tag{11}$$

(left as an exercise)

More Precise Analysis

$$\exists t \in (0, D_k - C_k] \text{ with } B_k + \sum_{\tau_i \in hp(\tau_k)} \left\lceil \frac{t}{T_i} \right\rceil C_i \leq t.$$

$$\exists t \in (0, D_k] \text{ with } C_k + \sum_{\tau_i \in hp(\tau_k)} \left\lceil \frac{t}{T_i} \right\rceil C_i \leq t.$$

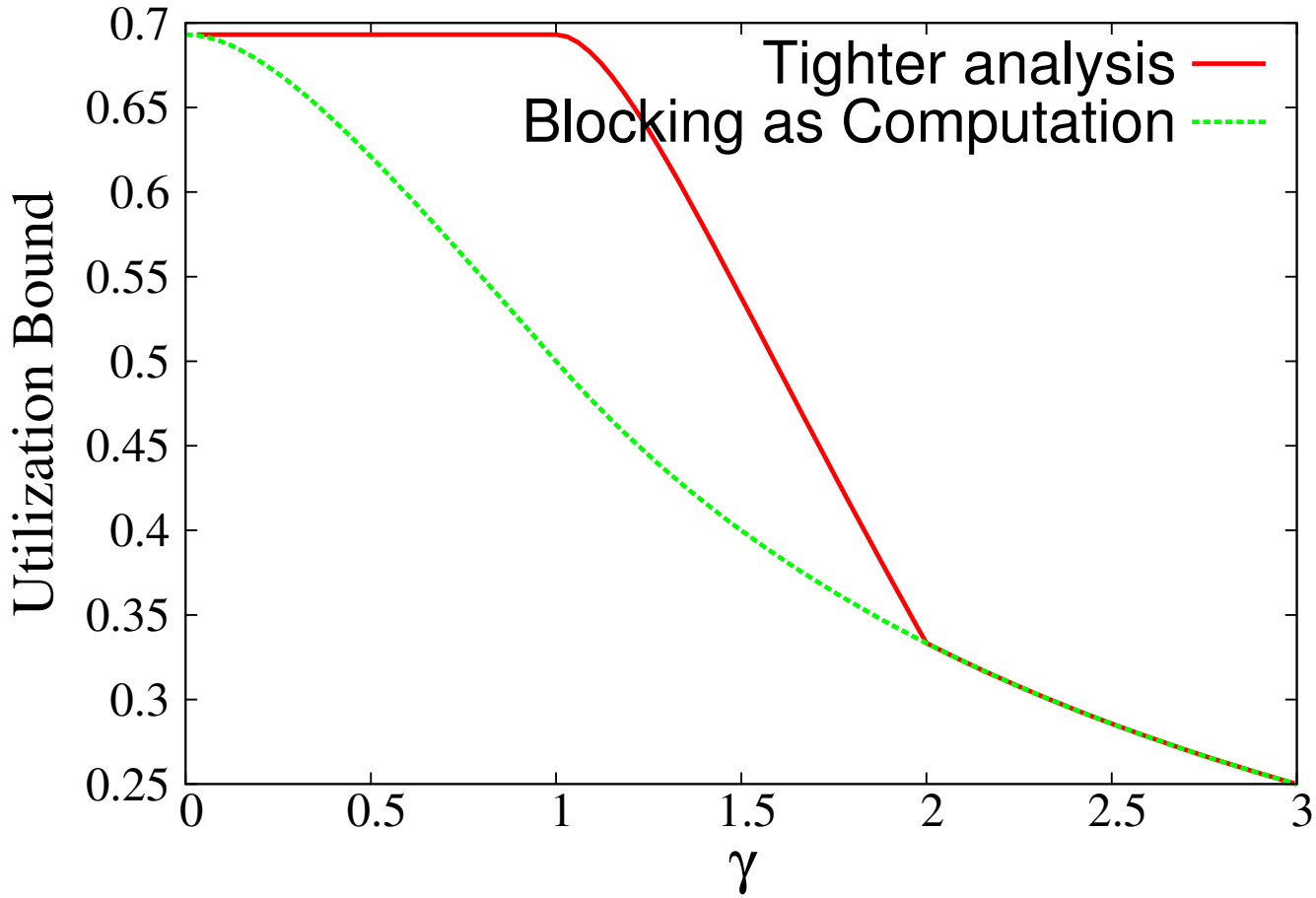
Theorem

Suppose that $\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in lp(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$. *A task set can be feasibly scheduled by RM-NP if*

$$U_{sum} \leq \begin{cases} ln(2) \approx 0.693 & \text{if } \gamma \leq 1 \\ \frac{\gamma - 1}{2\gamma - 1} + ln(\frac{2}{\gamma}) & \text{if } 1 < \gamma \leq 2 \\ \frac{1}{1+\gamma} & \text{if } 2 < \gamma \end{cases} \tag{12}$$

Uniprocessor Non-Preemptive Scheduling (cont.)

$$\gamma = \max_{\tau_k} \left\{ \max_{\tau_i \in lp(\tau_k)} \left\{ \frac{C_i}{C_k} \right\} \right\}$$



Self-Suspending Tasks

Task τ_k is schedulable under RM in a self-suspension task set, if

$$\exists t \text{ with } 0 < t \leq T_k \;\; \textit{and} \;\; C_k + S_k + \sum_{i=1}^{k-1} W_i(t) \leq t,$$

where

$$W_i(t) = \left(\left\lceil \frac{t}{T_i} \right\rceil - 1 \right) C_i + 2C_i.$$

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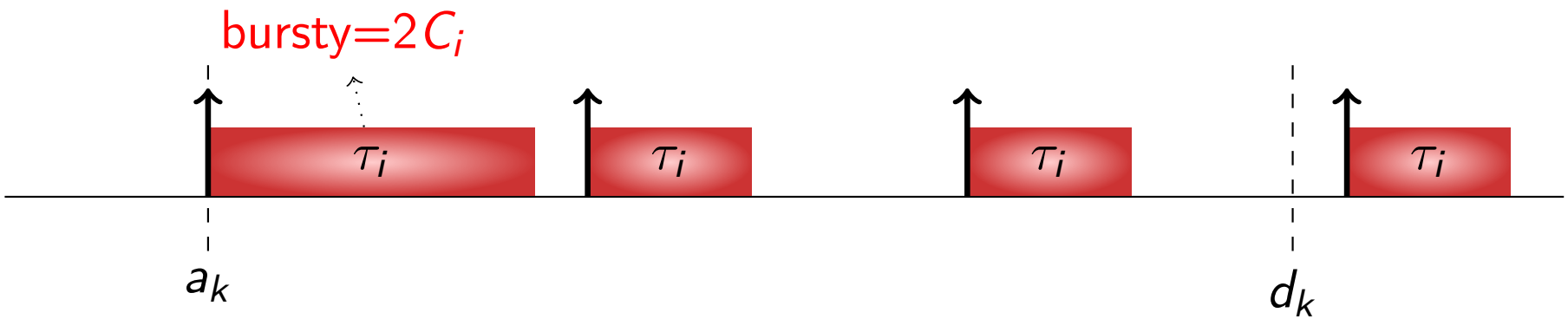


Utilization Bounds

Let $t_i = \left\lfloor \frac{T_i}{T_k} \right\rfloor$. Therefore, we have $\alpha_i \leq 2$ and $\beta_i \leq 1$.

Theorem (Liu and Chen in RTSS 2014)

Any implicit-deadline sporadic self-suspending task set is schedulable under RM if the following conditions hold:

$$\forall 1 \leq k \leq n, \boxed{U_k + \frac{S_k}{T_k}} \leq 1 - (\boxed{2} + 1) \cdot \left(1 - \frac{1}{\prod_{i=1}^{k-1}(U_i + 1)}\right). \quad (13)$$

Theorem (Liu and Chen in RTSS 2014)

Any implicit-deadline sporadic self-suspending task set is schedulable under RM if the following conditions hold:

$$\forall 1 \leq k \leq n, U_k + \boxed{\frac{S_k}{T_k}} + \sum_{i=1}^{k-1} U_i \leq k \left(\left(\frac{\boxed{2} + 1}{\boxed{2}} \right)^{\frac{1}{k}} - 1 \right) \quad (14)$$

Real-Time Systems with Mode Changes}

- Real-time tasks run in different modes over time to react to the change of physical environments
  - Avionic systems
  - Automotive systems

rotation (rpm)	functions to be executed
$[0, 2000]$	$f1(); f2(); f3(); f4();$
$(2000, 4000]$	$f1(); f2(); f3();$
$(4000, 6000]$	$f1(); f2();$
$(6000, 8000]$	$f1();$

Multi-Mode Task Model

- A multi-mode task τ_i is denoted by a set of triplets:

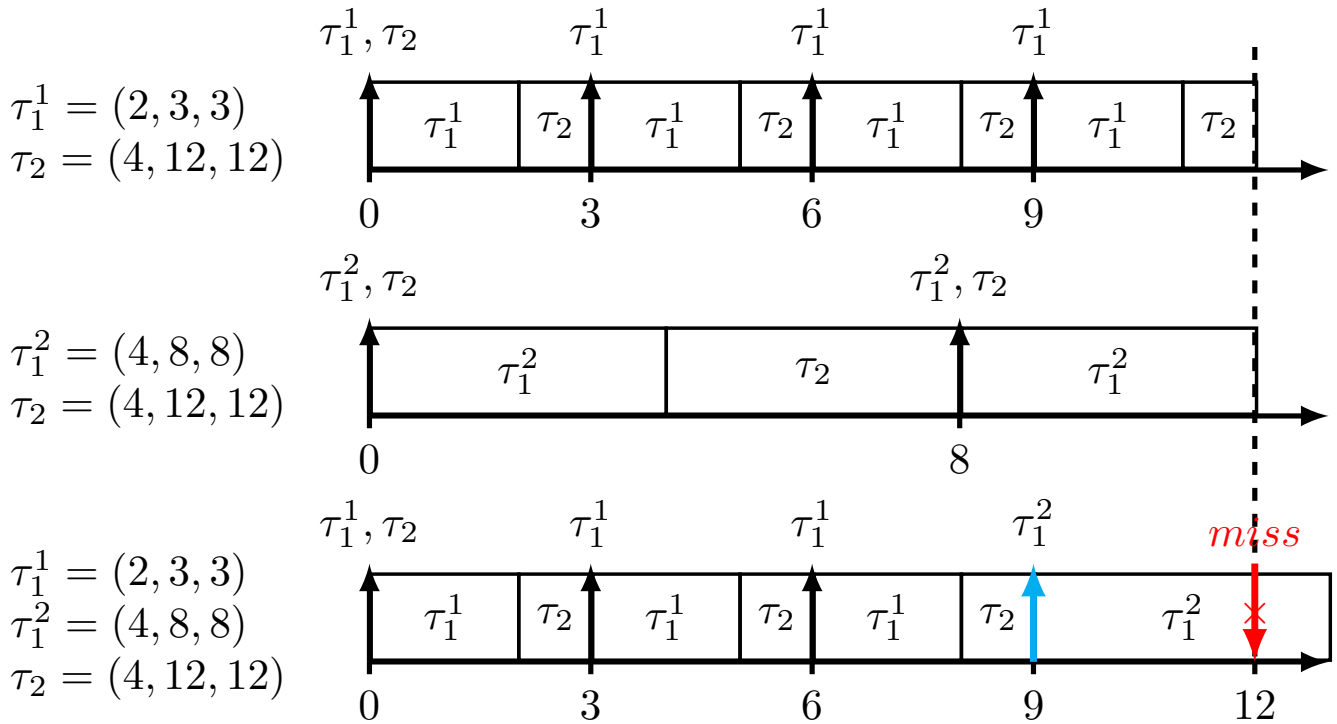
$$\tau_i = \{\tau_i^1 = (C_i^1, T_i^1, D_i^1),$$
$$\tau_i^2 = (C_i^2, T_i^2, D_i^2), ...,$$
$$\tau_i^{M_i} = (C_i^{M_i}, T_i^{M_i}, D_i^{M_i})\}$$

  - C_i^m denotes the *worst-case execution time* (WCET) of task τ_i under mode m
  - T_i^m denotes the *minimum inter-arrival time* of task τ_i under mode m
  - D_i^m denotes relative deadline (Constrained-deadline system ($D_i^m \leq T_i^m$))
- Fixed-priority scheduling
  - Fixed-priority task-level (FPT)
  - Fixed-priority mode-level (FPM)

Problems with a Naive Analysis

Deadline miss during mode transition under fixed-priority scheduling

- $\tau_1 = \{\tau_1^1 = (2, 3, 3), \tau_1^2 = (4, 8, 8)\}$
- $\tau_2 = (4, 12, 12)$



A Safe Exponential-Time Test

Theorem

Task mode τ_k^h is schedulable under an FPM scheduling if

$$\forall y \geq 0, \ \ \forall \textit{combinations of } t_i^* \textit{ with } 0 \leq t_i^* \leq t_{i+1}^*, \forall i = 1, 2, \ldots, k-1$$

$$\exists j = 1, 2, \ldots, k, s.t. \ \ C_k^h + y \cdot U_k^{max} + \sum_{i=1}^{k-1} (U_i^{max} \cdot t_i^*) + \sum_{i=1}^{j-1} C_i^{max} \leq t_j^* .$$

C_i^{max} is the maximum execution time among the modes of task τ_i with priority higher than or equals to task mode τ_k^h. U_i^{max} is the maximum utilization among the modes of task τ_i with priority higher than or equals to task mode τ_k^h. The constant t_k^* is defined as $T_k^h + y$.

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Hint: the above test also requires to enumerate all possible orderings. Under k^2Q, it is possible to safely only test one specific ordering.

Utilization Test under FPM-RM

For a given y, we have $C_k^h + y \cdot U_k^{max} \leq (T_k + y) \cdot U_k^{max}$. So, the remaining is a case with $\alpha_1 = 1$ and $\beta_i = 1$ in the k^2Q framework.

Theorem

Task τ_k^h in a multi-mode task system with implicit deadlines is schedulable by the mode-level RM scheduling algorithm on a uniprocessor system if the following condition holds

$$U_k^{\max} \leq 1 - 2\sum_{i=1}^{k-1} U_i^{\max} + 0.5\left((\sum_{i=1}^{k-1} U_i^{\max})^2 + (\sum_{i=1}^{k-1}(U_i^{\max})^2)\right), \quad (15)$$

or

$$\sum_{i=1}^{k-1} U_i^{\max} \leq \left(\frac{k-1}{k}\right)\left(2 - \sqrt{2 + 2U_k^{\max}\frac{k}{k-1}}\right), \quad (16)$$

or

$$U_k^{\max} + \sum_{i=1}^{k-1} U_i^{\max} \leq \begin{cases} \left(\frac{k-1}{k}\right)\left(2 - \sqrt{4 - \frac{2k}{k-1}}\right), & \text{if } k > 3 \\ 1 - \frac{(k-1)}{2k} & \text{otherwise.} \end{cases} \quad (17)$$

Outline

Other Applications

- Multi-frame task model
- VRB task models
- Digraph task models
- Uniprocessor/Multiprocessor scheduling with self-suspensions
- Multiprocessor global DM scheduling
- Multiprocessor partitioned RM/DM scheduling
- Multiprocessor scheduling with DAG structures
- etc.

Linear-time schedulability test framework

- Two applicable versions, depending upon the needs
- Basically handles the above problems very well with low time complexity
- The first evidence to translate *exponential-time schedulability tests* to *linear-time tests* with test quality guarantees