Scheduling Periodic Real-Time Tasks on Uniprocessor Systems

Prof. Dr. Jian-Jia Chen

LS 12, TU Dortmund

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Pseudo-code for this system

set timer to interrupt periodically with period $T$;

at each timer interrupt

do

• perform analog-to-digital conversion to get $y$;

• compute control output $u$;

• output $u$ and do digital-to-analog conversion;

od
Example: Sporadic Control System

Pseudo-code for this system

```
while (true)
    • start := get the system tick;
    • perform analog-to-digital conversion to get \( y \);
    • compute control output \( u \);
    • output \( u \) and do digital-to-analog conversion;
    • end := get the system tick;
    • \( \text{timeToSleep} := T - (end - start) \);
    • sleep \( \text{timeToSleep} \);
end while
```
Periodic and Sporadic Task Models

- When jobs (usually with the same computation requirement) are released recurrently, these jobs can be modeled by a recurrent task.
- **Periodic Task** $\tau_i$:
  - A job is released exactly and periodically by a period $T_i$.
  - A phase $\phi_i$ indicates when the first job is released.
  - A relative deadline $D_i$ for each job from task $\tau_i$.
  - $(\phi_i, C_i, T_i, D_i)$ is the specification of periodic task $\tau_i$, where $C_i$ is the worst-case execution time. When $\phi_i$ is omitted, we assume $\phi_i$ is 0.
Periodic and Sporadic Task Models

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  - A phase $\phi_i$ indicates when the first job is released
  - A relative deadline $D_i$ for each job from task $\tau_i$
  - $(\phi_i, C_i, T_i, D_i)$ is the specification of periodic task $\tau_i$, where $C_i$ is the worst-case execution time. When $\phi_i$ is omitted, we assume $\phi_i$ is 0.

- **Sporadic Task** $\tau_i$:
  - $T_i$ is the minimal time between any two consecutive job releases
  - A relative deadline $D_i$ for each job from task $\tau_i$
  - $(C_i, T_i, D_i)$ is the specification of sporadic task $\tau_i$, where $C_i$ is the worst-case execution time.
Examples of Recurrent Task Models

**Periodic task:** \((\phi_i, C_i, T_i, D_i) = (2, 2, 6, 6)\)

![Release and deadline for periodic task](image)

**Sporadic task:** \((C_i, T_i, D_i) = (2, 6, 6)\)

![Release and deadline for sporadic task](image)
Relative Deadline \( \iff \) Period

For a task set, we say that the task set is with

- \textit{implicit deadline} when the relative deadline \( D_i \) is equal to the period \( T_i \), i.e., \( D_i = T_i \), for every task \( \tau_i \),
- \textit{constrained deadline} when the relative deadline \( D_i \) is no more than the period \( T_i \), i.e., \( D_i \leq T_i \), for every task \( \tau_i \), or
- \textit{arbitrary deadline} when the relative deadline \( D_i \) could be larger than the period \( T_i \) for some task \( \tau_i \).

The response time of a job is its finishing time minus its arrival time. The worst-case response time of task \( \tau_i \) is \textit{the maximum response time} among the jobs of task \( \tau_i \).
Some Definitions for Sporadic/Periodic Tasks

- **Periodic Tasks:**
  - Synchronous system: Each task has a phase of 0.
  - Asynchronous system: Phases are arbitrary.
- **Hyperperiod:** Least common multiple (LCM) of $T_i$.
- **Task utilization of task $\tau_i$:** $U_i := \frac{C_i}{T_i}$.
- **System (total) utilization:** $U(\mathcal{T}) := \sum_{\tau_i \in \mathcal{T}} U_i$. 
Outline

Schedulability for Dynamic-Priority Scheduling

Schedulability for Static-Priority (or Fixed-Priority) Scheduling
Utilization-Based Test for EDF Scheduling

**Theorem**

Liu and Layland: A task set $\mathcal{T}$ of $n$ independent, preemptable, periodic tasks with implicit deadlines can be feasibly scheduled (under EDF) on one processor if and only if its total utilization $U$ is at most one.

**Proof**

- The *only if* part is obvious: If $U > 1$, then some task clearly must miss a deadline. So, we concentrate on the *if* part.
- Contrapositive: if $\mathcal{T}$ is not schedulable, then $U > 1$.
  - Let $J_{i,k}$ be the first job to miss its deadline
  - Let $d_{i,k}$ be the absolute deadline of $J_{i,k}$
  - Let $t_{-1}$ be the last instant before $d_{i,k}$, at which the processor is either idle or executing a job with absolute deadline larger than $d_{i,k}$

(cont.)
Proof of Utilization-Bound Test for EDF

Proof.

Because $J_{i,k}$ missed its deadline, we know that

\[ d_{i,k} - t_{-1} < \text{demand in } [t_{-1}, d_{i,k}) \]
by jobs with absolute deadline no more than $d_{i,k}$

\[
= \sum_{j=1}^{n} \left[ \frac{d_{i,k} - t_{-1}}{T_j} \right] C_j
\]

\[
\leq \sum_{j=1}^{n} \frac{d_{i,k} - t_{-1}}{T_j} C_j
\]

By cancelling $d_{i,k} - t_{-1}$, we conclude the proof by

\[ 1 < \sum_{j=1}^{n} \frac{C_j}{T_j} = U. \]
Relative Deadlines Less than Periods

Theorem

A task set $\mathcal{T}$ of $n$ independent, preemptable, periodic tasks with constrained deadlines can be feasibly scheduled (under EDF) on one processor if

$$\sum_{k=1}^{n} \frac{C_k}{\min\{D_k, T_k\}} \leq 1.$$ 

This theorem only gives a sufficient schedulability test.
Outline

Schedulability for Dynamic-Priority Scheduling

Schedulability for Static-Priority (or Fixed-Priority) Scheduling
Static-Priority (or Fixed-Priority) Scheduling

- Different jobs of a task are assigned the same priority.
  - Note: we will assume that no two tasks have the same priority.
- We will implicitly index tasks in decreasing priority order, i.e., $\tau_i$ has higher priority than $\tau_k$ if $i < k$. 

Which strategy is better or the best?
- largest execution time first?
- shortest job first?
- least-utilization first?
- most importance first?
- least period first?
Static-Priority (or Fixed-Priority) Scheduling

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- Which strategy is better or the best?
  - largest execution time first?
  - shortest job first?
  - least-utilization first?
  - most importance first?
  - least period first?
Rate-Monotonic (RM) Scheduling (Liu and Layland, 1973)

Priority Definition: A task with a smaller period has higher priority, in which ties are broken arbitrarily.

Example Schedule: $\tau_1 = (1, 6, 6), \tau_2 = (2, 8, 8), \tau_3 = (4, 12, 12)$. 

$[(C_i, T_i, D_i)]$
Deadline-Monotonic (DM) Scheduling (Leung and Whitehead)

Priority Definition: A task with a smaller relative deadline has higher priority, in which ties are broken arbitrarily.

Example Schedule: $\tau_1 = (2, 8, 4)$, $\tau_2 = (1, 6, 6)$, $\tau_3 = (4, 12, 12)$. 

$[(C_i, T_i, D_i)]$
Optimality (or not) of RM and DM

Example Schedule: \( \tau_1 = (2, 4, 4), \tau_2 = (5, 10, 10) \)

The above system is schedulable.
Optimality (or not) of RM and DM

Example Schedule: $\tau_1 = (2, 4, 4), \tau_2 = (5, 10, 10)$

The above system is schedulable.

No static-priority scheme is optimal for scheduling periodic tasks: However, a deadline will be missed, regardless of how we choose to (statically) prioritize $\tau_1$ and $\tau_2$.

Corollary

Neither RM nor DM is optimal.
Properties of Worst-Case Response Time

Suppose that we are analyzing the worst-case response time of task $\tau_i$. Let us assume that the $n-1$ tasks are already verified to meet their deadlines.

- Suppose $t'$ is the arrival time of a job of task $\tau_i$.
- A higher priority task $\tau_j$ may release a job before $t'$ and this job is executed after $t'$. 
Properties of Worst-Case Response Time (cont.)

Let \( t_j \) be the arrival time of the first job of task \( \tau_j \) after or at time \( t' \).

- \( t_j \geq t' \).
- The remaining execution time of the job of task \( \tau_j \) arrived before \( t' \) and unfinished at time \( t' \) is at most \( C_j \).

Since fixed-priority scheduling greedily executes an available job, the system remains busy from \( t' \) till the time instant \( f \) at which task \( \tau_n \) finishes the job arrived at time \( t' \). That is,

\[
\forall t' < t < f, \quad C_i + \sum_{j=1}^{i-1} C_j + \sum_{j=1}^{i-1} \max \left\{ \left\lceil \frac{t - t_j}{T_j} \right\rceil C_j, 0 \right\} > t.
\]

As a result, \((t - t'\) is replaced by \(t)\)

\[
\forall 0 < t < f - t', \quad C_i + \sum_{j=1}^{i-1} C_j + \sum_{j=1}^{i-1} \left\lceil \frac{t - t_j}{T_j} \right\rceil C_j > t.
\]
Properties of Worst-Case Response Time (cont.)

The minimum $t$ such that

$$\forall 0 < t \leq T_i, \quad C_i + \sum_{j=1}^{i-1} C_j + \sum_{j=1}^{i-1} \left\lfloor \frac{t - t_j}{T_j} \right\rfloor C_j = t.$$ 

is a safe upper bound on the worst-case response time of task $\tau_i$.

Why do we need to constrain $t \leq T_i$?
Aside: Critical Instants

**Definition**

The critical instant of a task $\tau_i$ is a time instant such that:

1. the job of $\tau_i$ released at this instant has the maximum response time of all jobs in $\tau_i$, if the response time of every job of $\tau_i$ is at most $T_i$, the period of $\tau_i$, and

2. the response time of the job released at this instant is greater than $T_i$ if the response time of some job in $\tau_i$ exceeds $T_i$.

Informally, a critical instant of $\tau_i$ represents a worst-case scenario from $\tau_i$’s standpoint when we use static-priority scheduling.
Critical Instants in Static-Priority Systems

Theorem

[Liu and Layland, JACM 1973] The critical instant of task $\tau_i$ for a set of independent, preemptable periodic tasks with implicit deadlines is to release the first jobs of all the higher-priority tasks at the same time.

We are not saying that $\tau_1, \ldots, \tau_i$ will all necessarily release their first jobs at the same time, but if this does happen, we are claiming that the time of release will be a critical instant for task $\tau_i$. 
Critical Instants: Informal Proof

Shifting the release time of tasks together will increase the response time of task $\tau_i$.

- Consider a job of $\tau_i$, released at time $t'$, with completion time $t_R$.
- Let $t_{-1}$ be the latest *idle instant* for $\tau_1, \ldots, \tau_{i-1}$ at or before $t_R$.
- Let $J$ be $\tau_i$’s job released at $t'$. 
We will show that shifting the release time of tasks together will increase the response time of task $\tau_i$.

- Moving $J$ from $t'$ to $t_{-1}$ does not decrease the completion time of $J$. 
We will show that shifting the release time of tasks together will increase the response time of task $\tau_i$.

- Releasing $\tau_1$ at $t_{-1}$ does not decrease the completion time of $J$. 
We will show that shifting the release time of tasks together will increase the response time of task $\tau_i$.

- Releasing $\tau_2$ at $t_{-1}$ does not decrease the completion time of $J$.
- Repeating the above movement and proves the criticality of the critical instant
Necessary and Sufficient (Exact) RM-Schedulability

- Time-demand analysis (TDA)
- TDA can be applied to produce a schedulability test for any fixed-priority algorithm that ensures that each job of every task completes before the next job of that task is released.
- For some important task models and scheduling algorithms, this schedulability test will be necessary and sufficient.
Schedulability Condition

According to the critical instant theorem, to test the schedulability of task $\tau_i$, we have to

1. release all the higher-priority tasks at time 0 together with task $\tau_i$
2. release all the higher-priority task instances as early as they can

We can simply simulate the above behavior to verify whether task $\tau_i$ misses the deadline.
Schedulability Test

The time-demand function $W_i(t)$ of the task $\tau_i$ is defined as follows:

$$W_i(t) = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{t}{T_j} \right\rceil C_j.$$

**Theorem**

A system $\mathcal{T}$ of periodic, independent, preemptable tasks is schedulable on one processor by algorithm A if

$$\forall \tau_i \in \mathcal{T} \exists t \text{ with } 0 < t \leq D_i \text{ and } W_i(t) \leq t$$

holds. This condition is also necessary for synchronous, periodic task systems and also sporadic task sets.

Note that this holds for implicit-deadline and constrained-deadline task sets. The sufficient condition can be proved by contradiction.
How to Use TDA?

The theorem of TDA might look strong as it requires to check all the time \( t \) with \( 0 < t \leq D_i \) for a given \( \tau_i \). There are two ways to avoid this:

- Iterate using \( t(k + 1) := W_i(t(k)) \), starting with \( t(0) := \sum_{j=1}^{i} C_j \), and stopping when, for some \( \ell \), \( t(\ell) = W_i(t(\ell)) \) or \( t(\ell) > D_i \).

- Only consider \( t \in \{ \ell T_j - \epsilon \mid 1 \leq j \leq i, \ell \in \mathcal{N}^+ \} \), where \( \epsilon \) is a constant close to 0. That is, only consider \( t \) at which a job of higher-priority tasks arrives.
Optimality Among Static-Priority Algorithms

**Theorem**

A system $\mathcal{T}$ of $n$ independent, preemptable, synchronous periodic tasks with implicit deadlines can be feasibly scheduled on one processor according to the RM algorithm whenever it can be feasibly scheduled according to any static priority algorithm.

The proof is omitted. It can be proved by using the critical instant theorem and the TDA analysis.
Harmonic Real-Time Systems

**Definition**

A system of periodic tasks is said with harmonic periods (also: *simply periodic*) if for every pair of tasks $\tau_i$ and $\tau_k$ in the system where $T_i < T_k$, $T_k$ is an integer multiple of $T_i$.

For example: Periods are 2, 6, 12, 24.

**Theorem**

[Kuo and Mok]: A system $\mathcal{T}$ of harmonic, independent, preemptable, and implicit-deadline tasks is schedulable on one processor according to the RM algorithm if and only if its total utilization $U(\mathcal{T}) = \sum_{\tau_j \in \mathcal{T}} \frac{C_j}{T_j}$ is less than or equal to 1.
Proof for Harmonic Systems

The case for the “only-if” part is skipped.

By using the contrapositive proof approach, suppose that \( T \) is not schedulable and \( \tau_i \) misses its deadline. We will prove that the utilization must be larger than 1.

- The response time of \( \tau_i \) is larger than \( D_i \).
- By critical instants, releasing all the tasks \( \tau_1, \tau_2, \ldots, \tau_i \) at time 0 will lead to a response time of \( \tau_i \) larger than \( D_i \).
Proof for Harmonic Systems (cont.)

As the schedule is workload-conserving, we know that from time 0 to time $D_i$, the whole system is executing jobs. Therefore,

$D_i < \text{the workload released in time interval } [0, D_i)$

$$= \sum_{j=1}^{i} C_j \cdot (\text{the number of job releases of } \tau_j \text{ in time interval } [0, D_i))$$

$$= \sum_{j=1}^{i} C_j \cdot \left\lfloor \frac{D_i}{T_j} \right\rfloor =^* \sum_{j=1}^{i} C_j \cdot \frac{D_i}{T_j},$$

where $=^* \text{ is because } D_i = T_i \text{ is an integer multiple of } T_j \text{ when } j \leq i.$
Proof for Harmonic Systems (cont.)

As the schedule is workload-conserving, we know that from time 0 to time $D_i$, the whole system is executing jobs. Therefore,

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\sum_{j=1}^{i} C_j \cdot (\text{the number of job releases of } \tau_j \text{ in time interval } [0, D_i))
\]

\[
= \sum_{j=1}^{i} C_j \cdot \left\lceil \frac{D_i}{T_j} \right\rceil =^* \sum_{j=1}^{i} C_j \cdot \frac{D_i}{T_j},
\]

where $=^*$ is because $D_i = T_i$ is an integer multiple of $T_j$ when $j \leq i$.

By canceling $D_i$, we reach the contradiction by having

\[
1 < \sum_{j=1}^{i} \frac{C_j}{T_j} \leq \sum_{\tau_j \in \mathcal{T}} \frac{C_j}{T_j}.
\]
Utilization-Based Schedulability Test

- Task utilization:
  \[ U_i := \frac{C_i}{T_i}. \]

- System (total) utilization:
  \[ U(\mathcal{T}) := \sum_{\tau_i \in \mathcal{T}} \frac{C_i}{T_i}. \]

A task system \( \mathcal{T} \) fully utilizes the processor under scheduling algorithm \( A \) if any increase in execution time (of any task) causes \( A \) to miss a deadline. In this case, \( U(\mathcal{T}) \) is an upper bound on utilization for \( A \), denoted \( U_{ub}(\mathcal{T}, A) \).

\( U_{lub}(A) \) is the least upper bound for algorithm \( A \):
\[ U_{lub}(A) = \min_{\mathcal{T}} U_{ub}(\mathcal{T}, A) \]
What is $U_{ub}(A)$ for?
Liu and Layland Bound

**Theorem**

[Liu and Layland] A set of $n$ independent, preemptable periodic tasks with implicit deadlines can be scheduled on a processor according to the RM algorithm if its total utilization $U$ is at most $n(2^{\frac{1}{n}} - 1)$. In other words,

$$U_{lub}(RM, n) = n(2^{\frac{1}{n}} - 1) \geq 0.693.$$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$U_{lub}(RM, n)$</th>
<th>$n$</th>
<th>$U_{lub}(RM, n)$</th>
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<tbody>
<tr>
<td>2</td>
<td>0.828</td>
<td>3</td>
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<tr>
<td>4</td>
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<td>5</td>
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<td>6</td>
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<td>8</td>
<td>0.724</td>
<td>9</td>
<td>0.720</td>
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<tr>
<td>10</td>
<td>0.717</td>
<td>$\infty$</td>
<td>0.693 = ln2</td>
</tr>
</tbody>
</table>
Least Upper Bound

![Graph showing least upper bound vs. n]

Prof. Dr. Jian-Jia Chen (LS 12, TU Dortmund)
Comparison between RM and EDF (Implicit Deadlines)

<table>
<thead>
<tr>
<th>RM</th>
<th>EDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Low run-time overhead: $O(1)$ with priority sorting in advance</td>
<td>• High run-time overhead: $O(\log n)$ with balanced binary tree</td>
</tr>
<tr>
<td>• Optimal for static-priority</td>
<td>• Optimal for dynamic-priority</td>
</tr>
<tr>
<td>• Schedulability test is $NP$-hard (even if the relative deadline = period)</td>
<td>• Schedulability test is easy (when the relative deadline = period)</td>
</tr>
<tr>
<td>• Least upper bound: 0.693</td>
<td>• Least upper bound: 1</td>
</tr>
<tr>
<td>• In general, more preemption</td>
<td>• In general, less preemption</td>
</tr>
</tbody>
</table>