Real-Time Calculus

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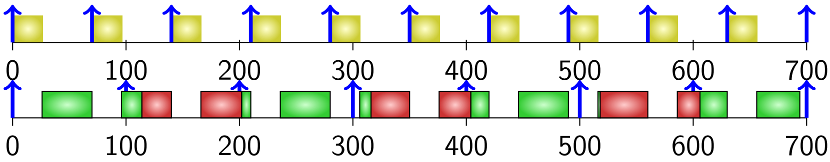
Arbitrary Deadlines

The worst-case response time of τ_i by only considering the first job of τ_i at the critical instant is too optimistic when the relative deadline of τ_i is larger than the period.



Consider two tasks:

- τ_1 has period 70 and execution time 26 and τ_2 is with period 100 and execution time 62.
- τ_2's seven jobs have the following response times, respectively: 114, 102, 116, 104, 118, 106, 94.
- Note that the first job's response time is not the longest.

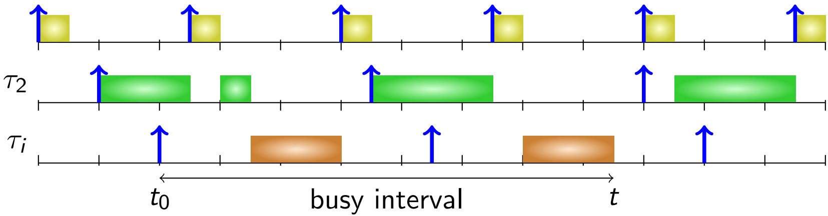
Busy Intervals

Definition

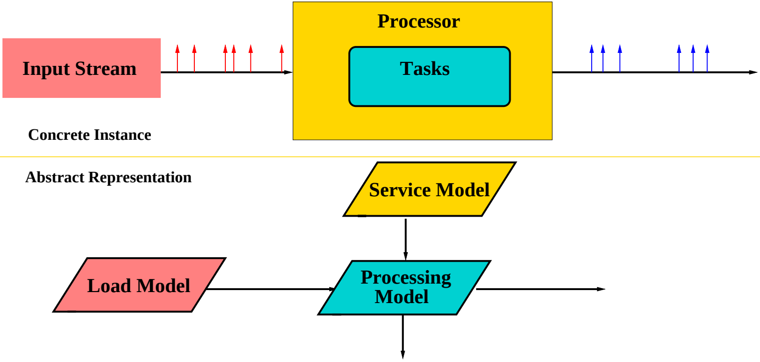
A τ_i-level busy interval $(t_0, t]$ of task τ_i begins at an instant t_0 when

1. all jobs in τ_i released before t have completed, and
2. a job of τ_i releases.

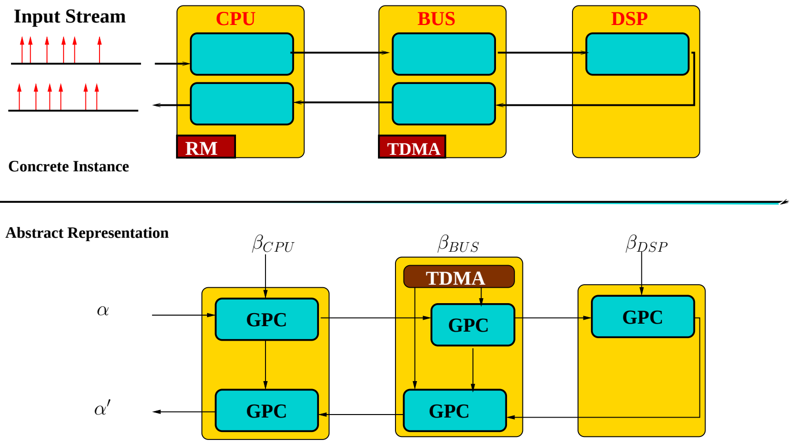
The interval ends at the first instant t after t0 when all jobs in τ_i released since t_0 are complete.



Abstract Models for Real-Time Calculus



Abstract Models for Module Performance Analysis



Overview

System View	Module Performance Analysis (MPA)
Math. View	Real-Time Calculus (RTC)
	Min-Plus Calculus, Max-Plus Calculus

Backgrounds

- Real-Time Calculus can be regarded as a worst-case/best-case variant of classical queuing theory. It is a formal method for the analysis of distributed real-time embedded systems.
- Related Work:
  - Min-Plus Algebra: F. Baccelli, G. Cohen, G. J. Olster, and J. P. Quadrat, Synchronization and Linearity —An Algebra for Discrete Event Systems, Wiley, New York, 1992.
  - Network Calculus: J.-Y. Le Boudec and P. Thiran, Network Calculus -A Theory of Deterministic Queuing Systems for the Internet, Lecture Notes in Computer Science, vol. 2050, Springer Verlag, 2001.

Definition of Arrival Curves and Service Curves

- For a specific trace:
  - Data streams: $R(t)$ = number of events in $[0, t)$
  - Resource stream: $C(t)$ = available resource in $[0, t)$
- For the worst cases and the best cases in any interval with length Δ:
  - Arrival Curve $[\alpha^l, \alpha^u]$:

$$\alpha^l(\Delta) = \inf_{\lambda \geq 0, \forall R} \{R(\Delta + \lambda) - R(\lambda)\}$$

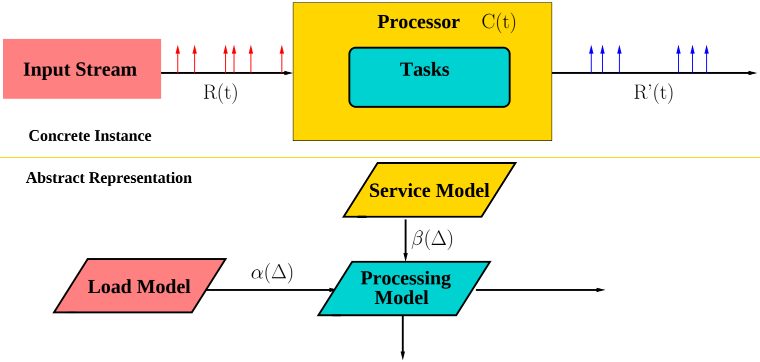
$$\alpha^u(\Delta) = \sup_{\lambda \geq 0, \forall R} \{R(\Delta + \lambda) - R(\lambda)\}$$

  - Service Curve $[\beta^l, \beta^u]$:

$$\beta^l(\Delta) = \inf_{\lambda \geq 0, \forall C} \{C(\Delta + \lambda) - C(\lambda)\}$$

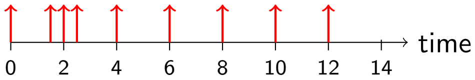
$$\beta^u(\Delta) = \sup_{\lambda \geq 0, \forall C} \{C(\Delta + \lambda) - C(\lambda)\}$$

Abstract Models for Real-Time Calculus



Arrival Curve: An Example

Use a sliding window to get the upper bound of the number of events in a specified interval length.



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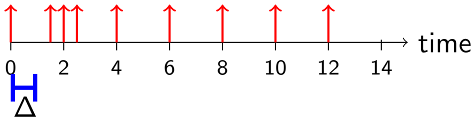

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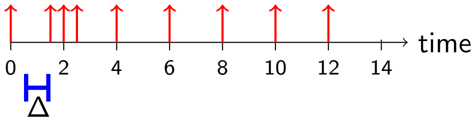

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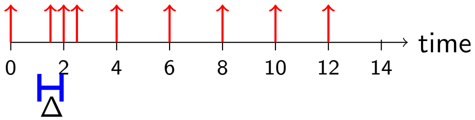

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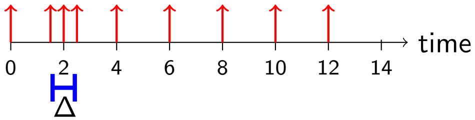

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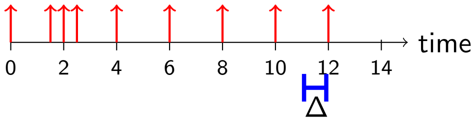

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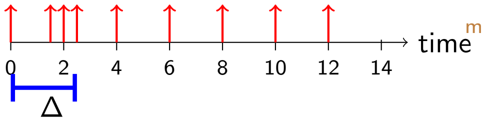

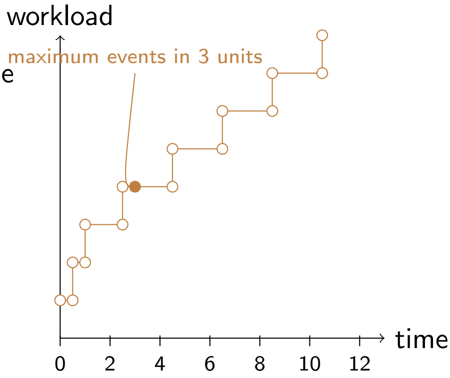

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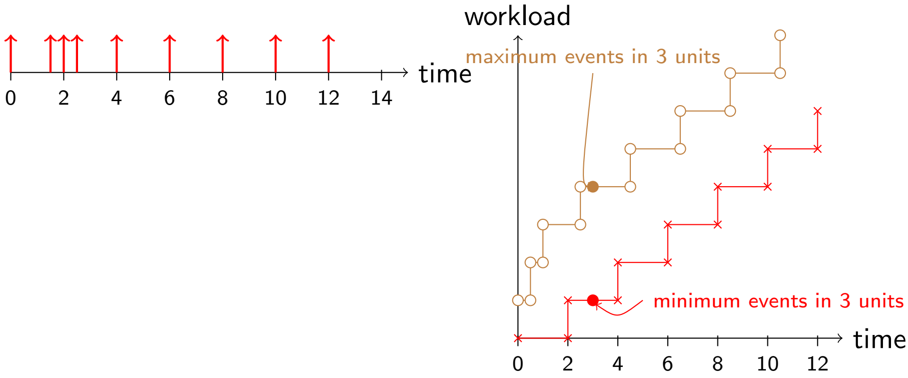

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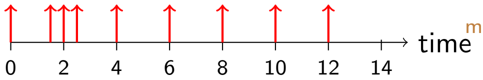
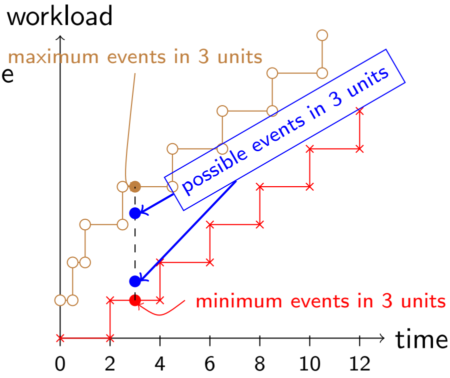
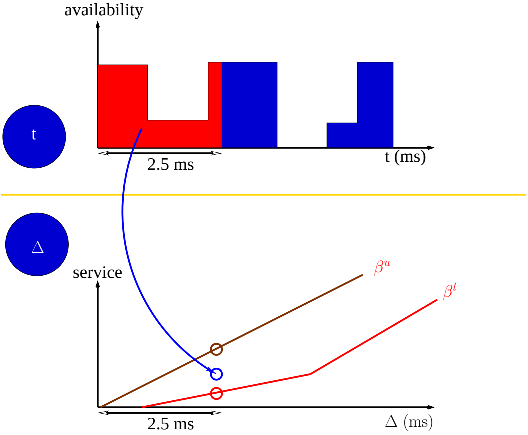




Service Curve: An Example

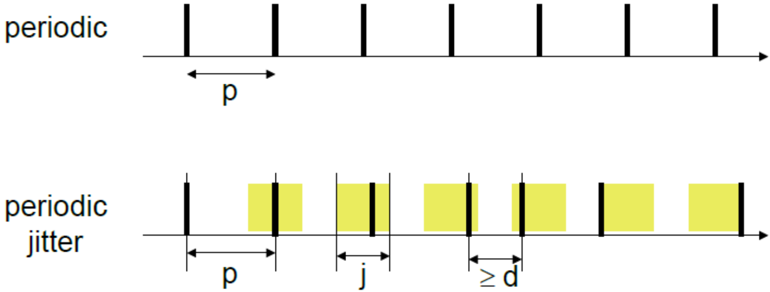
Resource Availability



Service Curves
$\beta = [\beta^l, \beta^u]$

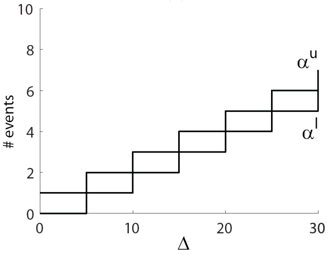
Example 1: Periodic with Jitter

A common event pattern that is used in literature can be specified by the parameter triple (p, j, d), where p denotes the period, j the jitter, and d the minimum inter-arrival distance of events in the modeled stream.



Example 1: Periodic with Jitter

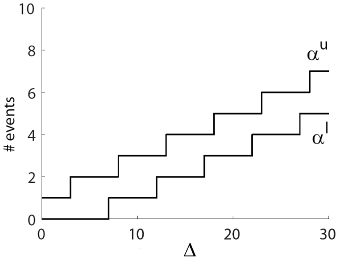
Periodic



$$\alpha^u(\Delta) = \left\lceil \frac{\Delta}{p} \right\rceil$$

$$\alpha^l(\Delta) = \left\lfloor \frac{\Delta}{p} \right\rfloor$$

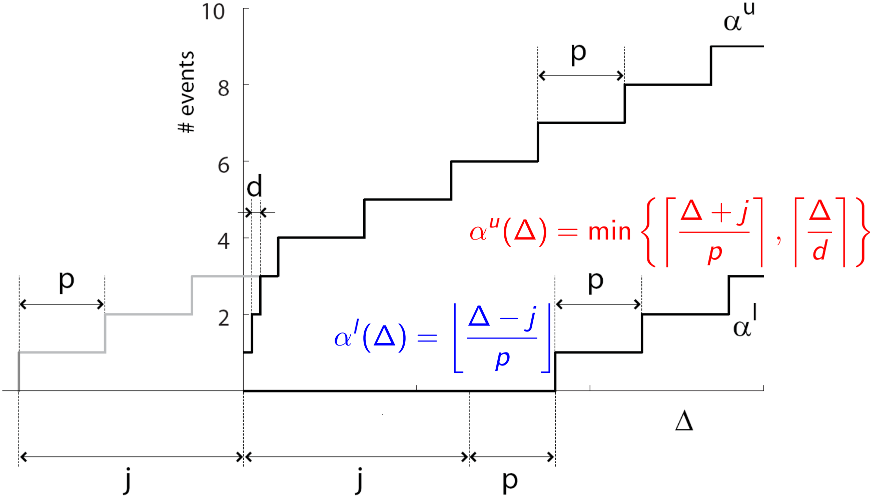
Periodic with Jitter



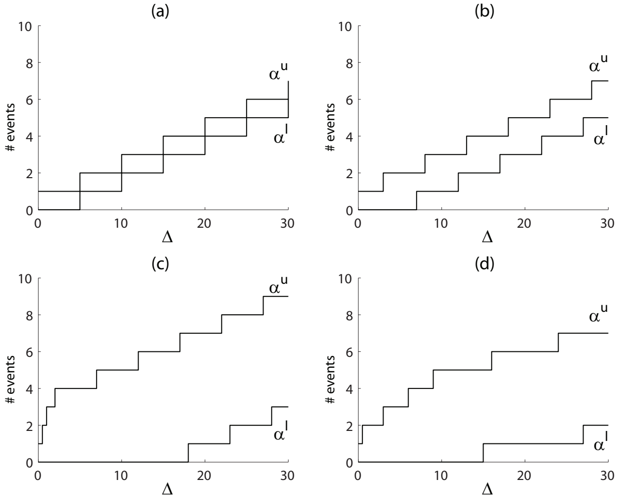
$$\alpha^u(\Delta) = \left\lceil \frac{\Delta + j}{p} \right\rceil$$

$$\alpha^l(\Delta) = \left\lfloor \frac{\Delta - j}{p} \right\rfloor$$

Example 1: Periodic with Jitter

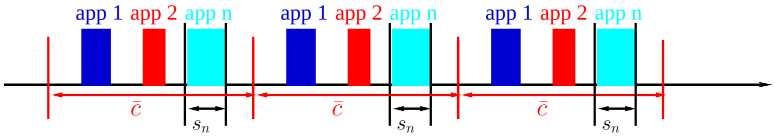


More Examples on Arrival Curves



Example 2: TDMA Resource

- Consider a real-time system consisting of n applications that are executed on a resource with bandwidth B that controls resource access using a TDMA (Time Division Multiple Access) policy.
- Analogously, we could consider a distributed system with n communicating nodes, that communicate via a shared bus with bandwidth B, with a bus arbitrator that implements a TDMA policy.
- TDMA policy: In every TDMA cycle of length $\bar{c}$, one single resource slot of length s_i is assigned to application i.



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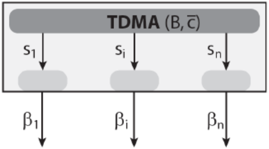
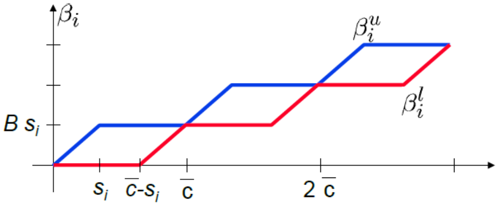
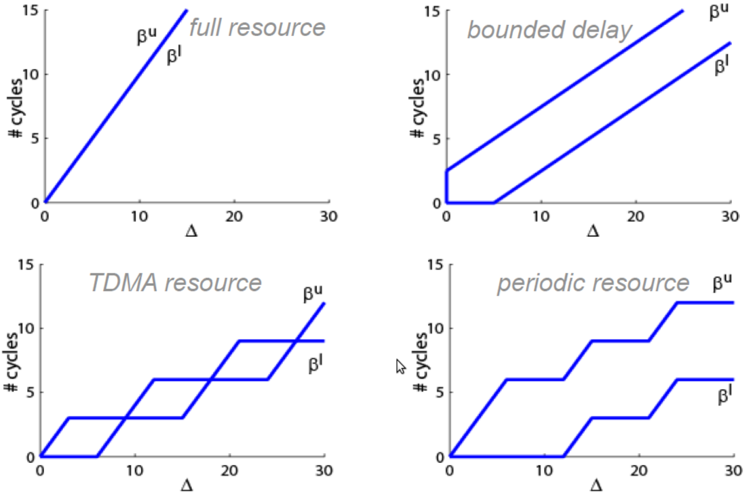




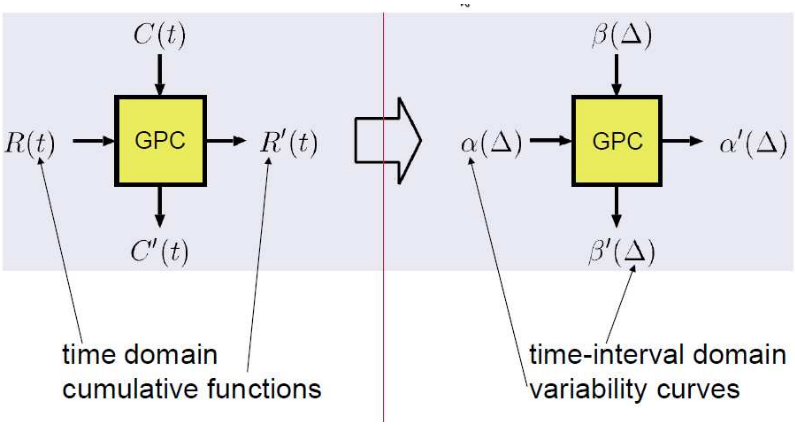
$$\beta^u(\Delta) = B \min\left\{ \left\lceil \frac{\Delta}{\bar{c}} \right\rceil s_i, \Delta - \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor (\bar{c} - s_i) \right\}$$

$$\beta^l(\Delta) = B \max\left\{ \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor s_i, \Delta - \left\lceil \frac{\Delta}{\bar{c}} \right\rceil (\bar{c} - s_i) \right\}$$

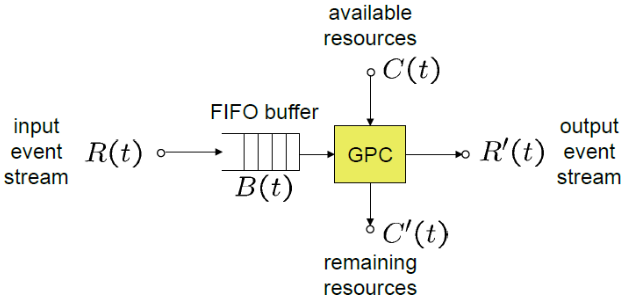
More Examples on Service Curves



Abstraction



Greedy Processing Component (GPC)



- Component is triggered by incoming events.
- A fully preemptable task is instantiated at every event arrival to process the incoming event.
- Active tasks are processed in a greedy fashion in FIFO order.
- Processing is restricted by the availability of resources.

Some Relations (only for your reference)

- The output upper arrival curve of a component satisfies

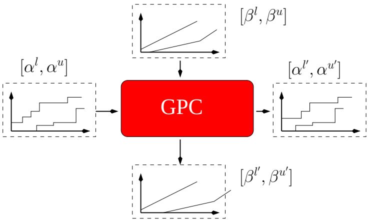
$$\alpha^{u\prime} \leq (\alpha^u \oslash \beta^l)$$

  with a simple and pessimistic calculation.
- The remaining lower service curve of a component satisfies

$$\beta^{l\prime}(\Delta) = \sup_{0 \leq \lambda \leq \Delta} (\beta^l(\lambda) - \alpha^u(\lambda))$$

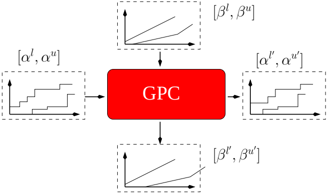
More Relations (only for your reference)

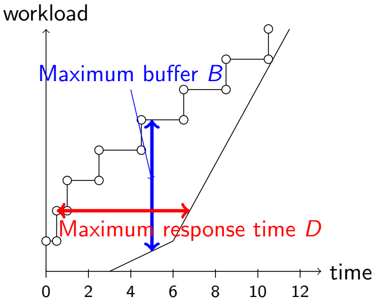
$$\alpha^{u'} = [(\alpha^u \otimes \beta^u) \oslash \beta^l] \wedge \beta^u$$
$$\alpha^{l'} = [(\alpha^u \oslash \beta^l) \otimes \beta^l] \wedge \beta^l$$
$$\beta^{u'} = (\beta^u - \alpha^l)\bar{\oslash}0$$
$$\beta^{l'} = (\beta^l - \alpha^u)\bar{\otimes}0$$



Without formal proofs....

Graphical Interpretation



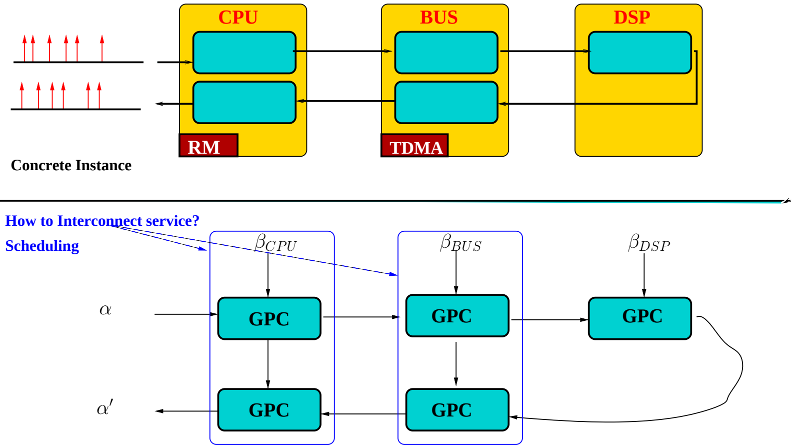


$$B = \sup_{t \geq 0}\{R(t) - R'(t)\} \leq \sup_{\lambda \geq 0}\{\alpha^u(\lambda) - \beta^l(\lambda)\}$$

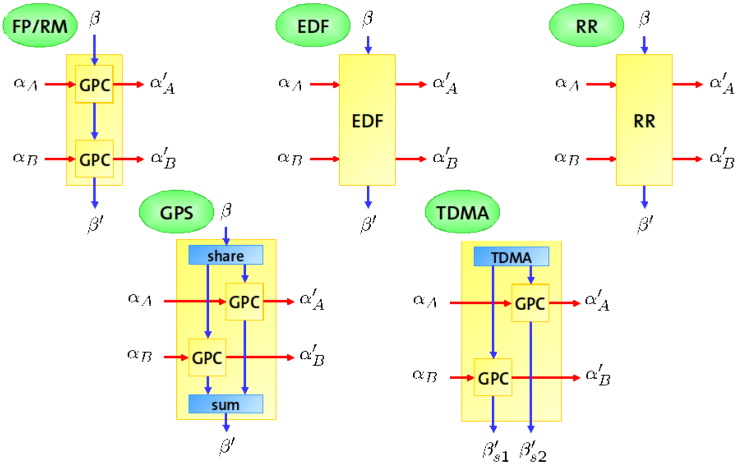
$$D = \sup_{t \geq 0}\{\inf\{\tau \geq 0 : R(t) \leq R'(t+\tau)\}\}$$

$$= \sup_{\Delta \geq 0}\{\inf\{\tau \geq 0 : \alpha^u(\Delta) \leq \beta^l(\Delta+\tau)\}\}$$

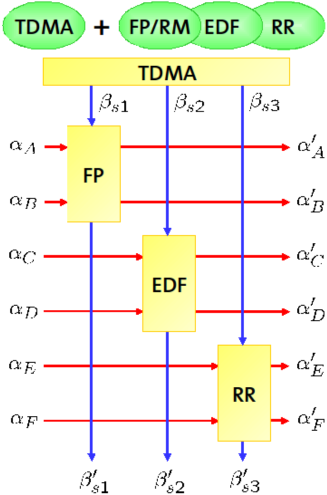
System Composition

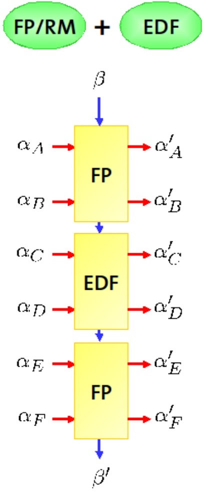


Scheduling and Arbitration



Mixed Hierarchical Scheduling

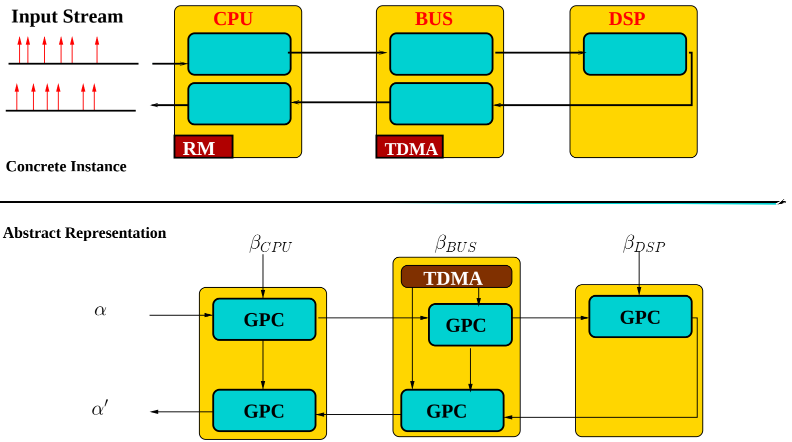




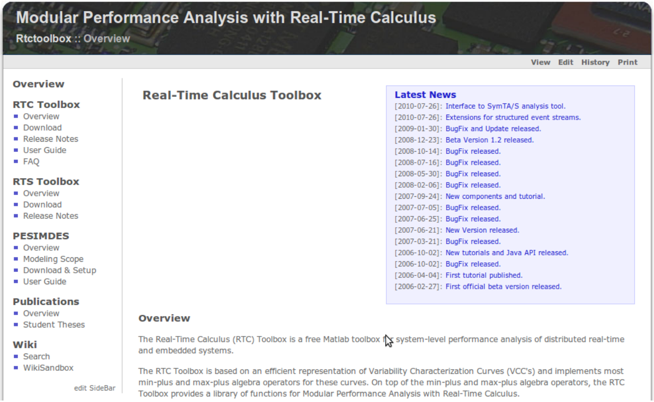
...and many other combinations:

- RR + EDF
- FP/RM + RR
- FP/RM + GPS
- GPS + EDF
- ...

Complete System Composition



RTC Toolbox (http://www.mpa.ethz.ch/Rtctoolbox)



Advantages and Disadvantages of RTC and MPA

- Advantages
  - More powerful abstraction than “classical” real-time analysis
  - Resources are first-class citizens of the method
  - Allows composition in terms of (a) tasks, (b) streams, (c) resources, (d) sharing strategies.
- Disadvantages
  - Needs some effort to understand and implement
  - Extension to new arbitration schemes not always simple
  - *Not applicable for schedulers that change the scheduling policies dynamically.*