Mapping of Applications to Platforms

Jian-Jia Chen
(slides are based on Peter Marwedel)
TU Dortmund, Informatik 12
Germany

2016年 01 月 26 日

These slides use Microsoft clip arts. Microsoft copyright restrictions apply.
Structure of this course

2: Specification
3: ES-hardware
4: system software (RTOS, middleware, …)

Design repository

6: Application mapping
7: Optimization
5: Evaluation & validation & (energy, cost, performance, …)

8: Test

Numbers denote sequence of chapters
Mapping of Applications to Platforms

© Renesas, Thiele
## Distinction between mapping problems

<table>
<thead>
<tr>
<th></th>
<th>Embedded</th>
<th>PC-like</th>
</tr>
</thead>
<tbody>
<tr>
<td>Architectures</td>
<td>Frequently heterogeneous very compact</td>
<td>Mostly homogeneous not compact (x86 etc)</td>
</tr>
<tr>
<td>x86 compatibility</td>
<td>Less relevant</td>
<td>Very relevant</td>
</tr>
<tr>
<td>Architecture fixed?</td>
<td>Sometimes not</td>
<td>Yes</td>
</tr>
<tr>
<td>Model of computation (MoCs)</td>
<td>C+multiple models (data flow, discrete events, ...)</td>
<td>Mostly von Neumann (C, C++, Java)</td>
</tr>
<tr>
<td>Optim. objectives</td>
<td>Multiple (energy, size, ...)</td>
<td>Average performance dominates</td>
</tr>
<tr>
<td>Real-time relevant</td>
<td>Yes, very!</td>
<td>Hardly</td>
</tr>
<tr>
<td>Applications</td>
<td>Several concurrent apps.</td>
<td>Mostly single application</td>
</tr>
<tr>
<td>Apps. known at design time</td>
<td>Most, if not all</td>
<td>Only some (e.g. WORD)</td>
</tr>
</tbody>
</table>
Problem Description

**Given**
- A set of applications
- Scenarios on how these applications will be used
- A set of candidate architectures comprising
  - (Possibly heterogeneous) processors
  - (Possibly heterogeneous) communication architectures
  - Possible scheduling policies

**Find**
- A mapping of applications to processors
- Appropriate scheduling techniques (if not fixed)
- A target architecture (if DSE is included)

**Objectives and constraints**
- deadlines, temperatures
- Cost, performance, energy, reliability
Related Work

- Mapping to ECUs in automotive design
- Scheduling theory:
  Provides insight for the mapping task $\rightarrow$ start times
- Hardware/software partitioning:
  Can be applied if it supports multiple processors
- High performance computing (HPC)
  Automatic parallelization, but only for
  - single applications,
  - fixed architectures,
  - no support for scheduling,
  - memory and communication model usually different
- High-level synthesis
  Provides useful terms like scheduling, allocation, assignment
- Optimization theory
Scope of mapping algorithms

Useful terms from hardware synthesis:

- **Resource Allocation**
  Decision concerning type and number of available resources

- **Resource Assignment**
  Mapping: Task $\rightarrow$ (Hardware) Resource

- **xx to yy binding:**
  Describes a mapping from behavioral to structural domain, e.g. task to processor binding, variable to memory binding

- **Scheduling**
  Mapping: Tasks $\rightarrow$ Task start times
  Sometimes, resource assignment is considered being included in scheduling.
Classes of mapping algorithms considered in this course

- Classical scheduling algorithms
  Mostly for independent tasks & ignoring communication, mostly for mono- and homogeneous multiprocessors (EDF, EDD, RM, DM, etc.)

- Dependent tasks as considered in architectural synthesis
  Initially designed in different context, but applicable

- Hardware/software partitioning
  Dependent tasks, heterogeneous systems, focus on resource assignment

- Design space exploration using genetic algorithms
  Heterogeneous systems, incl. communication modeling
Classes of mapping algorithms considered in this course

- **Classical scheduling algorithms**
  Mostly for independent tasks & ignoring communication, mostly for mono- and homogeneous multiprocessors (EDF, EDD, RM, DM, etc.)

- **Dependent tasks as considered in architectural synthesis**
  Initially designed in different context, but applicable

- **Hardware/software partitioning**
  Dependent tasks, heterogeneous systems, focus on resource assignment

- **Design space exploration using genetic algorithms**
  Heterogeneous systems, incl. communication modeling
Scheduling with precedence constraints

Task graph and possible schedule:
Simultaneous Arrival Times: The Latest Deadline First (LDF) Algorithm

LDF [Lawler, 1973]: reads the task graph and among the tasks with no successors inserts the one with the latest deadline into a queue. It then repeats this process, putting tasks whose successor have all been selected into the queue.

At run-time, the tasks are executed in an order opposite to the generated total order.

LDF is non-preemptive and is optimal for mono-processors.

If no local deadlines exist, LDF performs just a topological sort.
Asynchronous Arrival Times: Modified EDF Algorithm

This case can be handled with a modified EDF algorithm. The key idea is to transform the problem from a given set of dependent tasks into a set of independent tasks with different timing parameters [Chetto90]. This algorithm is optimal for mono-processor systems.

If preemption is not allowed, the heuristic algorithm developed by Stankovic and Ramamritham can be used.
Dependent tasks

The problem of deciding whether or not a schedule exists for a set of dependent tasks and a given deadline is NP-complete in a strong sense in general [Garey/Johnson].

Strategies:

1. Add resources, so that scheduling becomes easier

2. Split problem into static and dynamic part so that only a minimum of decisions need to be taken at run-time.

3. Use scheduling algorithms from high-level synthesis
Task graph

Assumption: execution time = 1 for all tasks
As soon as possible (ASAP) scheduling

**ASAP**: All tasks are scheduled as early as possible

Loop over (integer) time steps:
- Compute the set of unscheduled tasks for which all predecessors have finished their computation
- Schedule these tasks to start at the current time step.
As soon as possible (ASAP) scheduling: Example
As-late-as-possible (ALAP) scheduling

ALAP: All tasks are scheduled as late as possible

Start at last time step*:

Schedule tasks with no successors and tasks for which all successors have already been scheduled.

* Generate a list, starting at its end
As-late-as-possible (ALAP) scheduling: Example
(Resource constrained) List Scheduling

List scheduling: extension of ALAP/ASAP method

Preparation:

- Topological sort of task graph \( G=(V,E) \)
- Computation of priority of each task:

Possible priorities \( u \):

- Number of successors
- Longest path
- \textbf{Mobility} = \( \tau \) (ALAP schedule) - \( \tau \) (ASAP schedule)

Source: Teich: Dig. HW/ SW Systeme
Mobility as a priority function

Mobility is not very precise
Algorithm

List($G(V,E)$, $B$, $u$){
    $i := 0$;
    repeat {
        Compute set of candidate tasks $A_i$;
        Compute set of not terminated tasks $G_i$;
        Select $S_i \subseteq A_i$ of maximum priority $r$ such that
        $|S_i| + |G_i| \leq B$ (*resource constraint*)
        foreach ($v_j \in S_i$): $\tau(v_j):=i$; (*set start time*)
        $i := i + 1$;
    }
    until (all nodes are scheduled);
    return ($\tau$);
}
Example

Assuming $B = 2$, unit execution time and $u : \text{path length}$

$u(a) = u(b) = 4$
$u(c) = u(f) = 3$
$u(d) = u(g) = u(h) = u(j) = 2$
$u(e) = u(i) = u(k) = 1$

$\forall \ i : G_i = 0$

Modified example based on J. Teich
(Time constrained) Force-directed scheduling

- Goal: balanced utilization of resources
- Based on spring model;
- Originally proposed for high-level synthesis

Evaluation of HLS-Scheduling

- Focus on considering dependencies
- Mostly heuristics, few proofs on optimality
- Not using global knowledge about periods etc.
- Considering discrete time intervals
- Variable execution time available only as an extension
- Includes modeling of heterogeneous systems
### Overview

Scheduling of aperiodic tasks with real-time constraints:
Table with some known algorithms:

<table>
<thead>
<tr>
<th></th>
<th>Equal arrival times; non-preemptive</th>
<th>Arbitrary arrival times; preemptive</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Independent tasks</strong></td>
<td>EDD (Jackson)</td>
<td>EDF (Horn)</td>
</tr>
<tr>
<td><strong>Dependent tasks</strong></td>
<td>LDF (Lawler), ASAP, ALAP, LS, FDS</td>
<td>EDF* (Chetto)</td>
</tr>
</tbody>
</table>
Conclusion

- HLS-based scheduling
  - ASAP
  - ALAP
  - *List scheduling (LS)*
  - *Force-directed scheduling (FDS)*

- Evaluation
Classes of mapping algorithms considered in this course

- Classical scheduling algorithms
  Mostly for independent tasks & ignoring communication, mostly for mono- and homogeneous multiprocessors (EDF, EDD, RM, DM, etc.)

- Dependent tasks as considered in architectural synthesis
  Initially designed in different context, but applicable

- Hardware/software partitioning
  Dependent tasks, heterogeneous systems, focus on resource assignment

- Design space exploration using genetic algorithms
  Heterogeneous systems, incl. communication modeling
Hardware/software partitioning

No need to consider special hardware in the future?

Correct for fixed functionality, but wrong in general: “By the time MPEG-\(n\) can be implemented in software, MPEG-\(n+1\) has been invented” [de Man]

Functionality to be implemented in software or in hardware?
Functionality to be implemented in software or in hardware?

Decision based on hardware/software partitioning, a special case of hardware/software codesign.
Codesign Tool (COOL) as an example of HW/SW partitioning

Inputs to COOL:

1. Target technology
2. Design constraints
3. Required behavior
Hardware/software codesign: approach

Specification

Mapping

Steps of the COOL partitioning algorithm (1)

1. Translation of the behavior into an internal graph model
2. Translation of the behavior of each node from VHDL into C
3. Compilation
   • All C programs compiled for the target processor,
   • Computation of the resulting program size,
   • Estimation of the resulting execution time (simulation input data might be required)
4. Synthesis of hardware components:
   ∀ leaf nodes, application-specific hardware is synthesized.
   High-level synthesis sufficiently fast.
5. Flattening of the hierarchy:
   • Granularity used by the designer is maintained.
   • Cost and performance information added to the nodes.
   • Precise information required for partitioning is pre-computed

6. Generating and solving a mathematical model of the optimization problem:
   • Integer linear programming ILP model for optimization. Optimal with respect to the cost function (approximates communication time)
Steps of the COOL partitioning algorithm (3)

7. **Iterative improvements:**
   Adjacent nodes mapped to the same hardware component are now merged.

![Diagram showing iterative improvements in COOL partitioning algorithm]
Steps of the COOL partitioning algorithm (4)

8. Interface synthesis:
   After partitioning, the glue logic required for interfacing processors, application-specific hardware and memories is created.
Example

Hardware/Software Configurations

- Running on FPGA requires $C_i$ amount of configurable logic blocks (CLBs) and results in execution time $t_{i,h}$ (purely on FPGA)
- Running on the software (uniprocessor) requires $t_{i,s}$ (purely on software)

What is the minimum number of CLBs required for the task graph when the deadline is set to $D$?
An ILP model for HW/SW partitioning

- $X_v$: =1 if node $v$ is mapped to FPGA and 0 otherwise.
- Cost function: minimize $\sum_{v \in V} C_v \cdot X_v$
- Constraints:
  - Let $F_i = t_{i,h} \cdot X_i + t_{i,s} \cdot (1-X_i)$
  - If $X_2 = X_3 = 0$, then the finishing time is
    - $F_1 + F_2 + F_3 + F_4$
  - If $X_2 = X_3 = 1$, then the finishing time is
    - $F_1 + max\{F_2, F_3\} + F_4$
  - If $X_2 = 1$ and $X_3 = 0$, then the finishing time is
    - $F_1 + max\{F_2, F_3\} + F_4$
  - If $X_2 = 0$ and $X_3 = 1$, then the finishing time is
    - $F_1 + max\{F_2, F_3\} + F_4$
An ILP model for HW/SW partitioning

- $X_v$: =1 if node $v$ is mapped to FPGA and 0 otherwise.
- Cost function: minimize $\sum_v C_v X_v$
- Constraints:
  - Let $F_i = t_{i,h}X_i + t_{i,s}(1-X_i)$
  - If $X_2+X_3 = 0$, then the finishing time is
    - $F_1 + F_2 + F_3 + F_4$
  - If $X_2+X_3 \geq 1$, then the finishing time is
    - $F_1 + \max\{F_2, F_3\} + F_4$
- Logical Constraints:
  - $(X_2 \text{ OR } X_3)$ implies $F_1 + \max\{F_2, F_3\} + F_4 \leq D$
  - $\neg(X_2 \text{ OR } X_3)$ implies $F_1 + F_2 + F_3 + F_4 \leq D$
Transforming Nonlinear Operation “max” (only for your reference)

- \( G \leq \max\{F_2, F_3\} \)
  - Let \( f \) be a sufficiently large positive integer (i.e., \( 1000000D \))
  - Let \( z \) be a binary variable, either 0 or 1
  - It can be formulated by using the following four linear constraints:
    - \( F_2 \leq F_3 + fz \)
    - \( F_3 \leq F_2 + f(1-z) \)
    - \( G \leq F_3 + fz \)
    - \( G \leq F_2 + f(1-z) \)
Logical Operations “AND/OR/NOT/Implication” (only for your reference)

- Logical $x_1$ AND $x_2$:
  - Use the linear constraints $y_1 \geq x_1 + x_2 - 1$, $y_1 \leq x_1$, $y_1 \leq x_2$, $0 \leq y_1 \leq 1$, where $y_1$ is constrained to be an integer. This enforces the desired relationship.

- Logical $x_1$ OR $x_2$:
  - Use the linear constraints $y_2 \leq x_1 + x_2$, $y_2 \geq x_1$, $y_2 \geq x_2$, $0 \leq y_2 \leq 1$, where $y_2$ is constrained to be an integer.

- Logical NOT $x_1$:
  - Use $y_3 = 1 - x_1$.

- Logical implication: To express $y_4 = (x_1 \Rightarrow x_2)$ (i.e., $y_4 = \neg x_1 \lor x_2$), we can adapt the construction for logical OR.
  - Use the linear constraints $y_4 \leq 1 - x_1 + x_2$, $y_4 \geq 1 - x_1$, $y_4 \geq x_2$, $0 \leq y_4 \leq 1$, where $y_4$ is constrained to be an integer.
Separation of scheduling and partitioning

Combined scheduling/partitioning very complex;

Heuristic: Compute estimated schedule
- Perform partitioning for estimated schedule
- Perform final scheduling
- If final schedule does not meet time constraint, go to 1 using a reduced overall timing constraint.

Actual execution time
specification
approx. execution time

1st Iteration
2nd Iteration

New specification
approx. execution time

Actual execution time
HW/SW partitioning in the context of mapping applications to processors

- Handling of heterogeneous systems
- Handling of task dependencies
- Considers of communication (at least in COOL)
- Considers memory sizes etc (at least in COOL)
- For COOL: just homogeneous processors
- No link to scheduling theory
SPARE Slides
Phase 1: Generation of ASAP and ALAP Schedule

\[ \tau = 0 \]
\[ \tau = 1 \]
\[ \tau = 2 \]
\[ \tau = 3 \]
\[ \tau = 4 \]
\[ \tau = 5 \]

\[ a \]
\[ b \]
\[ c \]
\[ d \]
\[ e \]
\[ f \]
\[ g \]
\[ h \]
\[ i \]
\[ j \]
\[ k \]
\[ l \]
\[ m \]
\[ n \]
\[ z \]
Next: computation of “forces”

- Direct forces push each task into the direction of lower values of $D(i)$.
- Impact of direct forces on dependent tasks taken into account by indirect forces
- Balanced resource usage $\approx$ smallest forces
- For our simple example and time constraint=6: result = ALAP schedule
Scheduling – An example

Solve the differential equation
\[ y'' + 3zy' + 3y = 0 \]

This can be calculated using this iterative algorithm

\[
\text{while}(z < a) \text{ repeat} \\
\quad z_l := z + dz; \\
\quad u_l := u - (3 \cdot z \cdot u \cdot dz) - (3 \cdot y \cdot dz); \\
\quad y_l := y + (u \cdot dz); \\
\quad z := z_l; \\
\quad u := u_l; \\
\quad y := y_l;
\]
1. Compute time frames $R(j)$;
2. Compute “probability“ $P(j,i)$ of assignment $j \rightarrow i$

$R(j) = \{\text{ASAP-control step} \ldots \text{ALAP-control step}\}$

$$P(j,i) = \begin{cases} \frac{1}{|R(j)|} & \text{if } i \in R(j) \\ 0 & \text{otherwise} \end{cases}$$
3. Compute “distribution” $D(i)$
(# Operations in control step $i$)

\[ D(i) = \sum_{j, \text{type}(j) \in H} P(j, i) \]

\[ D(1) = 2 \frac{5}{6} \]
\[ D(2) = 2 \frac{2}{6} \]
\[ D(3) = 5 \frac{1}{6} \]
\[ D(4) = 0 \]
4. Compute direct forces (1)

- $\Delta P_i(j, i')$: $\Delta$ for force on task $j$ in time step $i'$, if $j$ is mapped to time step $i$.

The new probability for executing $j$ in $i$ is 1; the previous was $P(j, i)$.

The new probability for executing $j$ in $i' \neq i$ is 0; the previous was $P(j, i)$.

\[
\Delta P_i(j, i') = \begin{cases} 
1 - P(j, i) & \text{if } i = i' \\
-P(j, i') & \text{otherwise}
\end{cases}
\]
4. Compute direct forces (2)

- $SF(j, i)$ is the overall change of direct forces resulting from the mapping of $j$ to time step $i$.

$$SF(j, i) = \sum_{i' \in R(j)} D(i') \Delta P_i(j, i')$$

$$\Delta P_i(j, i') = \begin{cases} 1 - P(j, i) & \text{if } i = i' \\ -P(j, i') & \text{otherwise} \end{cases}$$

**Example**

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
\hline
D(1) &=& 2 & 5/6 \\
D(2) &=& 2 & 2/6 \\
D(3) &=& 5/6 \\
D(4) &=& 0 \\
\end{array}
\]

$$SF(1, 1) = 2 \frac{5}{6} (1 - 1/2) - 2 \frac{2}{6} (1/2) =$$

$$\frac{1}{2} (17/6 - 14/6) = \frac{1}{2} (3/6) = \frac{1}{4}$$
4. Compute direct forces (3)

Direct force if task/operation 1 is mapped to time step 2

\[
\begin{align*}
SF(1, 2) &= D(1) \times \Delta P_2(1, 1) + D(2) \times \Delta P_2(1, 2) \\
&= 2 \times \frac{5}{6} \times (-0, 5) + \frac{2}{6} \times 0.5 \\
&= -\frac{17}{12} + \frac{14}{12} \\
&= -\frac{3}{12} = -\frac{1}{4}
\end{align*}
\]
5. Compute indirect forces (1)

Mapping task 1 to time step 2 implies mapping task 2 to time step 3

Consider predecessor and successor forces:

\[ V F(j, i) = \sum_{j' \in \text{predecessor of } j} \sum_{i' \in I} D(i') \Delta P_{j,i}(j', i') \]

\[ N F(j, i) = \sum_{j' \in \text{successor of } j} \sum_{i' \in I} D(i') \Delta P_{j,i}(j', i') \]

\( \Delta P_{j,i}(j', i') \) is the \( \Delta \) in the probability of mapping \( j' \) to \( i' \) resulting from the mapping of \( j \) to \( i \)
5. Compute indirect forces (2)

\[ VF(j, i) = \sum_{j' \in \text{predecessor of } j} \sum_{i' \in I} D(i') \Delta P_{j,i}(j', i') \]

\[ NF(j, i) = \sum_{j' \in \text{successor of } j} \sum_{i' \in I} D(i') \Delta P_{j,i}(j', i') \]

Example: Computation of successor forces for task 1 in time step 2

\[ NF(1, 2) = D(2) \times \Delta P_{1,2}(2, 2) + D(3) \times \Delta P_{1,2}(2, 3) \]

\[ = \frac{2}{6} \times (-0, 5) + \frac{5}{6} \times 0.5 \]

\[ = \frac{-14}{12} + \frac{5}{12} \]

\[ = \frac{9}{12} = \frac{3}{4} \]
Overall forces

The total force is the sum of direct and indirect forces:

\[ F(j, i) = SF(j, i) + VF(j, i) + NF(j, i) \]

In the example:

\[ F(1, 2) = SF(1, 2) + NF(1, 2) = -\frac{1}{4} + (-\frac{3}{4}) = -1 \]

The low value suggests mapping task 1 to time step 2
Overall approach

```
procedure forceDirectedScheduling;
begin
  AsapScheduling;
  AlapScheduling;
  while not all tasks scheduled do
    begin
      select task \( T \) with smallest total force;
      schedule task \( T \) at time step minimizing forces;
      recompute forces;
    end;
  end;
end
```

May be repeated for different task/processor classes

Not sufficient for today’s complex, heterogeneous hardware platforms
An integer linear programming (ILP) model for HW/SW partitioning

Notation:

- Index set $V$ denotes task graph nodes.
- Index set $L$ denotes task graph node types e.g. square root, DCT or FFT
- Index set $M$ denotes hardware component types e.g. hardware components for the DCT or the FFT.
- Index set $J$ of hardware component instances
- Index set $KP$ denotes processors. All processors are assumed to be of the same type
An ILP model for HW/SW partitioning

- \( X_{v,m} \): =1 if node \( v \) is mapped to hardware component type \( m \in M \) and 0 otherwise.
- \( Y_{v,k} \): =1 if node \( v \) is mapped to processor \( k \in KP \) and 0 otherwise.
- \( NY_{l,k} \) =1 if at least one node of type \( l \) is mapped to processor \( k \in KP \) and 0 otherwise.
- Type is a mapping from task graph nodes to their types: \( Type : V \rightarrow L \)
- The cost function accumulates the cost of hardware units:
  \[ C = \text{cost(processors)} + \text{cost(memories)} + \text{cost(application specific hardware)} \]
Constraints

Operation assignment constraints

$$\forall v \in V : \sum_{m \in M} X_{v,m} + \sum_{k \in KP} Y_{v,k} = 1$$

All task graph nodes have to be mapped either in software or in hardware.

Variables are assumed to be integers.

Additional constraints to guarantee they are either 0 or 1:

$$\forall v \in V : \forall m \in M : X_{v,m} \leq 1$$

$$\forall v \in V : \forall k \in KP : Y_{v,k} \leq 1$$
Operation assignment constraints (2)

\[ \forall l \in L, \forall v: Type(v) = c_l, \forall k \in KP : NY_{l,k} \geq Y_{v,k} \]

For all types \( l \) of operations and for all nodes \( v \) of this type: if \( v \) is mapped to some processor \( k \), then that processor must implement the functionality of \( l \).

Decision variables must also be 0/1 variables:
\[ \forall l \in L, \forall k \in KP : NY_{l,k} \leq 1. \]
Resource & design constraints

- $\forall m \in M$, the cost (area) for components of type $m$ is equal to the sum of the costs of the components of that type. This cost should not exceed its maximum.
- $\forall k \in KP$, the cost for associated data storage area should not exceed its maximum.
- $\forall k \in KP$ the cost for storing instructions should not exceed its maximum.
- The total cost ($\sum_{m \in M}$) of HW components should not exceed its maximum.
- The total cost of data memories ($\sum_{k \in KP}$) should not exceed its maximum.
- The total cost instruction memories ($\sum_{k \in KP}$) should not exceed its maximum.
Scheduling

Processor

FIR\(_1\)  
FIR\(_2\)

ASIC \(h_1\)

Communication channel \(c_1\)

\(v_1\)  \(v_2\)  \(v_3\)  \(v_4\)

\(v_5\)  \(v_6\)  \(v_7\)  \(v_8\)

\(v_9\)  \(v_{10}\)  \(v_{11}\)

FIR\(_2\) on \(h_1\)

\(e_3\)  \(e_4\)

\(p_1\)

\(v_3\)  \(v_4\)

or

\(v_4\)  \(v_3\)

\(v_7\)  \(v_8\)

or

\(v_8\)  \(v_7\)

\(e_3\)  \(e_4\)

or

\(e_4\)  \(e_3\)
Scheduling / precedence constraints

- For all nodes $v_{i1}$ and $v_{i2}$ that are potentially mapped to the same processor or hardware component instance, introduce a binary decision variable $b_{i1,i2}$ with $b_{i1,i2}=1$ if $v_{i1}$ is executed before $v_{i2}$ and $= 0$ otherwise.

Define constraints of the type
- $(\text{end-time of } v_{i1}) \leq (\text{start time of } v_{i2})$ if $b_{i1,i2}=1$ and
- $(\text{end-time of } v_{i2}) \leq (\text{start time of } v_{i1})$ if $b_{i1,i2}=0$

- Ensure that the schedule for executing operations is consistent with the precedence constraints in the task graph.

- Approach fixes the order of execution
Other constraints

- **Timing constraints**
  These constraints can be used to guarantee that certain time constraints are met.
- Some less important constraints omitted ..
Example

HW types $H_1$, $H_2$ and $H_3$ with costs of 20, 25, and 30.
Processors of type $P$.
Tasks $T_1$ to $T_5$.
Execution times:

<table>
<thead>
<tr>
<th>$T$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$H_3$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>20</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Operation assignment constraints (1)

<table>
<thead>
<tr>
<th>T</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>12</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>12</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

∀v ∈ V : \( \sum_{m \in KM} X_{v,m} + \sum_{k \in KP} Y_{v,k} = 1 \)

\( X_{1,1} + Y_{1,1} = 1 \) (task 1 mapped to \( H1 \) or to \( P \))
\( X_{2,2} + Y_{2,1} = 1 \)
\( X_{3,3} + Y_{3,1} = 1 \)
\( X_{4,3} + Y_{4,1} = 1 \)
\( X_{5,1} + Y_{5,1} = 1 \)
Assume types of tasks are $ l = 1, 2, 3, 3, $ and $ 1 $. 

$ \forall \ l \in L, \ \forall \ v : Type(v) = c \ l, \ \forall \ k \in KP : NY_{l,k} \geq Y_{v,k} $
Other equations

Time constraints leading to: Application specific hardware required for time constraints ≤ 100 time units.

<table>
<thead>
<tr>
<th>T</th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>20</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>12</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>12</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Cost function:
\[ C = 20 \#(H1) + 25 \#(H2) + 30 \#(H3) + \text{cost(processor)} + \text{cost(memory)} \]
Result

For a time constraint of 100 time units and \(\text{cost}(P) < \text{cost}(H_3)\):

<table>
<thead>
<tr>
<th></th>
<th>(H_1)</th>
<th>(H_2)</th>
<th>(H_3)</th>
<th>(P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>20</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Solution (educated guessing):
\( T_1 \rightarrow H_1 \)
\( T_2 \rightarrow H_2 \)
\( T_3 \rightarrow P \)
\( T_4 \rightarrow P \)
\( T_5 \rightarrow H_1 \)
Application example

Audio lab (mixer, fader, echo, equalizer, balance units); slow SPARC processor
1µ ASIC library
Allowable delay of 22.675 µs (~ 44.1 kHz)

Outdated technology; just a proof of concept.
Only simple models can be solved optimally.
Deviation from optimal design

Hardly any loss in design quality.
Running time for heuristic
Design space for audio lab

Everything in software: 72.9 µs, 0 \( \lambda^2 \)
Everything in hardware: 3.06 µs, 457.9x10^6 \( \lambda^2 \)
Lowest cost for given sample rate: 18.6 µs, 78.4x10^6 \( \lambda^2 \),
Positioning of COOL

COOL approach:
- shows that a formal model of hardware/SW codesign is beneficial; IP modeling can lead to useful implementation even if optimal result is available only for small designs.

Other approaches for HW/SW partitioning:
- starting with everything mapped to hardware; gradually moving to software as long as timing constraint is met.
- starting with everything mapped to software; gradually moving to hardware until timing constraint is met.
- Binary search.