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Exercises for
Rechnerstrukturen
Winter Semester 2015

Exercise Sheet 5 (Block B - 1)

(16 points)

Submission until Wednesday, 25th November 2015, 16:00 pm
Discussion begins on Tuesday, 1st December 2015

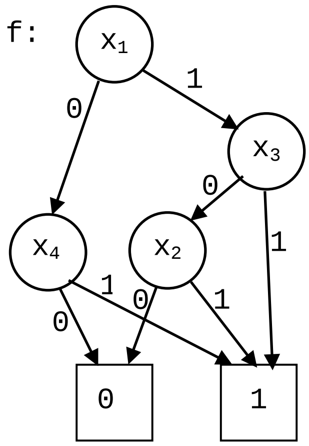
Please see notes at the end of the document for the submission.

5.1 Optimization of Digital Combinatorial Circuits (4 points)

a. In each of the four lines of the table a different function $g : B^3 \to B^1$ is given. Please decide, if the function is a Monomial, Polynomial, Minimal Polynomial, a disjunctive normal form (DNF) or a conjunctive normal form (KNF). In every line please mark all the the terms that hold true for the function.

Function	Monomial	Polynomial	Minimal Polynomial	DNF	KNF
$(x_1 \wedge x_2 \wedge x_3) \vee (\overline{x_1} \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge \overline{x_3})$					
$x_2 \wedge x_3$					
$(x_2 \vee x_3) \wedge (x_2 \vee \overline{x_3})$					
$x_1 \vee x_2 \vee \overline{x_3}$					

b. Let the Boolean function $f : B^4 \to B^1$ over the variables x_1, x_3, x_2, x_4 be given by the following $\pi OBDD$. Please determine the related Minimal Polynomial. As a first step please put all ones to the right places in the given KV-Diagram. Mark all prime implicants in the KV-Diagram and write them down next to the KV-Diagram. Please make sure that all prime implicants can be related to a marking easily, e.g., using colors or draw connection lines. Please also make sure your solutions remains easy to read. If you need more space please use an additional sheet of paper.



f

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00				
01				
11				
10				

c. Give a Minimal Polynomial for the function f from subtask b).

5.2 Practical Application of Optimization of Digital Combinatorial Circuits (4 points)

Assume you buy a chain of lights with 7 LEDs $x_1, \ldots, x_7$ that are placed side by side in one line. With these 7 LEDs you can display different patterns. As Christmas is approaching, you want to display a stylized Christmas Tree with your chain of lights. For this you need the following lighting patterns, that you name $0, 1, \ldots, 7$ depending on the number of shining LEDs.

LED	x_1	x_2	x_3	x_4	x_5	x_6	x_7
alle aus	○	○	○	○	○	○	○
	○	○	○	●	○	○	○
	○	○	●	○	●	○	○
	○	○	●	●	●	○	○
	○	●	●	○	●	●	○
	○	●	●	●	●	●	○
	●	●	●	○	●	●	●
alle an	●	●	●	●	●	●	●

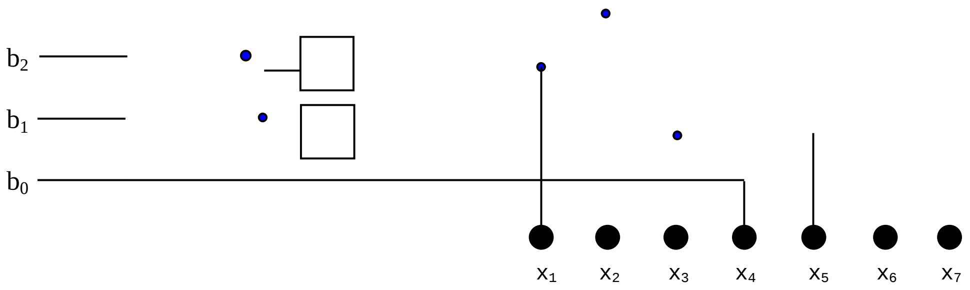
To get the stylized Christmas Tree, you want to use an clock-pulse generator to make sure the patterns $0, 1, \ldots, 7$ are shown in increasing order (from top to bottom). The clock-pulse generator counts the numbers 0 to 7 and outputs them binary on the 3 output lines b_2, b_1, b_0. You want to use this output to control the LEDs.

a. Code the 7 patterns $0, 1, \ldots, 7$ of the LEDs binary in relation to the 3 outputs b_2, b_1, b_0 in the table below. Code a shining LED with a 1 and an LED that is off with a 0. For pattern 0 all LEDs are off while for pattern 7 all LEDs are shining.

Zustand	b_2	b_1	b_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1							
2	0	1	0			1	0	1		
7	1	1	1	1	1	1	1	1	1	1

b. Determine the minimized Boolean Functions $f_i(b_2, b_1, b_0) = x_i$ for $i = 1, \ldots, 7$, representing the behavior of x_i in relation to b_2, b_1, b_0.

c. Complete the following Digital Combinatorial Circuit to ensure that the patterns for the stylized Christmas Tree are displayed correctly with relation to the b_2, b_1, b_0.



5.3 Algorithm of Quine and McCluskey (4 points)

Let the function $f : B^4 \to B^1$ over the variables a, b, c, d be given by the following vector of values:

$$F = (1,1,1,1,\ 0,0,0,1,\ 1,0,1,0,\ 1,0,1,1)$$

Use the algorithm of Quine and McCluskey to determine all prime implicants of f. Write down all sets L_i and mark the prime implicants that are moved to the set of prime implicants PI in the step. Create the PI-table and use it to determine one Minimal Polynomial.

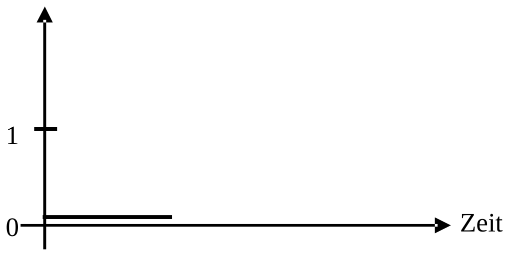
To determine the implicants of the set L_0 you can use the following table.

Vektorposition	a	b	c	d	Wert	kurz
0	0	0	0	0	1	$\bar{a}\bar{b}\bar{c}\bar{d}$
1	0	0	0	1	1	$\bar{a}\bar{b}\bar{c}d$
2	0	0	1	0	1	$\bar{a}\bar{b}c\bar{d}$
3	0	0	1	1	1	$\bar{a}\bar{b}cd$
13	1	1	0	1	0	$ab\bar{c}d$
14	1	1	1	0	1	$abc\bar{d}$
15	1	1	1	1	1	$abcd$

5.4 Hazards (4 points)

a. Display the chronological sequence of the function (zero and one level) of a Digital Combinatorial Circuit with one output line for a static (statischer Hazard) and for a dynamic hazard (dynamischer Hazard) by completing the following diagrams:

`statischer Hazard:`



`dynamischer Hazard:`

b. Let the function $f(x_1,x_2,x_3,x_4) : B^4 \to B^1$ be guveb by the vector $F = (1,1,0,1,\ 1,1,1,0,\ 0,1,0,0,\ 1,1,1,1)$.

Determine if hazards occur for the following four changes of input values. If a hazard occurs, please mark if it is a static or a dynamic hazard and display the input that leads to the hazard (the ”way” on which the hazards happens). Please use the KV-Diagram.

f		X_1 X_2			
		00	01	11	10
	00				
X_3 X_4	01				
	11				
	10				

1. (1011, . . . , 1100)
2. (0011, . . . , 1011)
3. (1111, . . . , 1001)
4. (1000, . . . , 0001)

Notes:
Submission until Wednesday, 25th November 2015, 16:00 pm in the mailbox number 46 at Otto-Hahn-Straße 12.

You can find the mailboxes in the first floor of the Otto-Hahn-Straße 12 near the transition to the ground floor of the Otto-Hahn-Straße 14. The mailboxes are labeled with “Rechnerstrukturen”, the exercise group number and time/place of the exercise. The English exercise group is number 30 and the mailbox is number 46.

Please write your **name**, your **student registration number** and your **exercise group number** at the top right corner of your submission. You can make submissions in teams with up to two more students. To make a team submission put names, student registrations numbers and group numbers of all members of the team on the submission. Only one submission per team has to be made.

Tack you submission. Please do not fold your submission and do not put it into an envelope. Use the correct mailbox, you will need your exercise group number for that.

In total there are 12 exercises in 3 blocks (A, B, C). In each block you have to achieve at least 30 points of 64 possible ones to get access to the exam.