Petri Nets

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(slides are based on
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Models of computation considered in this course

Communication/ local computations	**Shared memory**	**Message passing Synchronous**	**| Asynchronous**
Undefined components	Plain text, use cases	| (Message) sequence charts	
Communicating finite state machines	StateCharts		SDL
Data flow			Kahn networks, SDF
Petri nets		**C/E nets, P/T nets, …**	
Discrete event (DE) model	VHDL*, Verilog*, SystemC*, …	Only experimental systems, e.g. distributed DE in Ptolemy	
Von Neumann model	C, C++, Java	C, C++, Java with libraries CSP, ADA |	

* Classification is based on implementation of VHDL, Verilog, SystemC with central queue

Introduction

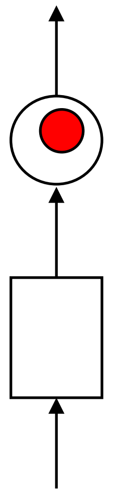
Introduced in 1962 by Carl Adam Petri in his PhD thesis.

Focus on modeling causal dependencies;

no global synchronization assumed (message passing only).

Key elements:

- **Conditions**
  Either met or not met.
- **Events**
  May take place if certain conditions are met.
- **Flow relation**
  Relates conditions and events.



Conditions, events and the flow relation form

a **bipartite graph** (graph with two kinds of nodes).

Interactive Example

http://www.informatik.uni-hamburg.de/TGI/PetriNets/introductions/aalst/

Condition/event nets

Def.: $N=(C,E,F)$ is called a **net**, if the following holds

1. C and E are disjoint sets
2. $F \subseteq (C \times E) \cup (E \times C)$; is binary relation, ("**flow relation**")

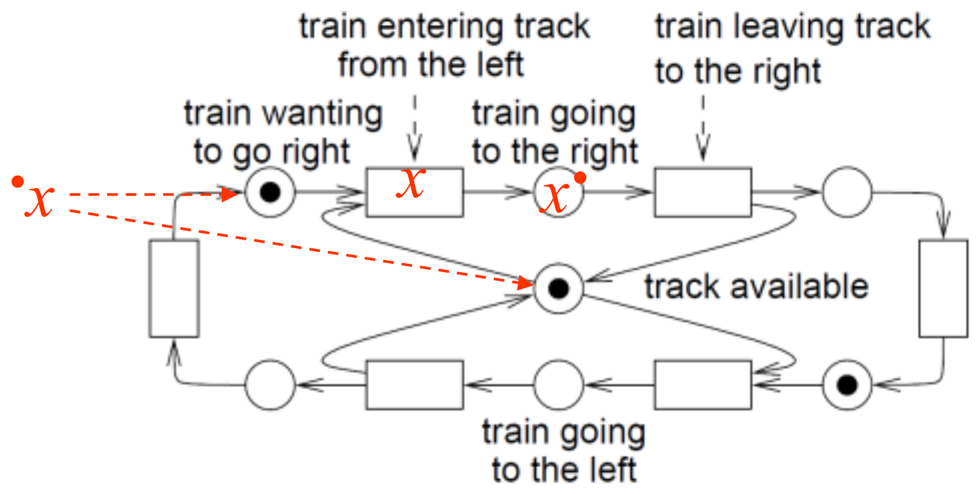
Pre- and post-sets

Def.: Let N be a net and let $x \in (C \cup E)$.

$^\bullet x := \{y \mid y\,F\,x\}$ is called the **pre-set** of x,
(or **preconditions** if $x \in E$)

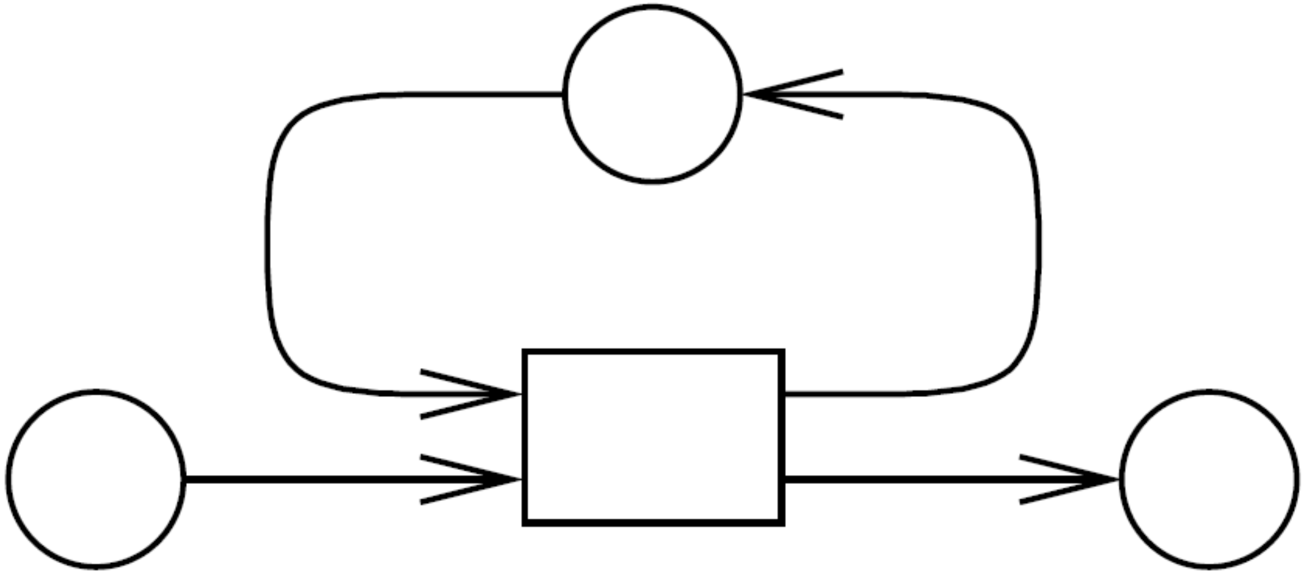
$x^\bullet := \{y \mid x\,F\,y\}$ is called the set of **post-set** of x,
(or **postconditions** if $x \in E$)

Example:


train entering track from the left
train leaving track to the right
train wanting to go right
train going to the right
•x
x
x•
track available
train going to the left

Loops and pure nets

Def.: Let $(c,e) \in C \times E$. (c, e) is called a **loop** if $cFe \wedge eFc$.

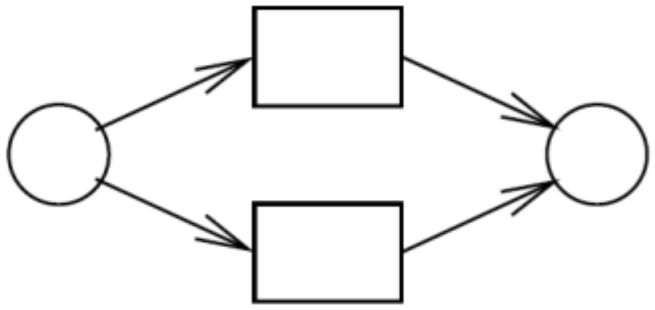


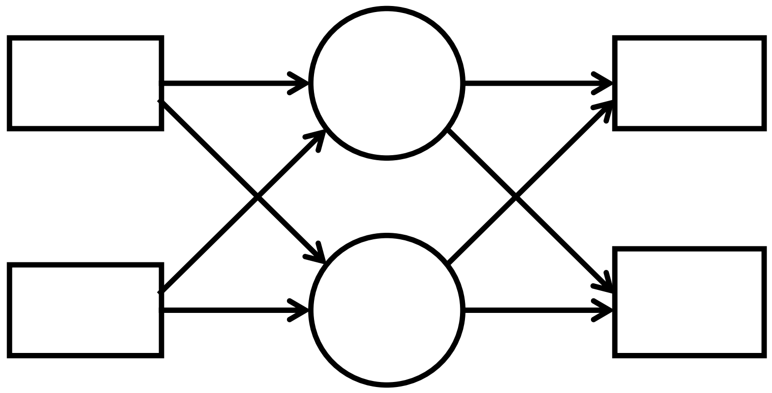
Def.: Net $N=(C,E,F)$ is called **pure**, if F does not contain any loops.

Simple nets

Def.: A net is called **simple** if no two nodes $n1$ and $n2$ have the same pre-set and post-set.

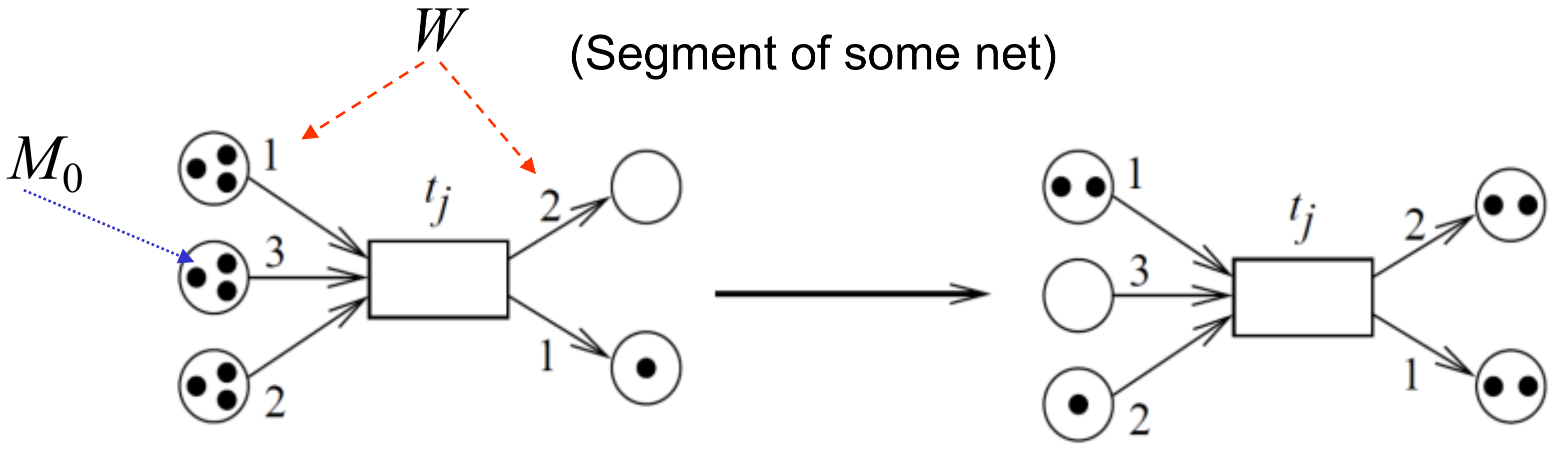
Example (not simple nets):





Def.: Simple nets with no isolated elements meeting some additional restrictions are called **condition/event nets (C/E nets)**.

Place/transition nets



defaults:
$K = \omega$
$W = 1$

Def.: (P, T, F, K, W, M_0) is called a **place/transition net** if

1. $N=(P, T, F)$ is a **net** with places $p \in P$ and transitions $t \in T$
2. $K: P \rightarrow (\mathbb{N}_0 \cup \{\omega\}) \setminus \{0\}$ denotes the **capacity** of places (ω symbolizes infinite capacity)
3. $W: F \rightarrow (\mathbb{N}_0 \setminus \{0\})$ denotes the **weight of graph edges**
4. $M_0: P \rightarrow \mathbb{N}_0 \cup \{\omega\}$ represents the **initial marking** of places

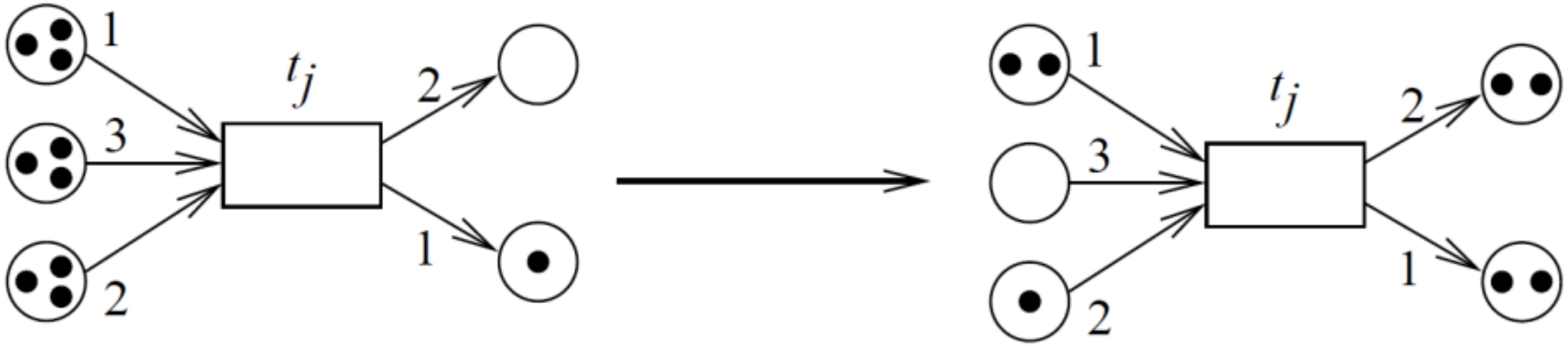
Applications

- Modeling of resources;
- modeling of mutual exclusion;
- modeling of synchronization.

Computing changes of markings

“Firing“ transitions t generate new markings on each of the places p according to the following rules:

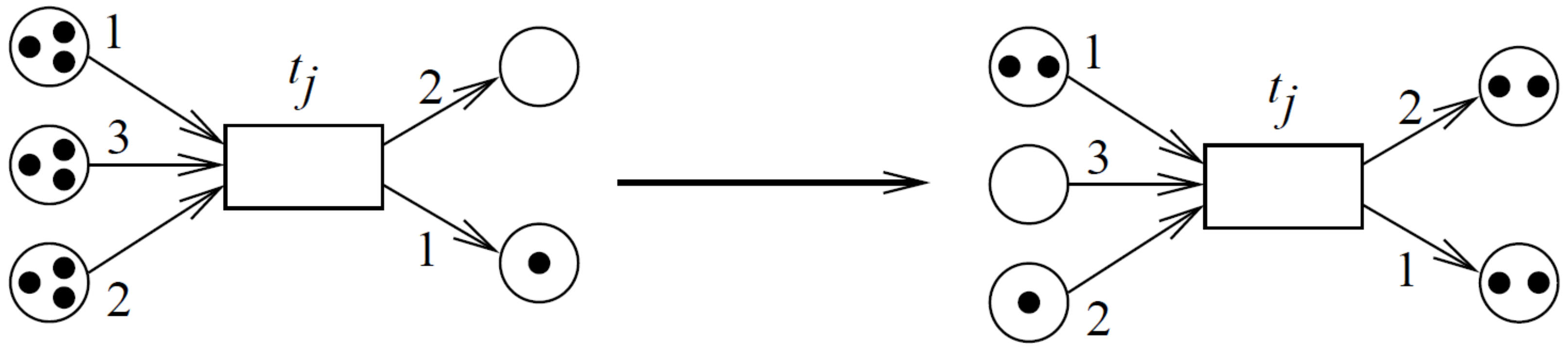
$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$



Activated transitions

Transition t is “activated“ if

$$(\forall p \in {}^{\bullet}t : M(p) \geq W(p,t)) \wedge (\forall p \in t^{\bullet} : M(p) + W(t,p) \leq K(p))$$



Activated transitions can “take place“ or “fire“, but don‘t have to.

We never talk about “time“ in the context of Petri nets.

The order in which activated transitions fire, is not fixed (it is non-determinate).

Shorthand for changes of markings

Slide 12:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

$\Rightarrow$ $\forall p \in P$: $M'(p) = M(p) + \underline{t}(p)$

$\Rightarrow$ $M' = M + \underline{t}$ +: vector add

Matrix $\underline{N}$ describing all changes of markings

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ +W(t,p) \text{ if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ -W(p,t) + W(t,p) \text{ if } p \in t^{\bullet} \cap {}^{\bullet}t \\ 0 \end{cases}$$

Def.: Matrix $\underline{N}$ of net N is a mapping

$$\underline{N}: P \times T \rightarrow \mathbb{Z} \text{ (integers)}$$

such that $\forall\, t \in T : \underline{N}(p,t) = \underline{t}(p)$

Component in column t and row p indicates the change of the marking of place p if transition t takes place.

For pure nets, $(\underline{N}, M_0)$ is a complete representation of a net.

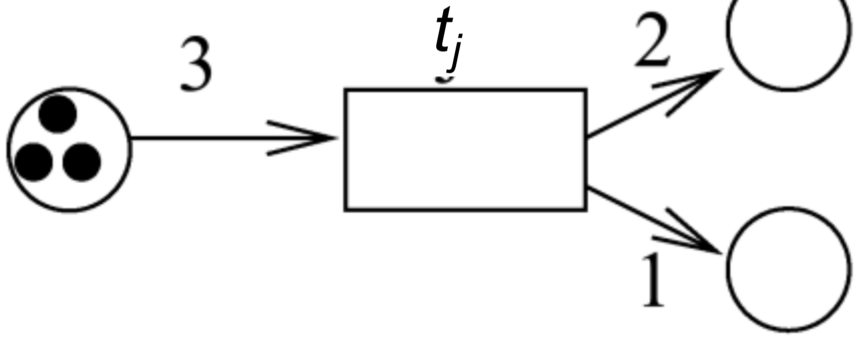
Place - invariants

Standardized technique for proving properties of system models

For any transition $t_j \in T$ we are looking for sets $R \subseteq P$ of places for which the accumulated marking is constant:

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Example:



Predicate/transition nets

Goal: compact representation of complex systems.

Key changes:

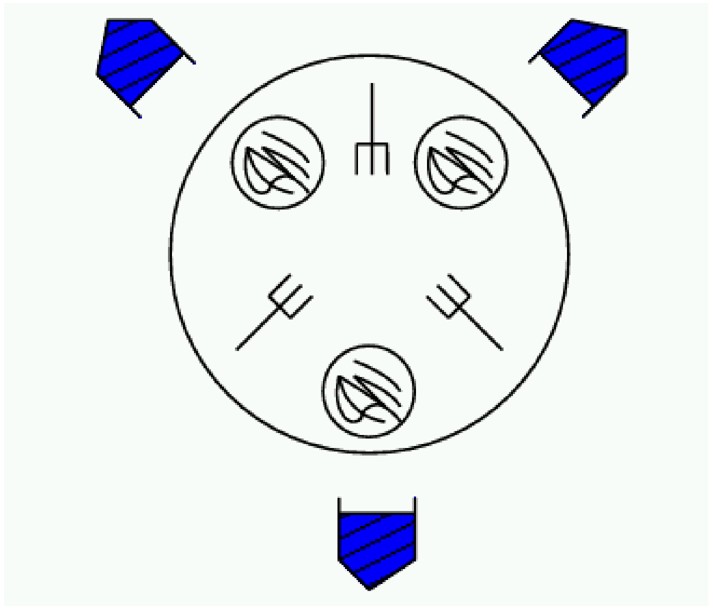
- Tokens are becoming individuals;
- Transitions enabled if functions at incoming edges true;
- Individuals generated by firing transitions defined through functions

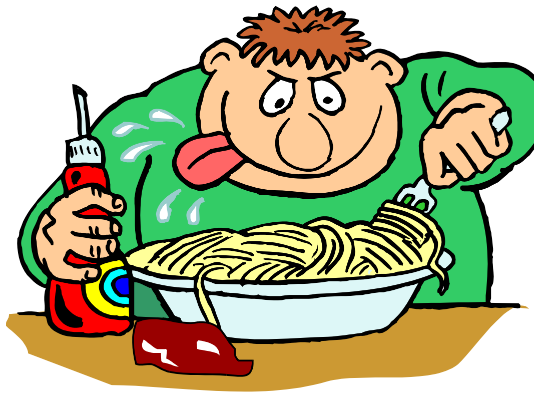
Changes can be explained by folding and unfolding C/E nets,

☞ semantics can be defined by C/E nets.

Example: Dining philosophers problem

n>1 philosophers sitting at a round table;
n forks,
n plates with spaghetti;
philosophers either thinking or eating spaghetti
(using left and right fork).



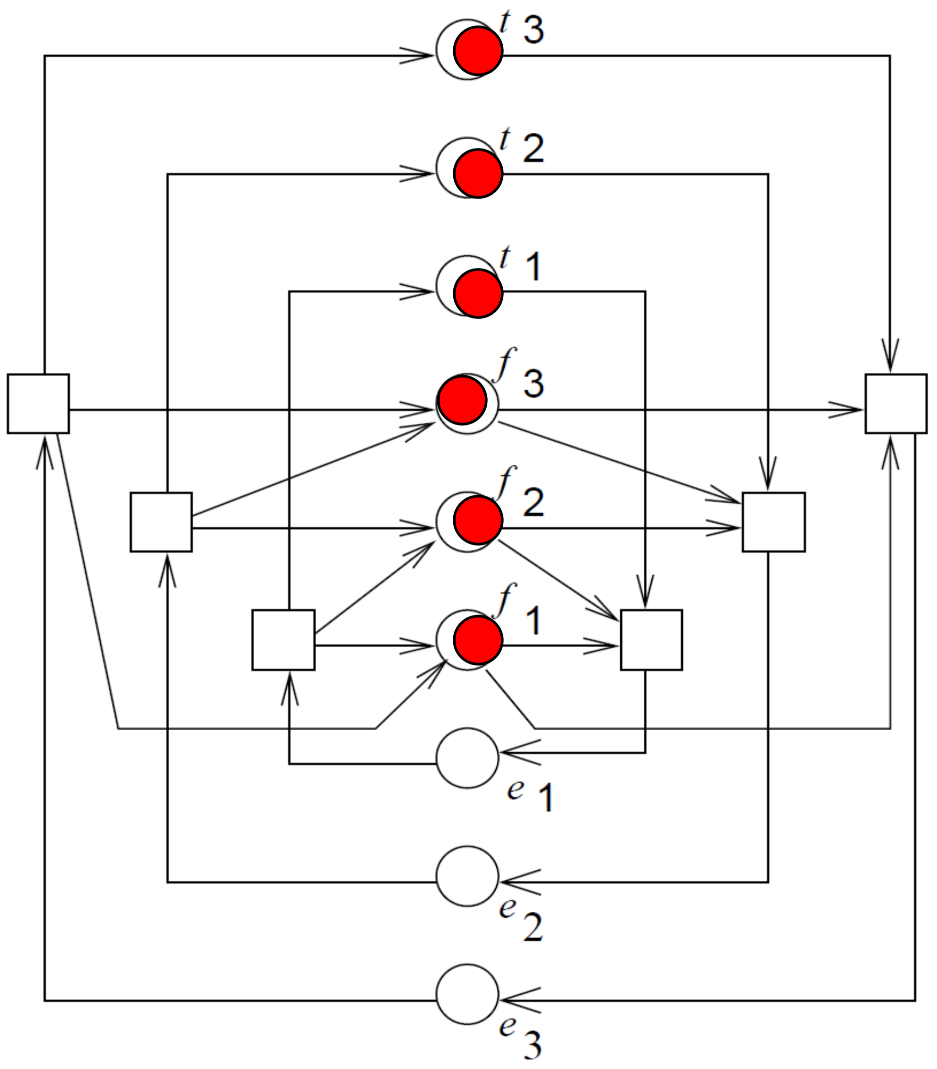


2 forks needed!

How to model conflict for forks?

How to guarantee avoiding starvation?

Condition/event net model of the dining philosophers problem



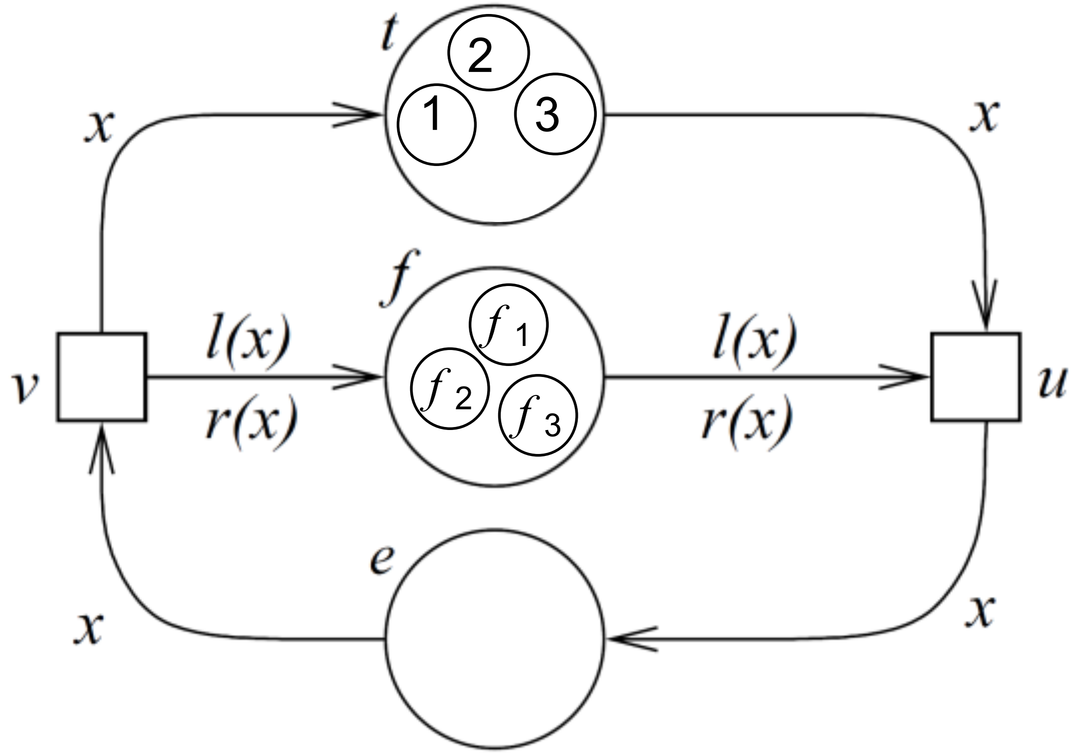
Let $x \in \{1..3\}$
t_x: x is thinking
e_x: x is eating
f_x: fork x is available

Model quite clumsy.

Difficult to extend to more philosophers.

Predicate/transition model of the dining philosophers problem (1)

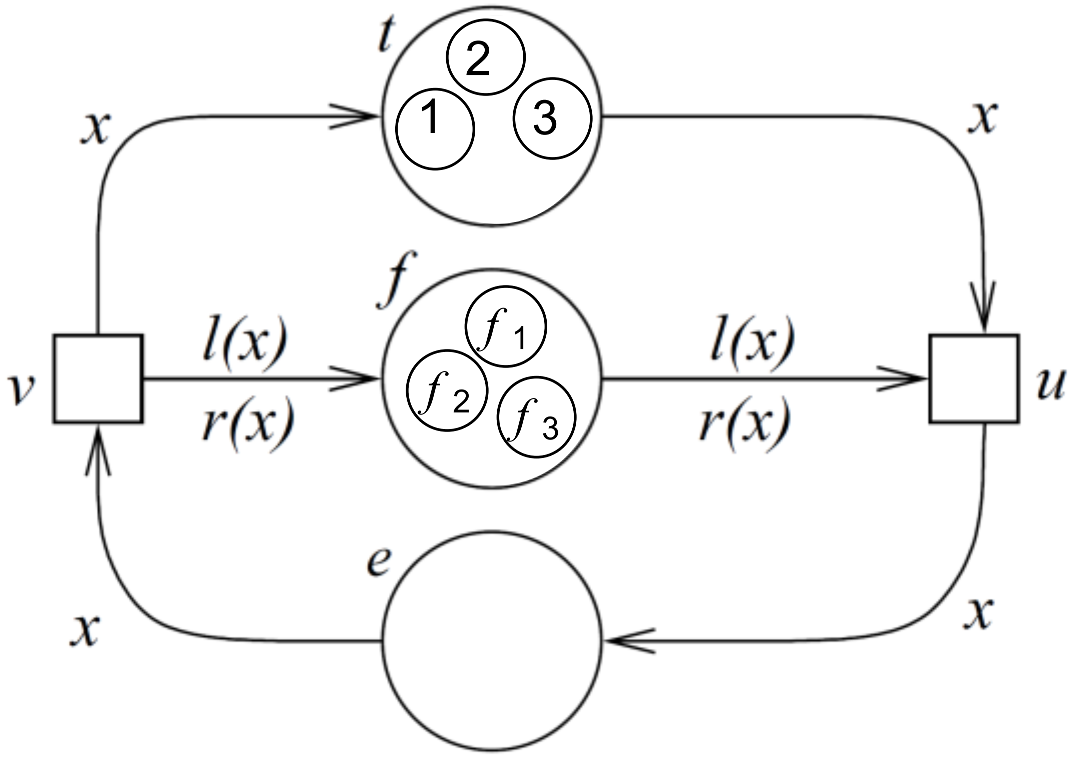
Let x be one of the philosophers,
let $l(x)$ be the left fork of x: f_x,
let $r(x)$ be the right fork of x: $f_{(x \bmod 3)+1}$.
$l(2)=f_2$, $r(2)=f_3$, $l(3)=f_3$, $r(3)=f_1$



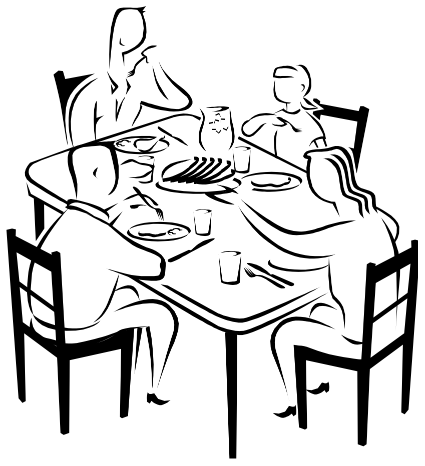
Tokens: individuals.

Semantics can be defined by replacing net by equivalent condition/event net.

Predicate/transition model of the dining philosophers problem (2)



Model can be extended to arbitrary numbers of people.



Evaluation

Pros:

- Appropriate for distributed applications,
- Well-known theory for formally proving properties,

Cons (for the nets presented) :

- problems with modeling timing,
- no programming elements,
- no hierarchy.

Extensions:

- Enormous amounts of efforts on removing limitations.

Summary

Petri nets: focus on causal dependencies

- Condition/event nets
  - Single token per place
- Place/transition nets
  - Multiple tokens per place
- Predicate/transition nets
  - Tokens become individuals
  - Dining philosophers used as an example