Abstract Models for Real-Time Calculus

Concrete Instance

Abstract Representation

Processor

Tasks

Service Model

Load Model

Processing Model
Abstract Models for Module Performance Analysis

RM: Rate-Monotonic (a fixed-priority scheduler, detailed in Chapter 6)
TDMA: Time Division Multiple Access (detailed later)
GPC: Greedy Processing Component (detailed later)
Overview

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<th>System View</th>
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<td>Module Performance Analysis (MPA)</td>
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Backgrounds

- Real-Time Calculus can be regarded as a worst-case/best-case variant of classical queuing theory. It is a formal method for the analysis of distributed real-time embedded systems.

- Related Work:
Definition of Arrival Curves and Service Curves

• For a specific trace:
  • Data streams: \( R(t) = \) number of events in \( [0, t) \)
  • Resource stream: \( C(t) = \) available resource in \( [0, t) \)

• For the worst cases and the best cases in any interval with length \( \Delta \):
  • Arrival Curve \([\alpha^l, \alpha^u]\):
    \[
    \alpha^l(\Delta) = \inf_{\lambda \geq 0, \forall R} \{ R(\Delta + \lambda) - R(\lambda) \}
    \]
    \[
    \alpha^u(\Delta) = \sup_{\lambda \geq 0, \forall R} \{ R(\Delta + \lambda) - R(\lambda) \}
    \]
  • Service Curve \([\beta^l, \beta^u]\):
    \[
    \beta^l(\Delta) = \inf_{\lambda \geq 0, \forall C} \{ C(\Delta + \lambda) - C(\lambda) \}
    \]
    \[
    \beta^u(\Delta) = \sup_{\lambda \geq 0, \forall C} \{ C(\Delta + \lambda) - C(\lambda) \}
    \]
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$R(t)$ $R'(t)$

$C(t)$

$\alpha(\Delta)$

$\beta(\Delta)$
Arrival Curve: An Example

Use a sliding window to get the upper bound of the number of events in a specified interval length.
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![Diagram showing arrival curve with time, workload, and possible events in 3 units]

\[ \Delta \]
Arrival Curve: An Example

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Arrival Curve: An Example

Use a sliding window to get the upper bound of the number of events in a specified interval length.

![Diagram showing an arrival curve with time and workload axes, illustrating maximum and minimum events in a 3-unit interval.](image)
Arrival Curve: An Example

Use a sliding window to get the upper bound of the number of events in a specified interval length.
Service Curve: An Example

Resource Availability

Service Curves \( \beta = [\beta^l, \beta^u] \)
Example 1: Periodic with Jitter

A common event pattern that is used in literature can be specified by the parameter triple \((p, j, d)\), where \(p\) denotes the period, \(j\) the jitter, and \(d\) the minimum inter-arrival distance of events in the modeled stream.
Example 1: Periodic with Jitter

\[ \alpha^u(\Delta) = \left\lceil \frac{\Delta + j}{p} \right\rceil \]

\[ \alpha^l(\Delta) = \left\lfloor \frac{\Delta - j}{p} \right\rfloor \]

\[ \alpha^u(\Delta) = \left\lceil \frac{\Delta}{p} \right\rceil \]

\[ \alpha^l(\Delta) = \left\lfloor \frac{\Delta}{p} \right\rfloor \]
Example 1: Periodic with Jitter

\[ \alpha^u(\Delta) = \min \left\{ \left\lceil \frac{\Delta + j}{p} \right\rceil, \left\lceil \frac{\Delta}{d} \right\rceil \right\} \]

\[ \alpha^l(\Delta) = \left\lfloor \frac{\Delta - j}{p} \right\rfloor \]

Graph showing the periodicity with jitter for \( \alpha^u \) and \( \alpha^l \) with equations for \( \Delta \) and \( j \).
More Examples on Arrival Curves
Example 2: TDMA Resource

• Consider a real-time system consisting of $n$ applications that are executed on a resource with bandwidth $B$ that controls resource access using a TDMA (Time Division Multiple Access) policy.

• Analogously, we could consider a distributed system with $n$ communicating nodes, that communicate via a shared bus with bandwidth $B$, with a bus arbitrator that implements a TDMA policy.

• TDMA policy: In every TDMA cycle of length $\bar{c}$, one single resource slot of length $s_i$ is assigned to application $i$. 

![Diagram of TDMA Resource with applications app1, app2, ..., appn and time slots $\bar{c}$ and $s_n$.]
Example 2: TDMA Resource

\[ \beta^u(\Delta) = B \min \left\{ \frac{\Delta}{\bar{c}}, \Delta - \left\lfloor \frac{\Delta}{\bar{c}} \right\rfloor (\bar{c} - s_i) \right\} \]

\[ \beta^l(\Delta) = B \max \left\{ \frac{\Delta}{\bar{c}}, \Delta - \left\lceil \frac{\Delta}{\bar{c}} \right\rceil (\bar{c} - s_i) \right\} \]
More Examples on Service Curves

- **Full resource**
  - \( \beta^u \)
  - \( \beta^l \)

- **Bounded delay**
  - \( \beta^u \)
  - \( \beta^l \)

- **TDMA resource**
  - \( \beta^u \)
  - \( \beta^l \)

- **Periodic resource**
  - \( \beta^u \)
  - \( \beta^l \)
Abstraction

\[ C(t) \rightarrow GPC \rightarrow R'(t) \rightarrow \alpha'(\Delta) \]

\[ R(t) \rightarrow C'(t) \rightarrow \beta'\Delta) \rightarrow GPC \rightarrow \alpha(\Delta) \]

time domain cumulative functions

time-interval domain variability curves
Greedy Processing Component (GPC)

- Component is triggered by incoming events.
- A fully preemptable task is instantiated at every event arrival to process the incoming event.
- Active tasks are processed in a greedy fashion in FIFO order.
- Processing is restricted by the availability of resources.
Some Relations (only for your reference)

- The output upper arrival curve of a component satisfies
  \[ \alpha^{u'} \leq (\alpha^{u} \bowtie \beta^{l}) \]
  with a simple and pessimistic calculation.
- The remaining lower service curve of a component satisfies
  \[ \beta^{l''}(\Delta) = \sup_{0 \leq \lambda \leq \Delta} (\beta^{l}(\lambda) - \alpha^{u}(\lambda)) \]
More Relations (only for your reference)

\[
\begin{align*}
\alpha^{u'} &= \left( (\alpha^u \otimes \beta^u) \oslash \beta^l \right) \land \beta^u \\
\alpha^{l'} &= \left( (\alpha^u \otimes \beta^l) \otimes \beta^l \right) \land \beta^l \\
\beta^{u'} &= (\beta^u - \alpha^l) \oslash 0 \\
\beta^{l'} &= (\beta^l - \alpha^u) \oslash 0
\end{align*}
\]

Without formal proofs....
Graphical Interpretation

\[ B = \sup_{t \geq 0} \{ R(t) - R'(t) \} \leq \sup_{\lambda \geq 0} \{ \alpha^u(\lambda) - \beta^l(\lambda) \} \]
\[ D = \sup_{t \geq 0} \{ \inf_{\Delta \geq 0} \{ \tau \geq 0 : R(t) \leq R'(t + \tau) \} \} \]
\[ = \sup_{\Delta \geq 0} \{ \inf_{\tau \geq 0} \{ \tau \geq 0 : \alpha^u(\tau) \leq \beta^l(\Delta + \tau) \} \} \]
Complete System Composition

Input Stream

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RM: Rate-Monotonic (a fixed-priority scheduler, detailed in Chapter 6)
TDMA: Time Division Multiple Access
GPC: Greedy Processing Component
RTC Toolbox (http://www.mpa.ethz.ch/Rtctoolbox)
Advantages and Disadvantages of RTC and MPA

- **Advantages**
  - Provides a powerful abstraction to model event arrivals and resource consumption
  - Considers resources as first-class citizens
  - Allows composition in terms of (a) tasks, (b) streams, (c) resources, (d) sharing strategies.

- **Disadvantages**
  - Needs some effort to understand and implement
  - Extension to new arbitration schemes not always simple
  - *Not applicable for schedulers that change the scheduling policies dynamically.*