Analysis of Deadline Miss Rates for Uniprocessor Fixed-Priority Scheduling

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Outline

Frequency of Deadline Misses

- Soft real-time systems
- Rare deadline misses often acceptable

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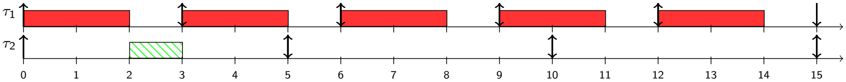
- How to quantify the frequency?

Probability of a Deadline Miss



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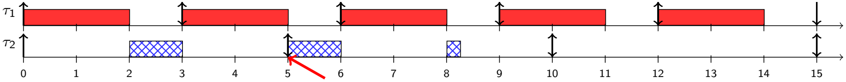

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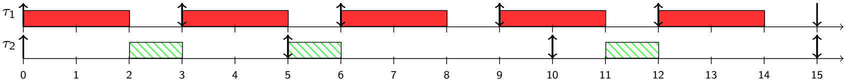

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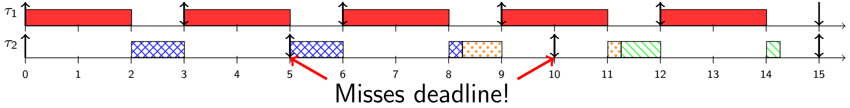

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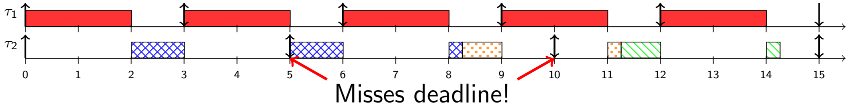

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- Miss rate $> 50\%$
- Simulation: 100 runs, each 5 million jobs of τ_2: 93.04%!

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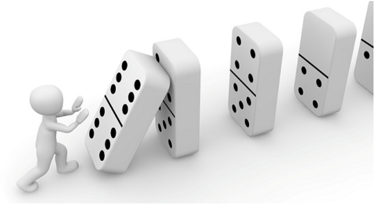

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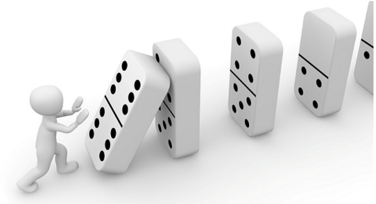
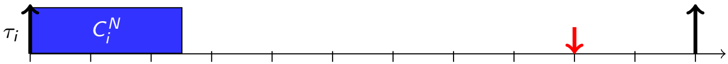


How to obtain Miss Rate analytically?

Task Model and Notation

$$\tau_i(C_i,\ D_i,\ T_i)$$



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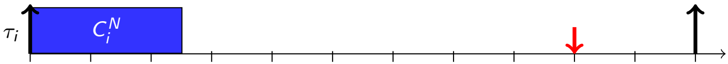

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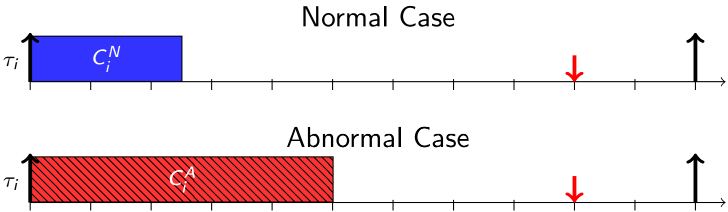

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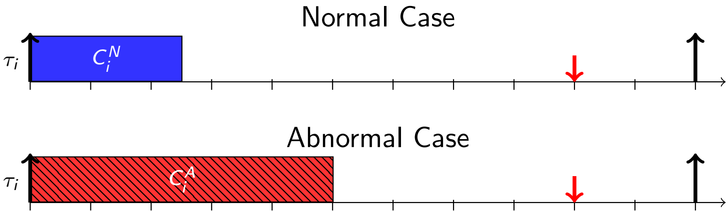

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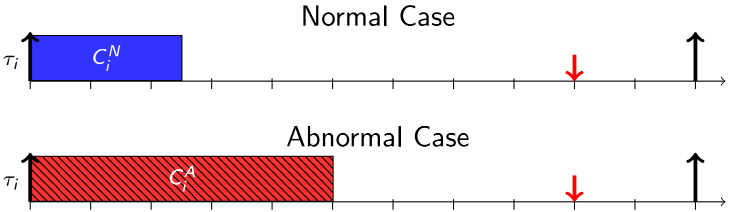

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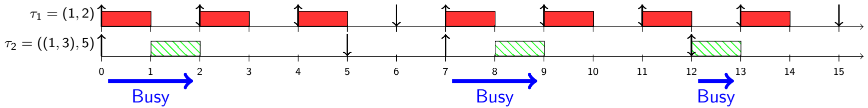
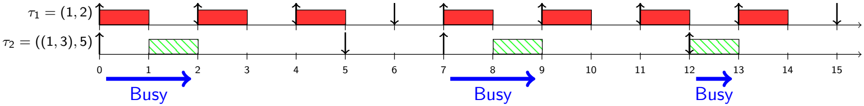
- $C_i^A \geq C_i^N$
- $P_i^A + P_i^N = 1 \rightarrow \sum_j^{v_i} P_i^j = 1$ (v_i is finite)
- Probabilities independent

Busy Interval



- Busy interval of τ_2: $[t_a, t_b]$
- t_a: Release time of a job of τ_2, no job of τ_2 in the system at t_a
- t_b: First time point after t_a where all jobs of τ_2 released since t_a are finished

Partition into Busy Intervals I_j



- Partition according to number of missed deadlines

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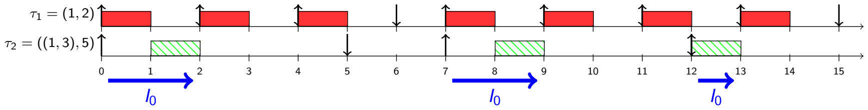

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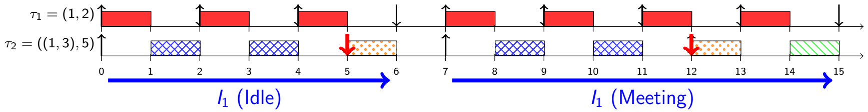

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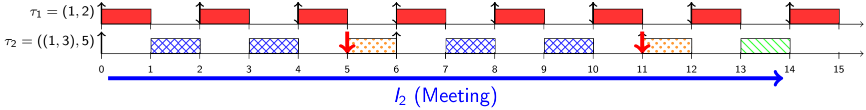

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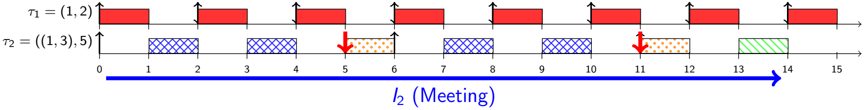

- Partition according to number of missed deadlines
- I_0: No deadline misses
- I_j: Exactly j deadline misses (first j jobs)
- Probability of I_j: $\psi(I_j)$
- $\sum_{j=0}^{\mathbb{J}} \psi(I_j) = 1$ (at most $\mathbb{J}$ consecutive misses)

Exact Miss Rate

Given a release pattern:

$$\text{Miss Rate} = \frac{\text{Number of deadline misses}}{\text{Number of released jobs}}$$

Expected Miss Rate

Given a release pattern:

$$\mathbb{E}_k = \frac{\text{Expected number of deadline misses}}{\text{Number of released jobs}}$$

Expected Miss Rate

Given a release pattern with at most $\mathbb{J}$ consecutive misses:

$$\mathbb{E}_k = \frac{\sum_{j=1}^{\mathbb{J}} \psi(I_j) \cdot j}{\sum_{j=1}^{\mathbb{J}} \psi(I_j) \cdot j + \psi(I_0) \cdot 1}$$

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Good news and bad news?

Upper Bound of $\psi(I_j)$

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Upper Bound of $\psi(I_j)$

- Calculating $\psi(I_j)$ exactly is challenging
- Upper bound probability $\psi(I_j)$
  - $\Phi_{k,j}$: at least j consecutive deadline misses
  - $\Phi_{k,j} \geq \psi(I_j)$ (exact)
  - Task-level Convolution (von der Brüggen et al. ECRTS'18)
  - Analytical bound approach (Chen and Chen SIES'17)

Upper Bounded Expected Miss Rate

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Upper Bounded Expected Miss Rate

Given a release pattern with at most $\mathbb{J}$ consecutive misses:

$$\mathbb{E}_k \leq \frac{\sum_{j=1}^{\mathbb{J}} \Phi_{k,j} \cdot j}{\sum_{j=1}^{\mathbb{J}} \psi(I_j) \cdot j + \psi(I_0) \cdot 1} \checkmark ?$$

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How large is the error?

Evaluation: Setup

- Utilization: 70%
- Periods: UUniFast, 1ms-100ms
- $P_i^A = 0.0001$, $P_i^N = 1 - P_i^A$
- Cardinality: $\{5, 10\}$ tasks
- For 5 tasks: 10 task sets
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- **AB**: Analytical approach with Chernoff bounds (Chen and Chen)
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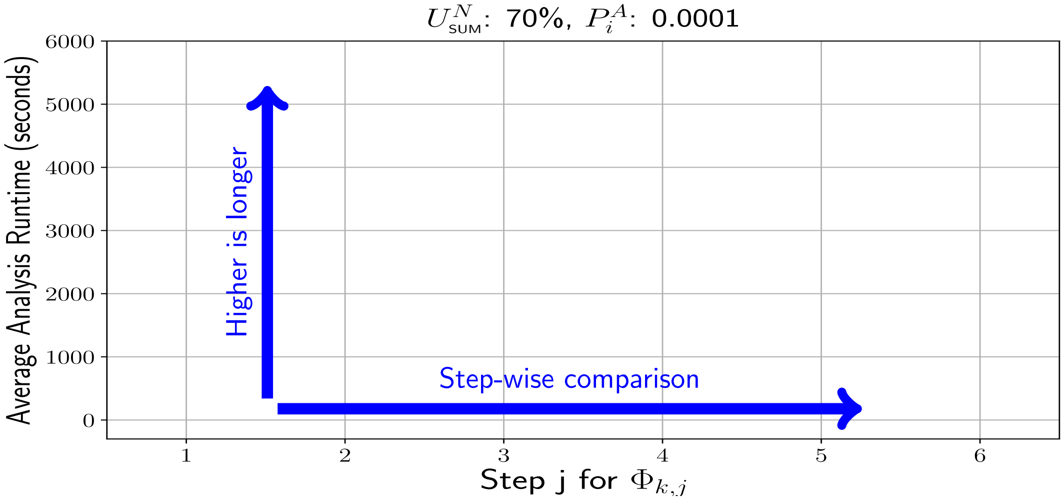
Focus on:

- Precision vs. Runtime for $\Phi_{k,j}$
- Expected Miss Rate among **AB**, **CON** and **SIM**

Runtime for $\Phi_{k,j}$



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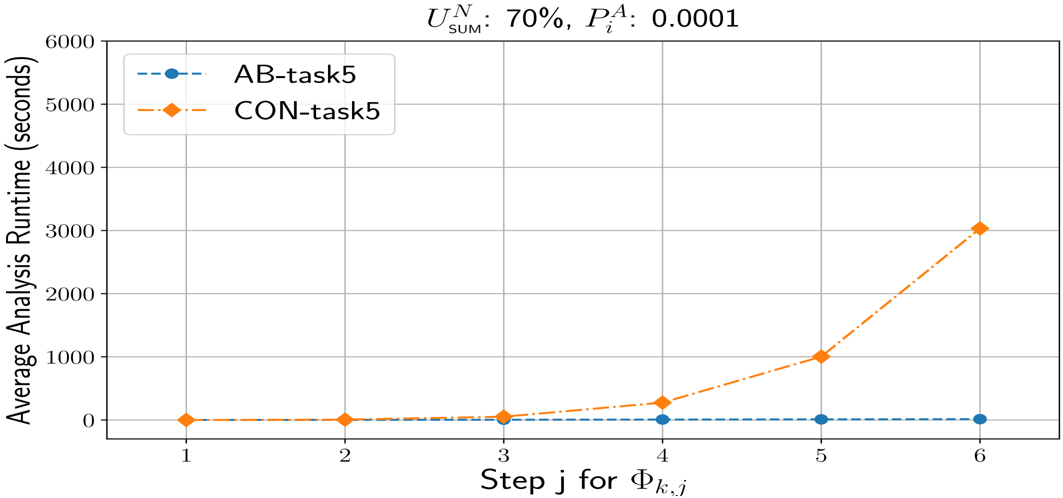

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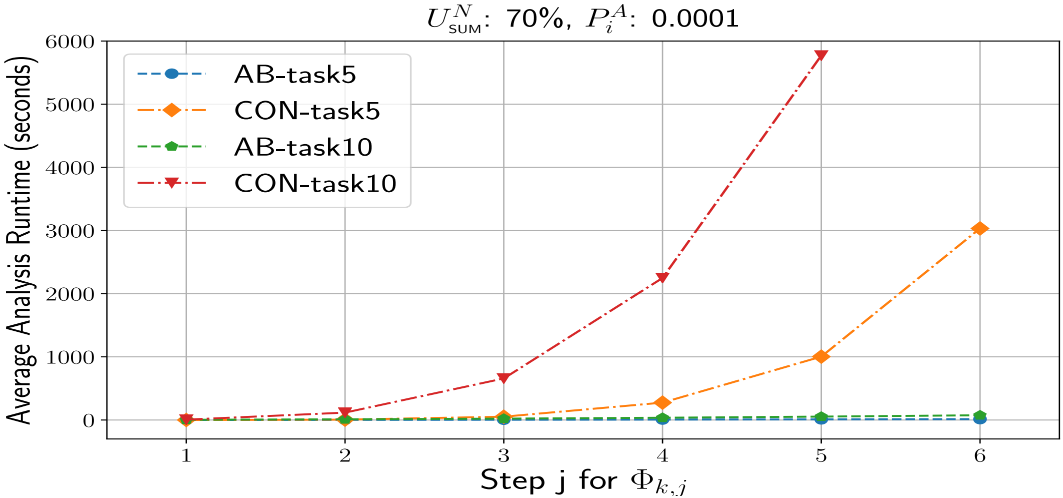

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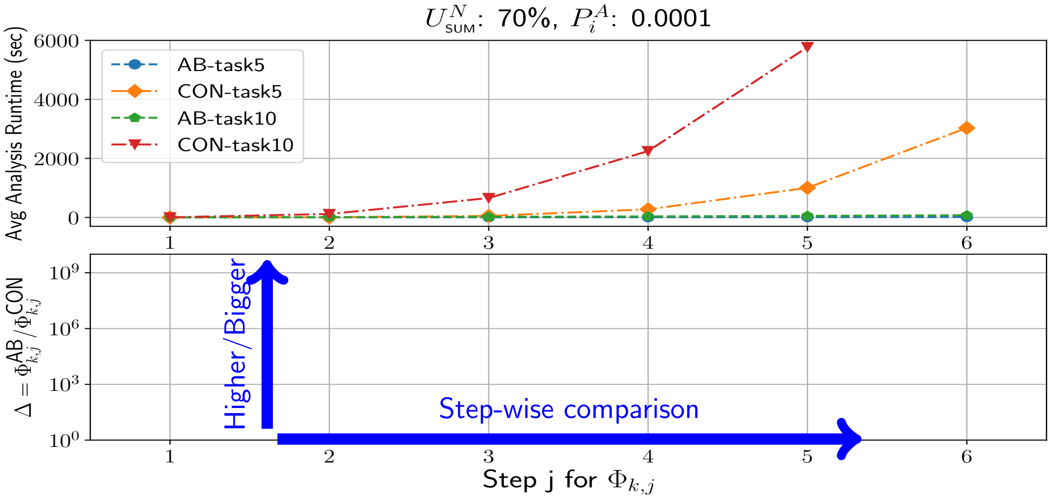

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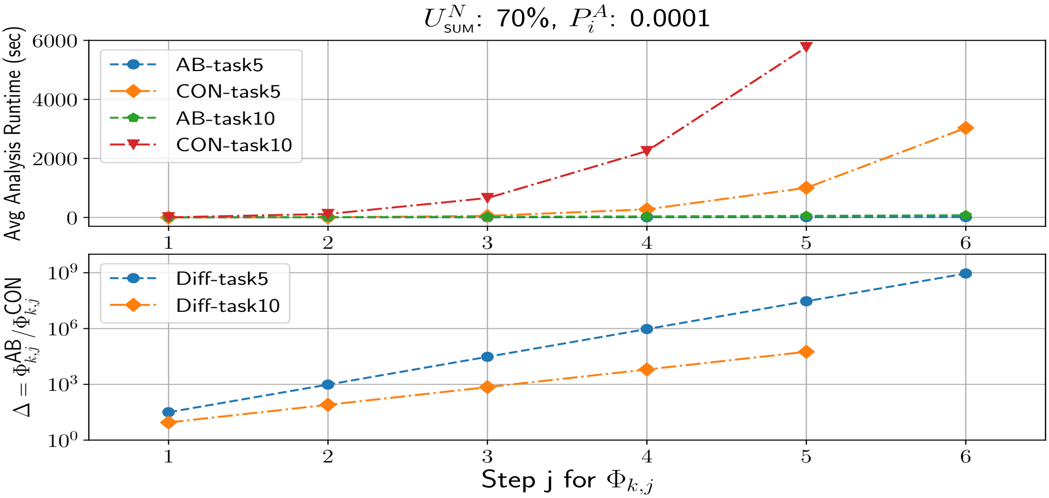

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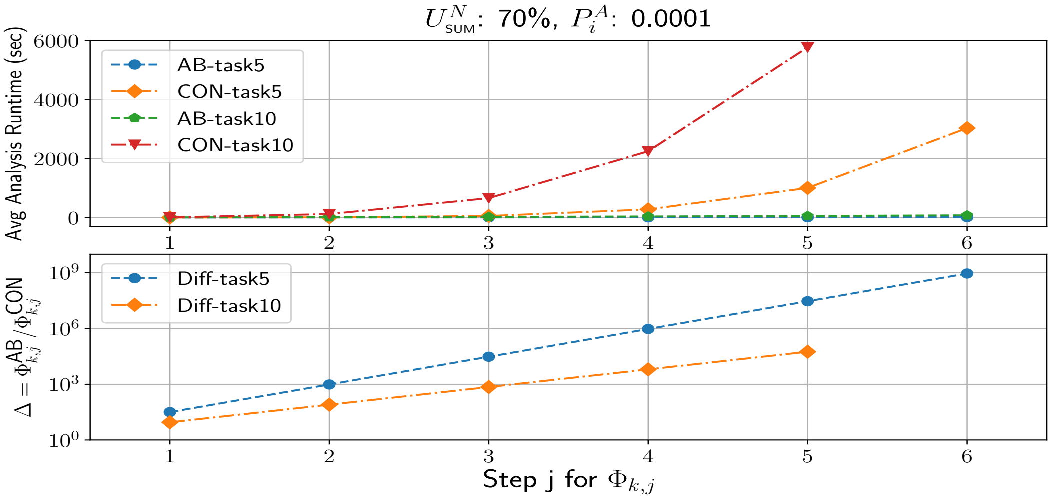
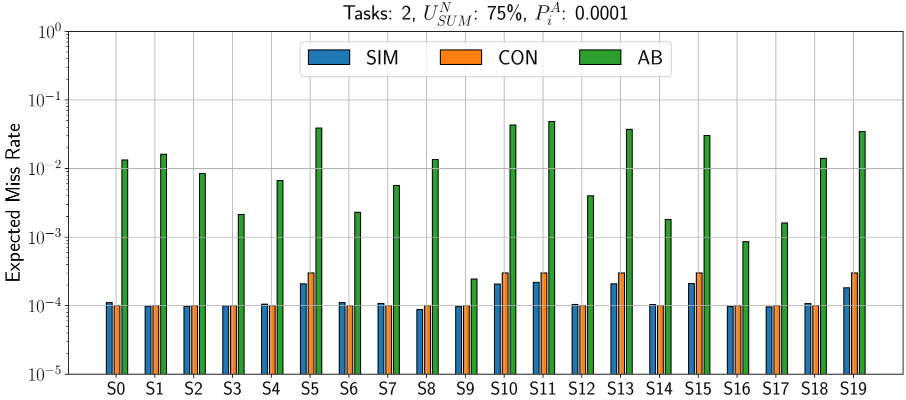


Trade-off: precision vs. runtime

Expected Miss Rate ($\mathbb{E}_k > 10^{-5}$)



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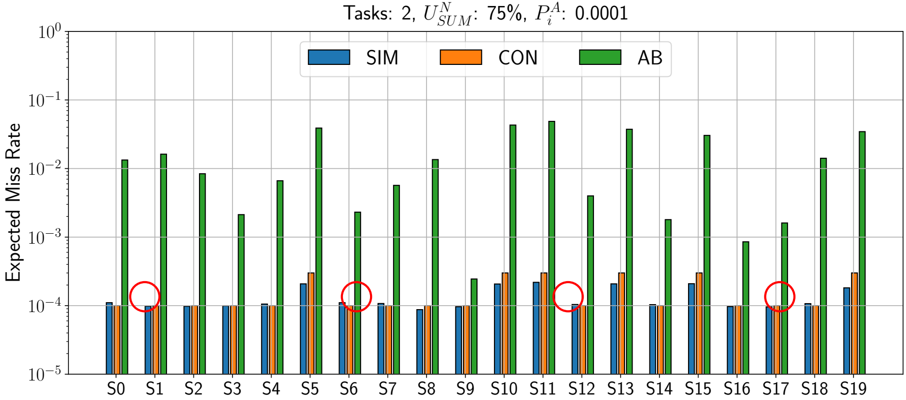

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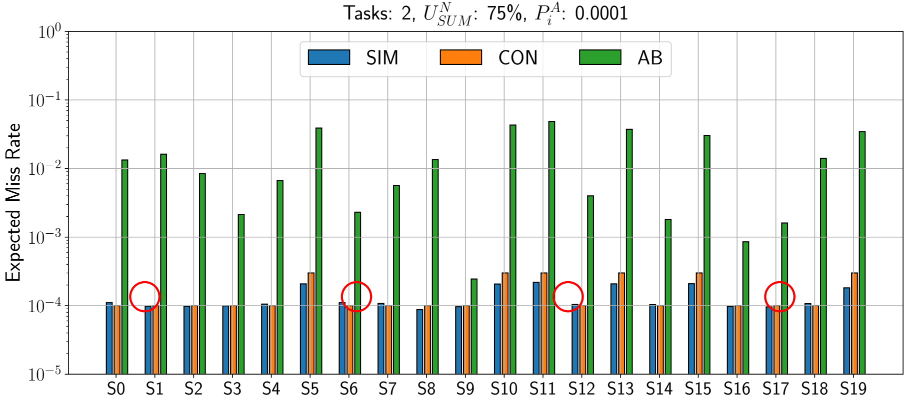

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